Tunable Angular Spatial Filter Based on 1D Magnetophotonic Crystal

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***Abstract*—One dimensional magnetophotonic crystal with gyrotropic plasma layers is considered. Theoretical investigation is performed within the framework of Floquet theory for 1D periodical structures. Dispersion equation, transmission coefficient and reflection one are obtained in explicit form. Scattering of the point source field on the bounded magnetophotonic crystal is investigated for TE waves. Field spatial distributions for different signs of plasma-like medium effective permittivity are calculated. Possibility of angular spatial filtering control is shown both inside magnetophotonic crystal and outside one.**

Keywords—one-dimensional magnetophotonic crystal; plasma layer; angular spatial filtering; Floquet theory

I. Introduction

Photonic crystals are 1D, 2D or 3D artificial dielectric periodic structures that have transmission and forbidden zones in spectral characteristics [1, 2]. These physical properties allow numerous applications of photonic crystal structures for generating, controlling and analyzing of optical, infrared and THz electromagnetic radiation [3-9]. Electrically and mechanically tuned photonic crystals open more wide possibilities for designing of various functional devices. Because of that intensive experimental and theoretical investigations of tunable periodical structures are performing in some last decades [10-14]. One dimensional magnetophotonic crystals (MPhCs) that contain elements sensitive to magnetic field provide perspectives on development of novel devices for spectroscopy, enhancing of magneto-optical effects (e. g. Faraday rotation and magneto-optical Kerr effect), filtering, optical isolating and other applications [15-21].

Spatial or direction filtering is one of important and interesting phenomena that can be used in different applications of 1D periodical structures. Dispersion properties of these structures determine the spatial distribution of electromagnetic field components depending on the radiation propagation direction. Moreover bounded photonic crystals can be used for control of electromagnetic radiation spatial distribution and incident wave front transformation [22-26]. However many applications require tunable control of field spatial distribution. In fact this requirement boils down to the possibility of photonic crystal dispersion characteristics changing by external fields. Therefore MPhCs are promising candidates for realization of tunable spatial filtering.

In this paper we consider 1D MPhC that contains alternating dielectric and plasma-like layers. Non-diagonal components of permittivity tensor of plasma medium depend on the external magnetic field. This fact provides possibility of dispersion diagram control and corresponding change of field spatial distribution of TE-polarized waves i.e. spatial filtering.

II. Theoretical Basis

Scheme of photonic crystal with appropriate coordinate system and external transversal magnetic field $\vec{H}_0$ is shown in Fig 1. $L = a+b$ is period of multilayer system, a and b are the

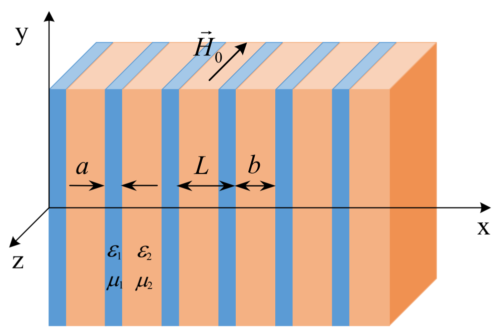

Fig. 1. Schematic of 1D magnetophotonic crystal.

thicknesses of plasma and dielectric layers respectively. Permittivity of plasma medium is well known tensor value:

$$\overset{\leftrightarrow}{\varepsilon_1} = \begin{Vmatrix} \varepsilon_1 & -i\varepsilon_{a1} & 0 \\ i\varepsilon_{a1} & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_z \end{Vmatrix}, \qquad (1)$$

where tensor components depend on frequency, external magnetic field induction and plasma frequency [27, 28]. Permeability of the plasma medium and material parameters of dielectric layer are scalar values.

We consider TE polarization of radiation with field components $(E_x,\ E_y,\ H_z)$. Then Helmholtz equation for the magnetic field component in j^{th} layer of structure period (j = 1, 2) can be written in such manner:

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_{\perp j}(x)}\frac{\partial H_z}{\partial x}\right)+\frac{1}{\varepsilon_{\perp j}(x)}\frac{\partial^2 H_z}{\partial y^2}+k^2\mu_j(x)H_z=0, \quad (2)$$

where $\varepsilon_{\perp j}(x)=\varepsilon_j\left(1-\varepsilon_{aj}^2/\varepsilon_j^2\right)$ – effective permittivity of j^{th} layer; $\mu_j(x)$ is permeability of layers; $k=\omega/c$.

Solution of the Helmholtz equation (2) is represented through the fundamental solutions of the third boundary value problem $\psi_1(x)$ and $\psi_2(x)$ with mixed boundary conditions (Cauchy conditions [29]):

$$\psi_1(0)=1, \quad \frac{1}{\varepsilon_{\perp 1}}\left[\frac{\partial\psi_1(0)}{\partial x}-\beta\frac{\varepsilon_{a1}}{\varepsilon_1}\psi_1(0)\right]=0, \qquad (3)$$

$$\psi_2(0)=0, \quad \frac{1}{\varepsilon_{\perp 1}}\left(\frac{\partial\psi_2(0)}{\partial x}-\beta\frac{\varepsilon_{a1}}{\varepsilon_1}\psi_2(0)\right)=1. \qquad (4)$$

Here β is longitudinal component of wave vector (along Oy axis). In accordance with the Floquet theory [30] one can always choose fundamental solutions which linear combination can take the simplest diagonal form:

$$\psi_1(x+L)=\rho_1\psi_1(x), \quad \psi_2(x+L)=\rho_2\psi_2(x). \qquad (5)$$

Here constants ρ_1 and ρ_2 (Floquet factors) are defined from characteristic equation:

$$\rho^2-2A\rho+Wr(\psi_1,\psi_2)=0, \qquad (6)$$

$$A=\psi_1(L)+\frac{1}{\varepsilon_{\perp 2}}\left[\psi_2'(L)-\beta\frac{\varepsilon_{a2}}{\varepsilon_2}\psi_2(L)\right],$$

$$Wr=\frac{1}{\varepsilon_{\perp 2}}\left(\psi_1(L)\psi_2'(L)-\psi_2(L)\psi_1'(L)\right)=1.$$

Cauchy boundary conditions (3)-(4) allow fundamental solutions $\psi_1(x)$ and $\psi_2(x)$ to be computed in 1D magnetophotonic crystal domains. Further these functions are used for obtaining of dispersion equation from (6), field spatial distribution in bounded MPhC that contains N periods. Furthermore these functions are used for obtaining transmission and reflection coefficients.

Solutions of the quadratic equation (6) satisfy the condition $\rho_1\rho_2=1$. Thus we find $\rho_{1,2}=e^{\pm iKL}$ where K is the Floquet wavenumber. Using condition

$$2\cos K_{TE}L=\left[\psi_2'(L)-\beta\frac{\varepsilon_{a2}}{\varepsilon_2}\psi_2(L)\right]/\varepsilon_{\perp 2}+\psi_1(L),$$

we obtain dispersion equation for TE polarized waves in 1D MPhC [23, 24]:

$$\cos KL=\cos\xi_1 a\cos\xi_2 b-\\ -\frac{1}{2}\left[\begin{array}{l}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\frac{\xi_1}{\xi_2}+\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\frac{\xi_2}{\xi_1}+\\ +\frac{\beta^2}{\xi_1\xi_2}\left(\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\right)\left(\frac{\varepsilon_{a1}}{\varepsilon_1}-\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2\end{array}\right]\sin\xi_1 a\sin\xi_2 b. \qquad (7)$$

Here $\xi_j=\sqrt{k^2\varepsilon_{\perp j}\mu_j-\beta^2}$ are transversal components of wave vector in layers of MPhC (along Ox axis).

Transmission and reflecting coefficients for bounded MPhC that contains N periods are derived using boundary conditions for tangential field components H_z and E_y at $x=0$ and $x=NL$:

$$1+R_N=H_z(0), \qquad i\frac{\xi_{in}}{\varepsilon_{in}}(1-R_N)=ikE_y(0), \qquad (8)$$

$$T_N=H_z(NL), \qquad i\frac{\xi_{ex}}{\varepsilon_{ex}}T_N=ikE_y(NL). \qquad (9)$$

Using transfer matrix W of MPhC period [31] we obtain analytical expressions for transmission and reflection coefficients:

$$R_N=1-T_N\left(\frac{\varepsilon_{in}}{\xi_{in}}\frac{\xi_{ex}}{\varepsilon_{ex}}M_{11}-\frac{\varepsilon_{in}}{\xi_{in}}kM_{21}\right), \qquad (10)$$

$$T_N=2\left[\frac{\varepsilon_{in}}{\xi_{in}}\frac{\xi_{ex}}{\varepsilon_{ex}}M_{11}+M_{22}+i\left(\frac{\varepsilon_{in}}{\xi_{in}}M_{21}-\frac{\xi_{ex}}{\varepsilon_{ex}}M_{12}\right)\right]^{-1}, \quad (11)$$

$$M=W\frac{\sin(NKL)}{\sin KL}-I\frac{\sin[(N-1)KL]}{\sin KL}.$$

Here I is unit matrix.

We assume that electromagnetic field of point source $\vec{j}(z,r,t)=\vec{z}_0\delta(r-0)\delta(\varphi-0)e^{ik_z z-i\omega t}/r$ (magnetic current wire) excites H_z-polarized wave within the framework of two-dimensional approach. Magnetic field component is defined by Hankel function of the first kind [32] $H_z(z,r,t) = H_0^{(1)}(ik_r r)e^{i(k_z z-\omega t)}$ where $k_r = \sqrt{k^2 - k_z^2}$. This nonuniform wave propagates in the z-direction with phase velocity $v_\varphi = \omega / k_z$ and transversal decay $\exp(-k_r r)/\sqrt{r}$. Then magnetic field expressions outside bounded MPhC can be written in such manner:

$$H_z^{ref}(z,r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(k_r) R_N(k_r) e^{ik_r r} dk_r e^{ik_z z - i\omega t}, \quad (12)$$

$$H_z^{tr}(z,r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(k_r) T_N(k_r) e^{ik_r r} dk_r e^{ik_z z - i\omega t}, \quad (13)$$

where $h(k_r)$ is the Fourier amplitude of Hankel function expansion. Magnetic field component calculations in j^{th} layer of MPhC are performed using representation of the first kind Hankel function [33]:

$$H_0^{(1)}(ik_r r) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{e^{ik_x x + ik_y |y|}}{k_y} dk_x . \quad (14)$$

It is clear that (14) is the plane-wave expansion of a cylindrical wave. Moreover this expression can be used for angular spectrum decomposition of the wave beams [34, 35].

III. Discussion

Fig. 2 shows projected dispersion diagram of the infinite MPhC for positive value of effective permittivity $\varepsilon_{\perp 1}$. In this case such parameters are used for calculations: $a/L = 0.1$; $\varepsilon_1 = 12$; $\varepsilon_2 = 3.75$; $\varepsilon_{a1} = 11$; $\varepsilon_{a2} = 0$; $\mu_1 = \mu_2 = 1$. Shaded and unshaded areas indicate transmission zones where TE electromagnetic waves propagate through MPhC and forbidden zones where radiation cannot propagate in periodic structure respectively. Tilted solid lines show light lines for vacuum and MPhC dielectric layer. It should be noted that region of transmission zone under the dielectric light line is the area of extraordinary gyrotropic surface waves existence [20]. Dashed horizontal lines marked by "1" and "2" on dispersion diagram represent two values of frequency. It is apparent that electromagnetic waves with these frequencies are subject to spatial selection due to transmission and forbidden zones configuration. Propagation direction normal to MPhC layers (direction of *Ox* axis) is allowed. In this case we have zero longitudinal wave number ($\beta = 0$). On the other hand waves decay arises due to forbidden zones when the propagating wave deviates from the normal direction. Vertical dotted lines show boundary between transmission and forbidden zones for two chosen frequencies. Therefore "propagating" cone for lower frequency should be wider than for higher frequency. This is well known angular filtering of electromagnetic waves verified experimentally for all-dielectric 1D photonic crystal in [25].

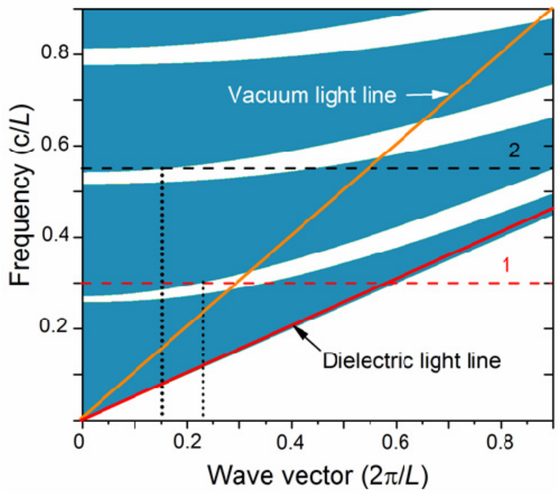

Fig. 2. Dispersion diagram of the magnetophotonic crystal.

Fig. 3 and 4 show results of magnetic field (H_z) spatial distribution calculations for frequencies ω_1 and ω_2 respectively. MPhC slab consists ten periods in this case. Point source of H_z-polarized cylindrical electromagnetic waves propagating in MPhC under different angles lies within first structure period in dielectric layer. Indeed in Fig. 4 we obtain narrower wave beam on the periodic structure output than in Fig. 3. However we can see some allowed directions (not only normal) for wave's propagation within the MPhC slab in accordance with dispersion diagram in Fig. 2. These additional directions are due to transmission zones existence for different values of longitudinal wave number β for fixed frequency. Therefore in fact effective angular filtering realizes outside the MPhC structure only. Near-normally propagating waves passes the multilayer periodic structure and outputted in free space. Another waves propagating within MPhC under defined angles are not able to leave multilayer stack due to total internal reflection. Thus MPhC operates as planar waveguide for these angular components of point source radiation.

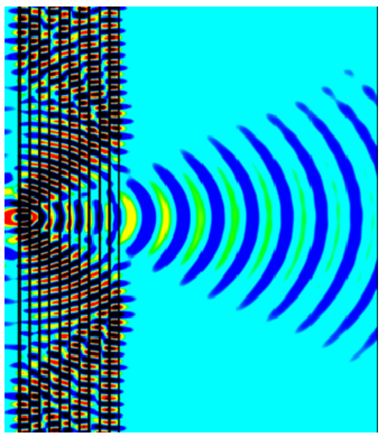

Fig. 3. Magnetic field spatial distribution for normalized frequency of 0.3.

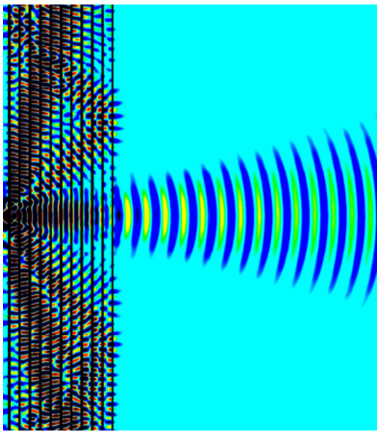

Fig. 4. Magnetic field spatial distribution for normalized frequency of 0.56.

Varying of gyrotropic layers effective permittivity by external magnetostatic field results in dispersion properties changing of entire magnetophotonic structure. This fact allows to control field spatial distribution as within MPhC as outside one. For example shift of band gap edges along frequency axis with changing of permittivity tensor non-diagonal components can result in transformation of wave beam after waves passing through MPhC. Fig. 5 illustrates this assumption. Here we can see spatial field distribution for frequency ω_2 and value $\varepsilon_{a1} =$ 9.8 instead $\varepsilon_{a1} =$ 11 for pattern on Fig. 4. Increase of beam width at outer boundary of multilayer structure results in decrease of beam divergence.

Increase of ε_{a1} leads to changing of effective permittivity sign. In turn sign of the radicand in the formula for the transverse wave numbers ξ_j is changing too. Therefore we obtain imaginary transverse wave number in gyrotropic layer and corresponding decay of the wave at this part of structure period. Naturally conditions for surface waves realization are arise in this case.

Projected dispersion diagram for negative value of gyrotropic layer effective permittivity ($\varepsilon_{a1} = 14$) is shown in Fig. 6. Computed diagram represents a significant expansion

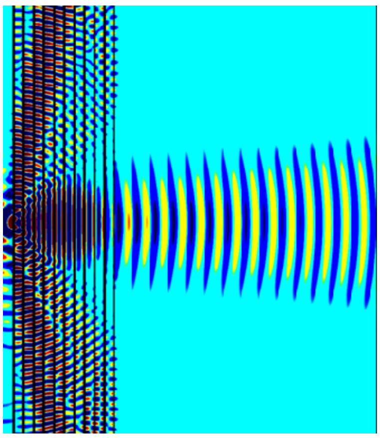

Fig. 5. Magnetic field spatial distribution.Normalized frequency of 0.56; effective permittivity of 3.997.

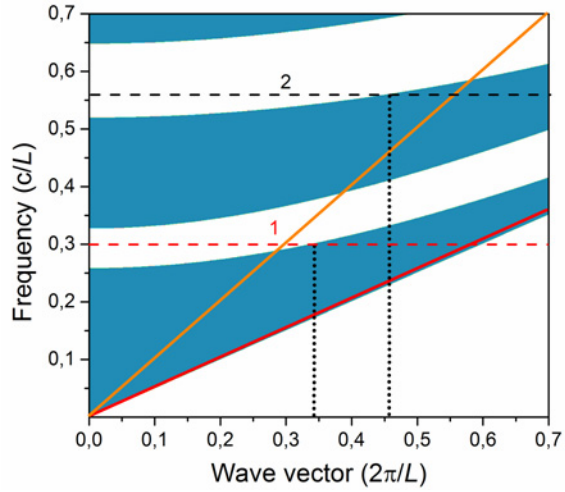

Fig. 6. Dispersion diagram of the magnetophotonic crystal for negative value of effective permittivity.

of forbidden zones in comparison to Fig. 2. We note that near-normal propagation for two marked frequencies have no ability in this case because of these frequencies lay within forbidden zones for zero value of longitudinal wave number. But tilted electromagnetic waves propagation in the MPhC is allowed according to dispersion properties of MPhC. Moreover different angles for propagation condition edges realize for frequencies ω_1 and ω_2. Vertical dotted lines indicate these edges i. e. boundaries between transmission and forbidden zones for two fixed frequencies. It should be noted that cut-off angle increases with frequency decrease as for above considered case of positive effective permittivity. Results of the field spatial distribution calculations represented in Fig. 7 and 8 confirm these reasoning. Fig. 7 shows calculations results for normalized frequency $\omega_2 L/2\pi c = 0.56$. Normal wave's propagation is not realized in this case and field pattern outside MPhC formed only by waves that propagate under defined angles to axis *Ox* within periodic structure. It is apparent that we obtain “non-propagating” cone instead “propagating” one for positive value of effective permittivity of gyrotropic layer. Furthermore this cone is expanded with frequency decrease.

Comparing of field patterns in Fig. 7 and 8 shows significant decrease of transmittance for lower frequency. The reasons for this are related to the increase of wave propagation angle in MPhC slab and enhancement of total internal reflection effect on the wave scattering on the MPhC outer boundaries.

Therefore external magnetic field varying allows to control the field spatial distribution within MPhC by means of dispersion properties changing. Spatial angular filtering of radiation outside periodical structure realizes due to not only dispersion diagram configuration but regularities of wave scattering on the MPhC outer boundaries.

It should be noted that nonreciprocity phenomenon is illustrated by field spatial distributions within MPhC. This is expected result because periodical structure under study contains gyrotropic media with corresponding permittivity

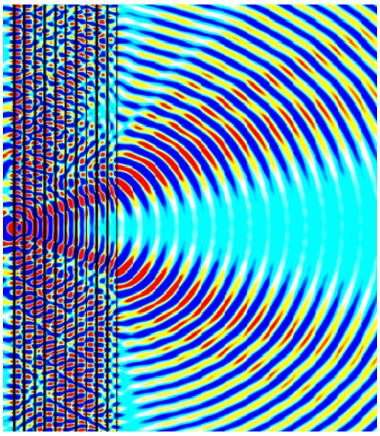

Fig. 7. Magnetic field spatial distribution.Normalized frequency of 0.56; effective permittivity of -2.08.

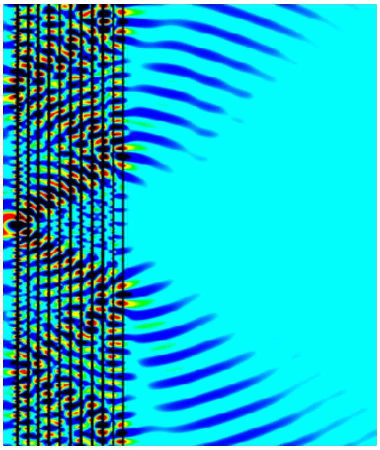

Fig. 8. Magnetic field spatial distribution.Normalized frequency of 0.3; effective permittivity of -4.33.

tensor. In this case we obtained dispersion equation (7) that does not depend on longitudinal wave number sign but expressions for electromagnetic field components contain longitudinal wave number in power one. Thus field spatial distribution depends on direction of wave propagation along MPhC layers that confirmed by calculations results for example on Fig. 4. These regularities allow additional possibilities for spatial filtering and in general controlling the optical and THz radiation.

IV. Conclusions

Angular filtering of the TE-polarized electromagnetic waves inside and outside bounded magnetophotonic crystal is considered. Dispersion properties of magnetophotonic crystal consisting of dielectric and gyrotropic plasma-like layers are investigated for different values of external dc magnetic field. Spatial distributions of the electromagnetic field components are calculated for different signs of gyrotropic layers effective permittivity. Angular filtering occurs both inside and outside magnetophotonic crystal. But field spatial distribution is formed under the influence of not only filtering effect but also total internal reflection at the magnetophotonic crystal outer boundaries.

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