Spectral Features of a Dielectric Layer in Paraxial Approximation

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***Abstract*—Spectrum of an Airy pulse scattered by a dielectric layer is investigated in paraxial approximation using an integral equation. Peculiarities of transmitted and reflected spectra are explored.**

***Keywords*—Airy pulse; reflection and transmission; dielectric layer**

I. Introduction

Interest to Airy beams motivated by their unusual features, the most interesting of which is accelerating motion [1-4], makes reasonable their investigations more profound. Especially it concerns temporal behaviour of the Airy pulses in an inhomogeneous medium. Time dependent Airy pulses have very complex behaviour in temporal domain [5, 6] and this behaviour can change significantly when the pulse propagates in the inhomogeneous medium. The main features of such changes can be revealed even in a simple problem of a pulse reflection from and transmission through a dielectric layer. Besides of the initial pulse form spectral characteristics of the layer itself have sufficient influence on a process of pulse scattering. It seems convenient to solve the problem of the pulse interaction with the layer using integral equation methods describing the problem as a whole including initial and boundary conditions and allowing to take into account contribution of the pulse form and scattering features of the layer separately. Here such an approach is applied to a problem when an electromagnetic pulse generated by an external source is described without making any assumptions on the temporal dependence of the pulse field but with the assumption of the paraxial approximation.

II. The Method For Problem Solution In Paraxial Approximation

A. The Integral Equation Method

Let the pulse be generated by an extrinsic electric current $j(t,x)$, located at the point $x = x_0$, and the current can have an arbitrary form but the form determined by the Airy function $j(t) = \mathrm{Ai}(t)$ will be considered with special interest. Transformation of an electromagnetic Airy pulse spectrum by a dielectric layer is the main goal of this consideration. A dielectric medium in a layer is characterized by the permittivity ε_1, the permeability μ_1 and the conductivity σ_1. The medium outside the layer is characterised by the permittivity ε, the permeability μ and the conductivity σ. For investigation of an electric field in an inhomogeneous dielectric medium we describe it by an inhomogeneous wave equation which right-hand side besides of the source is determined by an inhomogeneity.

$$\frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} + \gamma\frac{\partial E}{\partial t} - \frac{\partial^2 E}{\partial x^2} = \left(\frac{1}{v^2} - \frac{1}{{v_1}^2}\right)\chi(t,x)\frac{\partial^2 E}{\partial t^2} + (\gamma - \gamma_1)\chi(t,x)\frac{\partial E}{\partial t} - \mu_0\mu\frac{\partial j}{\partial t} \quad (1)$$

where $v = c/\sqrt{\varepsilon\mu}$, $v_1 = c/\sqrt{\varepsilon_1\mu_1}$, c is the velocity of light in vacuum, $\gamma = \mu_0\mu\sigma$, $\gamma_1 = \mu_0\mu_1\sigma_1$, μ_0 is the permeability of vacuum, σ and μ, σ_1 and μ_1 are the conductivity and the permeability outside and inside the inhomogeneity respectively. The inhomogeneity is represented by the function $\chi(t,x)$ which is equal to one inside the layer and to zero outside of it.

The main assumption in this consideration is that in the representation of the field by $E = F(t,x)e^{-ik|x|}$ the paraxial approximation $\left|F''_{xx}\right| \ll \left|2ikF'_x\right|$ for the envelope $F(t,x)$ is applicable. In this case it satisfies the equation followed form (1)

$$\frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} + \gamma\frac{\partial F}{\partial t} + 2ik\,\mathrm{sign}(x)\frac{\partial F(t,x)}{\partial x} + 2ik\delta(x)F(t,x) + k^2F(t,x) = \chi(t,x)\left[\frac{{v_1}^2 - v^2}{v^2{v_1}^2}\frac{\partial^2 F}{\partial t^2} + (\gamma - \gamma_1)\frac{\partial F}{\partial t}\right] - \mu_0\mu\frac{\partial j}{\partial t}e^{ik|x|} \quad (2)$$

Further consideration will be implemented for the case when the location of the layer is attached to the spatial interval $[0,a]$.

By virtue of the Green's function [5]

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$$G(t,x) = -\frac{(1+i)v}{4\sqrt{\pi k|x|}} e^{i[k\frac{|x|}{2} + \frac{v^2(2kt + i\gamma|x|)^2}{8k|x|}]} \quad (3)$$

the equation (2) is transformed to the integral equation

$$F(t,x) = F_0(t,x) - \frac{(1+i)v}{4\sqrt{\pi k}} \int_a^b dx' \int_{-\infty}^{\infty} dt' \frac{e^{i\frac{k}{2}[|x-x'| + \frac{v^2(2k(t-t') + i\gamma|x-x'|)^2}{4k^2|x-x'|}]}}{\sqrt{|x-x'|}} \times \left[\frac{{v_1}^2 - v^2}{v^2 {v_1}^2} \frac{\partial^2 F}{\partial t'^2} + (\gamma - \gamma_1) \frac{\partial F}{\partial t'}\right] \quad (4)$$

This equation describes the problem as a whole, i.e. the field inside as well outside the layer, and contains boundary conditions on the layer boundaries. The source current

$$j_0(t,x) = \delta(x - x_0) \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) e^{i\omega t} d\omega \quad (5)$$

determines the free term in this equation

$$F_0(t,x) = \int_{-\infty}^{\infty} J_0(\omega) e^{i(k - \frac{\omega^2 - i\omega v^2 \gamma}{v^2 k})\frac{x}{2}} e^{i\omega t} d\omega \quad (6)$$

where

$$J_0(\omega) = -\frac{\mu_0 \mu}{4\pi k} e^{ik(|x_0| - \frac{x_0}{2}) + i\frac{\omega^2 - i\omega v^2 \gamma}{v^2 k}\frac{x_0}{2}} \omega \Phi(\omega) \quad (7)$$

The Fourier transform of the envelope $f(\omega,x) = \int_{-\infty}^{\infty} F(t,x) e^{-i\omega t} dt$ allows to obtain the integral equation for the field spectra.

The spectrum for the inner field satisfies the integral equation

$$f_{in}(\omega,x) = j_0(\omega) e^{ik\Omega\frac{x}{2}} + \frac{i}{2k} \int_a^b dx' e^{ik\Omega(\omega)\frac{|x-x'|}{2}} \{\omega^2 \frac{{v_1}^2 - v^2}{v^2 {v_1}^2} - i\omega(\gamma - \gamma_1)\} f_{in}(\omega,x') , \quad (8)$$

where $\Omega(\omega) = 1 - \frac{\omega^2}{k^2 v^2} + i\gamma \frac{\omega}{k^2}$. The spectrum of the reflected pulse is calculated over the inner field by the integral

$$f_R(\omega,x) = j_0(\omega) e^{ik\Omega\frac{x}{2}} + \frac{i}{2k} e^{-ik\Omega\frac{x}{2}} \int_a^b dx' e^{ik\Omega\frac{x'}{2}} \{\omega^2 \frac{{v_1}^2 - v^2}{v^2 {v_1}^2} - i\omega(\gamma - \gamma_1)\} f_{in}(\omega,x') \quad (9)$$

and for the transmitted pulse by the integral

$$f_T(\omega,x) = j_0(\omega) e^{ik\Omega\frac{x}{2}} + \frac{i}{2k} e^{ik\Omega\frac{x}{2}} \int_a^b dx' e^{-ik\Omega\frac{x'}{2}} \{\omega^2 \frac{{v_1}^2 - v^2}{v^2 {v_1}^2} - i\omega(\gamma - \gamma_1)\} f_{in}(\omega,x') . \quad (10)$$

B. The spectra conditioned by a layer itself

Solution of the equation (8) and calculation of the integrals (9) and (10) gives the spectrum for the reflected

$$f_R(\omega,x) = J_0(\omega) e^{ik\Omega x/2} + J_0(\omega) e^{ik\Omega(a - x/2)} [e^{ik\sqrt{\Omega(2\Omega_1 - \Omega)}(b-a)} - 1](\Omega_1 - \Omega) D \quad (11)$$

and the transmitted pulses

$$f_T(\omega,x) = 2J_0(\omega) e^{ik\Omega x/2} e^{i(\sqrt{\Omega(2\Omega_1 - \Omega)} - \Omega)k\frac{(b-a)}{2}} D\sqrt{\Omega(2\Omega_1 - \Omega)} \quad (12)$$

Here, $\Omega_1(\omega) = 1 - \frac{\omega^2}{k^2 {v_1}^2} + i\gamma_1 \frac{\omega}{k^2}$ and

$$D = \frac{\Omega_1 + \sqrt{\Omega(2\Omega_1 - \Omega)}}{[\Omega_1 + \sqrt{\Omega(2\Omega_1 - \Omega)}]^2 - (\Omega_1 - \Omega)^2 e^{ik\sqrt{\Omega(2\Omega_1 - \Omega)}(b-a)}} \quad (13)$$

As it is seen from above the spectra of scattered waves are described by the expressions consisting of two multipliers one of which is determined by an inhomogeneity itself and the other corresponds to a kind of a scattering initial wave.

The expressions (11) and (12) allow to write the part of the spectrum determined by the layer itself:

- for the reflection effect

$$f_{RL}(\omega) = [e^{ik\sqrt{\Omega(2\Omega_1 - \Omega)}(b-a)} - 1](\Omega_1 - \Omega) D \quad (14)$$

- and for the transmission effect

$$f_{TL}(\omega) = e^{i(\sqrt{\Omega(2\Omega_1 - \Omega)} - \Omega)k\frac{(b-a)}{2}} D\sqrt{\Omega(2\Omega_1 - \Omega)} . \quad (15)$$

The radical $\sqrt{\Omega(2\Omega_1 - \Omega)}$ in the expressions above becomes imaginary if $\Omega(2\Omega_1 - \Omega) < 0$, which is met in the hatched region in Fig. 1. The critical frequency for the spectra is determined by equating the value of the expression under the radical in formulas (14) and (15) to zero: $2\Omega_1 - \Omega = 0$. It gives the value $\omega_c = kv\sqrt{2(v/v_1)^2 - 1}$ for the critical frequency if one neglects dissipation.

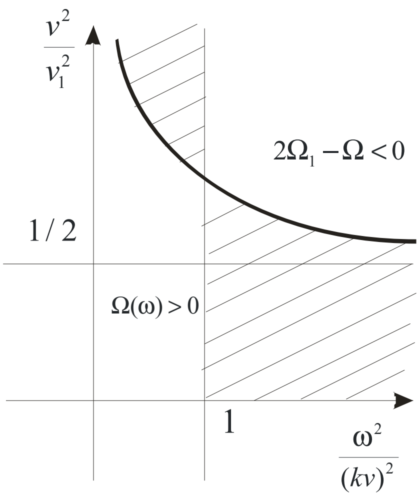


Fig. 1. The region hatched on the diagram for the relation between the frequency and the ratio of refractive indices corresponds to the imaginary wavenumber, $\Omega(2\Omega_1 - \Omega) < 0$.

We consider two cases of the medium parameters: a) a more transparent layer when $v / v_1 = 1.25$ and $\omega_c / (kv) \approx 0,68599$, and b) a less transparent layer when $v / v_1 = 0.75$ and $\omega_c / (kv) \approx 2,82842$. Further the dimensionless frequency $w = \omega / (kv)$ and the normalized layer width $L = k(b - a)$ are used.

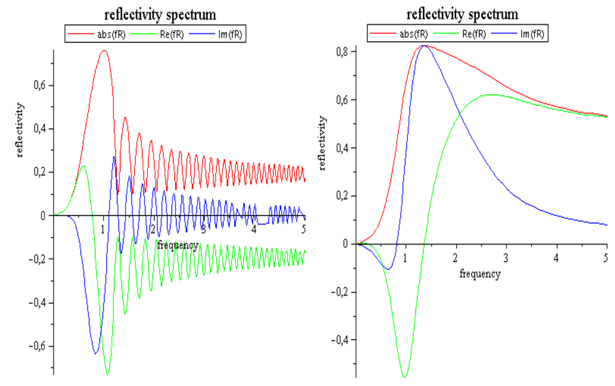
The reflected spectra raised by the inhomogeneity itself are shown in Fig. 2-4.

The module of the spectrum and its real and imaginary parts for the reflection effect $f_{RL}(\omega)$ are shown in Fig. 2 for two cases of the medium parameters. One can see that the greatest values of the spectrum are achieved at the critical frequency in both cases: near 1 for the more transparent layer and near 2 for the less transparent layer.

The dependence of the reflection effect on the layer width is shown in Fig. 3 and on the dissipation degree in Fig . 4. One can see much more influence of the layer width on the spectrum in both cases of the transparency as well as qualitative change of the layer width influence on the spectrum with transparency.

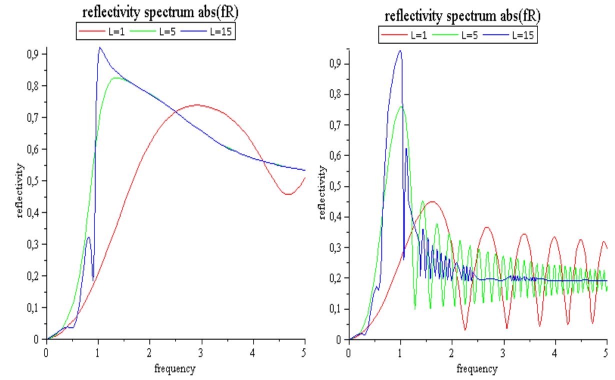
The module of the spectrum and its real and imaginary parts for the transmittance effect $f_{TL}(\omega)$ are shown in Fig. 4 for two cases of the medium parameters. One can see that the greatest values of the spectrum are achieved also at the critical frequency in both cases: near 1 for the more transparent layer and near 2 for the less transparent layer.

The module of the spectrum and its real and imaginary parts for the transmission effect $f_{TL}(\omega)$ are shown in Fig. 5. The dependence of the transmission effect on the layer width is shown in Fig. 6. The dependence of the reflection effect on the dissipation degree is shown in Fig. 7.



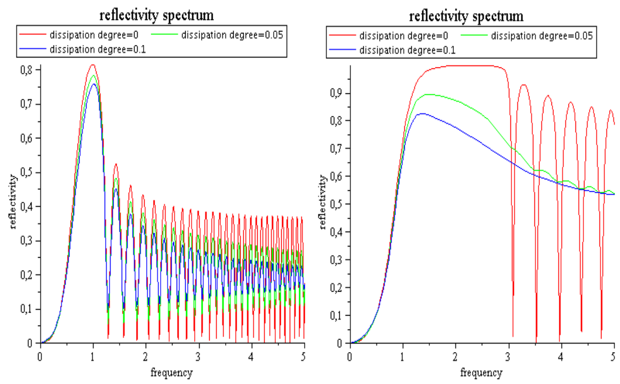
a) b)

Fig. 2. The spectra of the reflection effect for the layer width $L = 5$ and the dissipation degree $\gamma_1 \mu_0 \mu_1 = 0.1$: $|f_{RL}|$ - red, $\mathrm{Re}|f_{RL}|$ - green, $\mathrm{Im}|f_{RL}|$ - blue.



a) b)

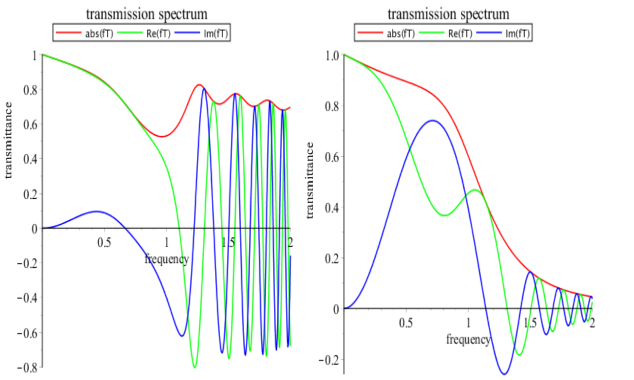
Fig. 3. The dependence of the reflection effect spectrum on the layer width: $L = 1$ - red, $L = 5$ - green, $L = 15$ - blue, the dissipation degree $\gamma_1 \mu_0 \mu_1 = 0.1$ in all cases.



a) b)

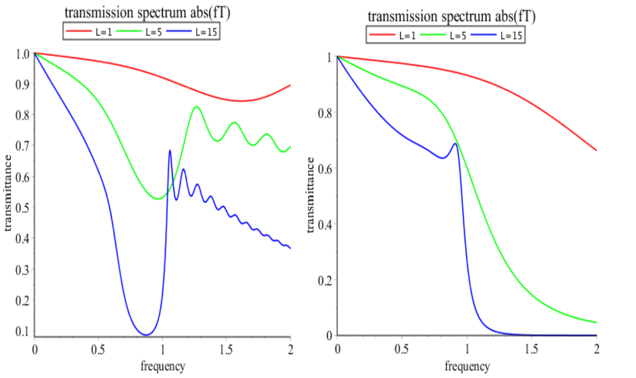
Fig. 4. The dependence of the reflection effect spectrum on the dissipation degree for the layer width $L = 5$: $\gamma_1 \mu_0 \mu_1 = 0$ - red, $\gamma_1 \mu_0 \mu_1 = 0.05$ - green, $\gamma_1 \mu_0 \mu_1 = 0.1$ - blue.

If $\Omega(2\Omega_1 - \Omega) > 0$ then the absolute value of the fraction in (16) is less than one and this function can be expanded into the series



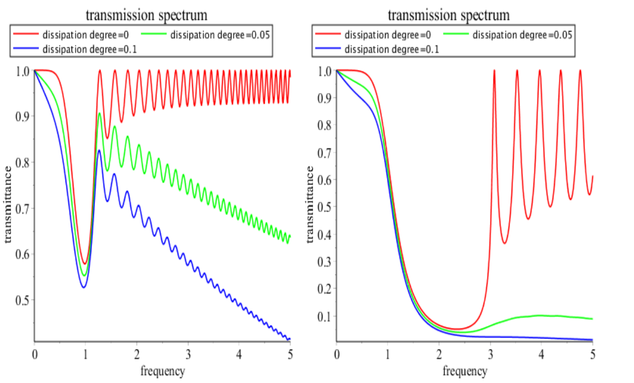
a) b)

Fig. 5. The spectra for the transmittance effect for the layer width $L = 5$ and dissipation degree $\gamma_1 \mu_0 \mu_1 = 0.1$: $|f_{RL}|$ - red, $\mathrm{Re}|f_{RL}|$ - green, $\mathrm{Im}|f_{RL}|$ - blue.



a) b)

Fig. 6. The dependence of the transmittance effect spectrum on the layer width: $L = 1$ - red, $L = 5$ - green, $L = 15$ - blue, the dissipation degree $\gamma_1 \mu_0 \mu_1 = 0.1$.



a) b)

Fig. 7. The dependence of the transmission effect spectrum on the dissipation for the layer width $L = 5$: $\gamma_1 \mu_0 \mu_1 = 0$ - red, $\gamma_1 \mu_0 \mu_1 = 0.05$ - green, $\gamma_1 \mu_0 \mu_1 = 0.1$ - blue.

$$D = \sum_{n=0}^{\infty} \left[\frac{\Omega_1 - \sqrt{\Omega(2\Omega_1 - \Omega)}}{\Omega_1 + \sqrt{\Omega(2\Omega_1 - \Omega)}} \right]^n e^{ink\sqrt{\Omega(2\Omega_1 - \Omega)}(b-a)} \quad (16)$$

The denominators in the series terms become equal to zero only in the trivial case $\nu_1 = \nu$ but in this case there is no layer and there is no diffraction. If $\Omega(2\Omega_1 - \Omega) < 0$ then the radicals in (16) are imaginary and $\frac{\Omega_1 - \sqrt{\Omega(2\Omega_1 - \Omega)}}{\Omega_1 + \sqrt{\Omega(2\Omega_1 - \Omega)}} = e^{i\varphi}$ with the absolute value equal to one, $\left| \frac{\Omega_1 - \sqrt{\Omega(2\Omega_1 - \Omega)}}{\Omega_1 + \sqrt{\Omega(2\Omega_1 - \Omega)}} \right| = 1$. In this case the series (16) consists of evanescent terms

$$D = \sum_{n=0}^{\infty} e^{in\varphi - nk\left|\sqrt{\Omega(2\Omega_1 - \Omega)}\right|(b-a)} \quad (17)$$

where φ is the argument of the fraction. In both cases the terms in the series (16) and (17) decrease as power functions.

III. Conclusion

An integral equation describing in paraxial approximation the interaction of an electromagnetic pulse with an obstacle in the form of a dielectric layer is derived. The initial pulse is taken in the form of the Airy pulse and two kinds of an inhomogeneity are considered. The first one is in the case when the inhomogeneity is optically more transparent than the environment and the second one concerns the contrary case. The spectra of the scattered pulses are obtained and analyzed.

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