# What Types of Space Charge Are Inherent to Different Models of Crossed-Field Devices?

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***Abstract*—In this article four models of crossed-field devices: planar magnetron diode, planar magnetron, magnetron diode and magnetron were described. As well known in crossed-field devices electrons moved by cycloid-like trajectories and formed space charge. The shape of the space charge depends on the model under consideration. In a planar magnetron diode the space charge formed as a flow that is parallel to the electrodes. In a planar magnetron the space charge is not formed and is determined by the anode current only. In a magnetron diode the space charge formed as a flow that is parallel to the electrodes. In a magnetron the space charge is not formed and is determined by the anode current only.**

***Index Terms*—planar magnetron diode, planar magnetron, magnetron diode, magnetron, space charge**

## I. Introduction

Early authors' works proposed to these conferences [1] were explained why magnetrons always have anode current and have no diocotron effect. It is well known a magnetron is electron device with space charge. Here we discussed what types of space charge exist in different magnetron configurations. To solve these tasks we observed such magnetron configurations: planar (magnetron diode and magnetron) and cylindrical (magnetron diode and magnetron).

The crossed-field devices are high-power ones. These devices are associated with high efficiency, small weight, adequate bandwidth and good performance as a regards various parameters such as phase coherence, jitter, fast frequency sweep and phase versus frequency [2].

The crossed-field devices such as magnetron diode, magnetron, magnetron amplifier etc. were used in industry, science, medicine and domestic [1], [3].

Magnetron generators were designed back in the 20s of the twentieth century and are one of the most widely used microwave oscillators now.

Magnetron generator is a combination of three blocks such as cathode junction, anode with cavity resonators, RF energy output joint [1].

Electrons move in crossed static electrical and magnetic fields and interact with RF electromagnetic fields in the magnetrons [1].

The main interaction in such a crossed-field device is the wave particle interaction (diocotron). This interaction leads to the classical Brillouin flow nonlinearly unstable in the presence of high-frequency (RF) waves, propagating in the slow-wave structures that this devices contain [4].

Such systems have been analyzed with PIC (particle in cell) codes used in most modern electronic device simulation packages for example CST Microwave Studio and guiding-centre theory that is used much less often. The wavelike operating density profile has been achieved, the RF wave grows until it saturates. The main RF wave will beat against any other RF waves (noise) in the crossed-field device, which will generate all possible harmonics and beat waves. To study this phenomenon, we shall develop here, the small-signal response for these noise modes, their dispersion characteristics, their modal profiles, and their energy characteristics. Then we allow these waves to beat against the main RF wave, and determine their nonlinear growth characteristics.

In most studies, it is assumed that the space charge is an integral part of the operation of such devices. But it is not true.

Thus this work purpose is determinate what type of space charge is inherent for each magnetron device configuration. To achieve this goal, we must necessary to consider the following models of crossed-field devices: planar magnetron diode, planar magnetron, magnetron diode and magnetron.

## II. Main Part

In this work we shall consider four different constructions of magnetron: planar magnetron diode, magnetron diode, planar magnetron and magnetron. Schematic images were shown in figure 1.

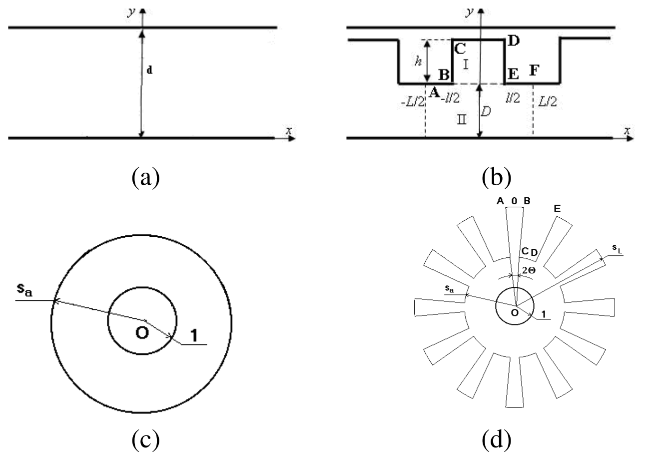


Fig. 1. Schematic images of magnetrons. (a) – planar magnetron diode; (b) – planar magnetron; (c) – magnetron diode; (d) – magnetron

### *A. Main equations*

Processes of electrons and waves interaction in systems with crossed-fields lead to the calculation of the trajectories of electrons that move practically without collisions with each other, and the fields affecting them. The equations of electrons' motion in electromagnetic fields have the form

$$\frac{d^2\vec{r}}{dt^2} = \frac{e}{m}\vec{E}(\vec{r},t) + \frac{e}{m}\left[\frac{d\vec{r}}{dt},\vec{B}\right], \tag{1}$$

where $\vec{r}$ — radius vector;
$e, m$ — electron charge and mass;
$\vec{E}$ — electric field density;
$\vec{B}$ — magnetic field density.

The equation (1) breaks down into two ones depending on the coordinate systems.

For the Cartesian coordinate system $(x, y)$, equation (1) will take the form

$$\frac{d^2x}{dt^2} = \eta E_x + \omega_H\frac{dy}{dt} \tag{2}$$

$$\frac{d^2y}{dt^2} = \eta E_y + \omega_H\frac{dx}{dt} \tag{3}$$

where $\eta = 1.76\cdot 10^{11} C/kg$ — the specific charge of electron;
$\omega_H = \eta B$ — cyclotron frequency;
$B$ — axial magnetic field density;
$E_x$ — electrostatic field density along the abscissa axis;
$E_y$ — electrostatic field density along the ordinate axis.
with initial conditions

$$x(0) = 0 \tag{4}$$

$$\left.\frac{dx}{dt}\right|_{t=0} \tag{5}$$

$$y(0) = 0 \tag{6}$$

$$\left.\frac{dy}{dt}\right|_{t=0} \tag{7}$$

Equations (2) and (3) describe the electrons' movement in planar devices: a planar magnetron diode and a planar magnetron.

For the polar coordinate system $(s, \varphi)$, equation (1) will take the form

$$\frac{d^2s}{dt^2} - s\left(\frac{d\varphi}{dt}\right)^2 = \eta\left(E_s + Bs\frac{d\varphi}{dt}\right) \tag{8}$$

$$s\frac{d^2\varphi}{dt^2} + 2\frac{ds}{dt}\frac{d\varphi}{dt} = \eta\left(E_\varphi - B\frac{ds}{dt}\right), \tag{9}$$

where $s$ — dimensionless radius $r/r_c$;
$E_s$ — electrostatic field density along the radial coordinate;
$E_\varphi$ — electrostatic field density in azimuthal coordinate.
with initial conditions

$$s(0) = 1; \quad \left.\frac{ds}{dt}\right|_{t=0} = 0; \tag{10}$$

$$\varphi(0) = 0; \quad \left.\frac{d\varphi}{dt}\right|_{t=0} = 0. \tag{11}$$

Equations (8) and (9) describe the electrons' movement in cylindtical devices: a magnetron diode and a magnetron.

### *B. Planar constructions*

From equations (2) and (3) we obtained working motion equations for a planar magnetron diode

$$\frac{d^2x}{dt^2} = \omega_H\frac{dy}{dt} \tag{12}$$

$$\frac{d^2y}{dt^2} = \eta\frac{U_a}{d} + \omega_H\frac{dx}{dt}. \tag{13}$$

It is well know the solutions of equations (12) and (13) with initial conditions (4)–(7) are parametric notation of the cycloid

$$x(t) = \frac{\eta U_a}{\omega_H^2}(\omega_H t - \sin\omega_H t) \tag{14}$$

$$y(t) = \frac{\eta U_a}{\omega_H^2}(1 - \cos\omega_H t). \tag{15}$$

For a planar magnetron the equations (2) and (3) were used directly.

In planar magnerton components of the electrical field densities were defined as [1]

$$E_x = \frac{4\pi h}{L}A_0\sum_{n=1}^{\infty} nA_n \sin\frac{2\pi nx}{L}\sinh\frac{ny}{D}$$

$$E_y = A_0\left(1 - \frac{4h}{D}\sum_{n=1}^{\infty} nA_n \cos\frac{2\pi nx}{L}\cosh\frac{ny}{D}\right),$$

where

$$A_0 = \frac{U_a}{D + \frac{lh}{L}}$$

$$A_n = \frac{sin\frac{\pi nl}{L}}{\left(\frac{\pi nl}{L} + \sin\frac{2\pi nl}{L}\right)\left(\sinh\frac{n(h+D)}{D} - \sinh n\right) + \pi \sinh n}.$$

Unfortunately, the solutions of equations (2) and (3) with initial conditions (4)–(7) cannot be obtained analytically.

Nevertheless, the trajectories of electrons have a cycloid-like appearance.

### C. Cylindrical constructions

From equations (8) and (9) we obtained working motion equations for a magnetron diode

$$\frac{d^2 s}{dt^2} = -\frac{s}{4} + \frac{b}{s} + \frac{1}{s^3} \tag{16}$$

$$\frac{d\varphi}{dt} = \frac{1}{2}\left(1 - \frac{1}{s^2}\right), \tag{17}$$

where $b = \frac{\eta U_a}{r_c^2 \omega_H^2 \ln s_a}$.

Unfortunately, the solutions of equations (16) and (17) with initial conditions (10) and (11) cannot be obtained analytically.

Nevertheless, the trajectories of electrons have a quasi epicycloid-like appearance.

From equations (8) and (9) we obtained working motion equations for a magnetron

$$\frac{d^2 s}{dt^2} + \left(1 - \frac{d\varphi}{dt}\right)\frac{d\varphi}{dt} s = \eta E_s \tag{18}$$

$$\frac{d^2 \varphi}{dt^2} + \frac{1}{s}\frac{ds}{dt}\left(2\frac{d\varphi}{dt} - 1\right) = \eta E_\varphi, \tag{19}$$

where

$$E_s = \frac{A}{s}\left(1 - 2N \ln\frac{s_L}{s_a}\sum_{n=1}^{\infty} n a_n cors^{Nn} \cos Nn\varphi\right)$$

$$E_\varphi = -\frac{2AN}{s}\ln\frac{s_L}{s_a}\sum_{n=1}^{\infty} n a_n sirs^{Nn} \sin Nn\varphi$$

$$A = \frac{\eta U_a}{\left(\frac{N\theta}{\pi}\ln\frac{s_L}{s_a} + \ln s_a\right) r_c^2 \omega_H^2}$$

$$a_n = \frac{\sin nN\theta}{(nN\theta + \sin 2nN\theta)(sirs_L^{nN} - sirs_a^{nN}) + \pi sirs_a^{nN}}$$

$$sirx = \frac{x - x^{-1}}{2}$$

$$corx = \frac{x + x^{-1}}{2}.$$

Unfortunately, the solutions of equations (18) and (19) with initial conditions (10) and (11) cannot be obtained analytically.

Nevertheless, the trajectories of electrons have a quasi epicycloid-like appearance similar to the trajectories in a magnetron diode.

It follows from the solutions of the equations (2), (3), (12), (13), (16) – (19) that the electron trajectories in all models of crossed-fields devices had cycloid-like trajectories. From the shape of above-mentioned trajectories, it is not possible to draw a conclusion about the formation of a space charge. In order to consider the process of space charge formation in crossed-field devices, it is necessary to take into account dissipation which has a diverse physical nature, in all the above-mentioned models.

## III. Results

Taking into account the dissipation in all crossed-field device models leads to a radical change in the electrons' trajectories.

A planar magnetron diode model has a dissipative term in equation (13).

In this case electrons' trajectories have shape imaging in figure 2

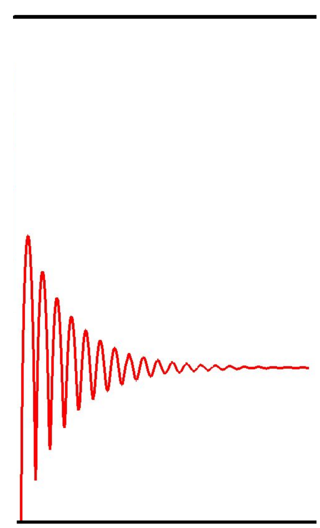

Fig. 2. Motion trajectory in a planar magnetron diode in the dissipation presence

Taking dissipation into account leads to the fact that electrons begin to move parallel to the electrodes. Thus, the space charge in the planar magnetron diode takes the shape of a flow that is parallel to the electrodes.

A planar magnetron model has a dissipative term in equation (3).

In this case electrons' trajectories have shape imaging in figure 3

Taking dissipation into account leads to the fact that electrons begin to move to the anode. The space charge in the

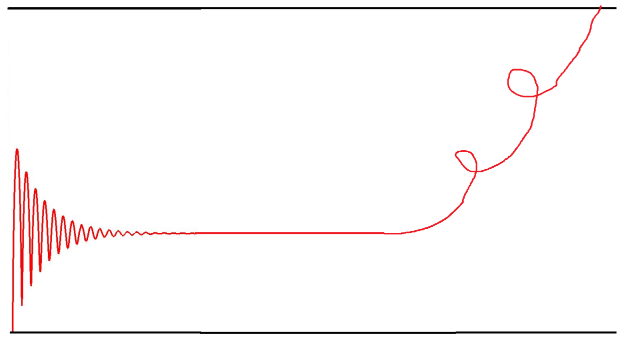

Fig. 3. Motion trajectory in a planar magnetron in the dissipation presence

planar magnetron takes the shape of a flow that moves to the potential energy minimum. Thus, in the planar magnetron the space charge is caused only by the anode current.

A magnetron diode model has a dissipative term in equation (13).

In this case electrons' trajectories have shape imaging in figure 4.

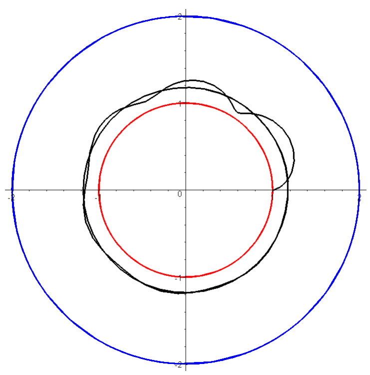

Fig. 4. Motion trajectory in a magnetron diode in the dissipation presence

Taking dissipation into account leads to the fact that electrons begin to move parallel to the electrodes in a circle with a radius of $s_0 = \sqrt{2b + \sqrt{4b^2 + 1}}$. Thus, the space charge in the magnetron diode takes the shape of a flow that is parallel to the electrodes in a circle.

A magnetron model has a dissipative term in equation (18).

In this case electrons' trajectories have shape imaging in figure 5

Taking dissipation into account leads to the fact that electrons begin to move to the anode. The space charge in the magnetron takes the shape of a flow that moves to the potential energy minimum. Thus, in the magnetron the space charge is caused only by the anode current.

The practical significance value of the proposed models of crossed-fields devices lies in the adequate choice of taking into account the space charge field during the modeling of such devices.

## IV. Conclusion

Four models of crossed-field devices: planar magnetron diode, planar magnetron, magnetron diode, magnetron were described.

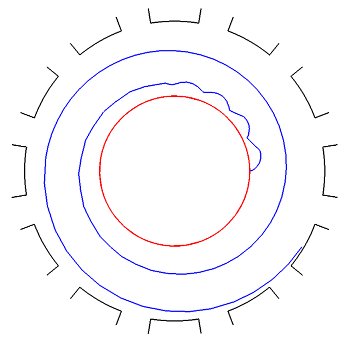

Fig. 5. Motion trajectory in a magnetron in the dissipation presence

The space charge in the planar magnetron diode formed as a flow that is parallel to the electrodes and located at a distance of $\frac{\eta U_a}{d\omega_H^2}$ from the cathode.

In a planar magnetron the space charge is not formed and is determined by the anode current only.

The space charge in the magnetron diode formed as a flow that is parallel to the electrodes and located near radius $\sqrt{2b + \sqrt{4b^2 + 1}}$.

In the magnetron the space charge is not formed and is determined by the anode current only.

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