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PHYSICAL AND MATHEMATICAL FOUNDATIONS OF MEASUREMENTS IN NONLINEAR DYNAMIC SYSTEMS*

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Physical and mathematical bases of measurements in physical non-linear dynamical systems are formulated. Physical properties, common to different systems, include: interval physical values; different modes of dynamics (including chaotic); strong dependence on the initial conditions; exposure to noise. We have used mathematical tools and methods of the dynamic chaos theory, open systems theory, fractal analysis: intervals of quantities values, fractal dimension, predictability time, model equation and others. New results and models are important for creating lasers with high stabilization of characteristics and practical application of solitons.

KEY WORDS: *nonlinearity, dynamic variable, chaotic mode, laser, nonlinear measurements*

1. INTRODUCTION

Academician V.L. Ginzburg notes in [1] that the attention to nonlinear physics increases continually and nonlinear physics, turbulence, solitons, chaos, strange attractors are especially important and interesting problems. Indeed, the solution of the problems of ultrashort-pulse propagation in media, the appearance and control of a chaotic mode in optoelectronic devices (lasers) and other physical systems, the creation and use of solitons with specified characteristics, and the stabilization of the characteristics of laser radiation are of great importance for applied optics, laser physics, developing soliton telecommunication systems and networks [2].

Lasers, solitons generated by fiber lasers, and other objects with nonlinearly varying characteristics refer to nonlinear dynamic systems [3]. The difficulties at developing and controlling nonlinear dynamic systems with specified characteristics are explained by such properties as: the dissipativity, nonlinear dynamics of characteristics of nonlinear dynamic systems (dynamic variables); a strong dependence on changes in initial conditions and noise; the possibility of a chaotic mode; the

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evolution and a short prediction time. Conservative and dissipative nonlinear dynamic systems are marked out, although the conservatism is often conditional.

The progress toward developing and controlling nonlinear dynamic systems depends on the models and the principles of measurement of their characteristics. At the same time, there are obvious contradictions between the classical theory of measuring [4] determined in its basis and the stochastic or chaotic dynamics of nonlinear dynamic systems [5]. The paper [6] devoted to solving the problem of laser frequency stabilization is one of the first papers in which a nondetermined approach to measurement in nonlinear dynamic systems has been proposed. It offers a fractal method for classifying the dynamics of the laser frequency, which makes it possible to draw a conclusion about the regularity, stochasticity, or chaotic condition of frequency oscillations. The nondetermined approach to the measurement of dynamic variables of nonlinear dynamic systems has been developed in [7,8]. It became obvious that a fundamentally new theory of measuring (the theory of nonlinear measuring or nonlinear metrology) in nonlinear dynamic systems is needed in order to eliminate the contradiction between the general principles of the classical theory of measuring and the real dynamics of nonlinear dynamic systems [8]. It is based on the principles of interdisciplinary information theories, open systems, dynamic chaos, fractal, entropy and interval analysis and a number of other principles.

The purpose of the paper is to formulate the physical and mathematical foundations of measuring in physical nonlinear dynamical systems.

2. LASER AS A NONLINEAR DYNAMIC SYSTEM

Today more and more objects of the surrounding world are considered as nonlinear dynamic systems. In the monograph [5], devoted to synergistic effects in various nonlinear dynamic systems, an analogy is made between the dynamics of processes occurring in lasers, nonlinear optics, in chemical reaction models, in biological organisms and populations. The nonlinear dynamic systems are considered as continuously distributed systems with fluctuations of dynamic variables $X_i(t)$. In the case of a laser or a soliton, the dynamic variables characterizes: the intensity of the electromagnetic field, the peak power, the frequency and polarization of the radiation, the pulse duration. In nonlinear optics they describe the amplitudes of several interacting modes or the dielectric permeability of the medium. In this case, the dynamic variables can be described by the following nonlinear model equation

$$\frac{\partial X_i}{\partial t} = G_i\left(\nabla, X_i\right) + D_i \nabla^2 X_i + F_i\left(t\right), \tag{1}$$

where G_i is the nonlinear function of $X_i(t)$ and the gradient ∇X; D_i is the coefficient described the diffusion (the real value) or the wave propagation (the

3. PHYSICAL AND MATHEMATICAL BASIS OF NONLINEAR MEASUREMENTS

We shall consider the physical properties which are common to various nonlinear dynamical systems, and choose the mathematical tools for analysis and presentation of measurement results of dynamic variables.

The state of the nonlinear dynamical systems at the time t is characterized by the n-dimensional vector $X\left[X_1(t),...,X_n(t)\right]$, where $X_i(t)$ is the i-th dynamic variable. With time the value $X_i(t)$ changes, but it is in the interval $X_i^{\min} \le X_i \le X_i^{\max}$. This interval is stipulated by the possibilities of the system functioning. Overrunning the value of the dynamic variable means the destruction of the system. Therefore, when carrying out measurements of an individual dynamic variable, a sufficiently long time series should be formed, covering all possible values of the dynamic variables:

$$x_i^{\mathrm{j}}\left(t_{\mathrm{j}}\right),...,x_i^{\mathrm{n}}\left(t_n\right), \tag{3}$$

where $x_i^j\left(t_j\right)$ is the result of measuring the dynamic variables $X_i(t)$ at the time t_i.

The minimum number $n_{\min}$ of the measurement experiments which is necessary for the formation of the attractor is estimated by the formula [10]:

$$n_{\min} \ge 10^{2+0.4D}, \tag{4}$$

where D is the fractal dimension of the attractor.

Note that in this case the fractal dimension can be estimated from above by taking D equal to the dimension of the vector of the state of the nonlinear dynamical system.

The results of measuring dynamic variables are taken into account the uncertainty of measurement:

$$Y_i(t) = \left(y_i^{\min} - u_i^{\min},\quad y_i^{\max} + u_i^{\max}\right), \tag{5}$$

where $Y_i(t)$ is the result of measurement of $X_i(t)$; $y_i^{\min}$, $y_i^{\max}$ are the estimations of measuring the minimum and maximum values of (3), and $u_i^{\min}$, $u_i^{\max}$ are their uncertainties of the type A [11].

The spread of values in the interval (5) is caused both by the imperfection of the measurement procedures and by the dynamics of the dynamic variable itself, while the contribution of the dynamics to the measurement uncertainty is dominant. The dynamics of the dynamic variable is complex and in the process of the system evolution it can be regular (deterministic), random or chaotic [12].

To classify the dynamics, a fractal scale is used with the reference points $D = 1$, $D = 1.5$, $D = 2$, separating different dynamics nature. At $D = 1$ the behavior of the system is strictly deterministic. At $D = 1.5$ the process is random. At $D = 2$ the solitons, we can relate: the interval of values of dynamic variables; different modes of dynamics; a strong dependence on the initial conditions, and the susceptibility to noise. The application of the described mathematical methods and tools makes it possible not only to obtain the measurement result at a particular moment in time, but also to obtain the basic scientific data in the form of previously unknown mathematical models and to perform their interpretation. The obtained data and models are important for stabilizing the characteristics of lasers and solitons.

4. CONCLUSIONS

The physical and mathematical bases of measurements in physical nonlinear dynamical systems have been formulated. The laser is considered as an example of nonlinear systems. The physical properties common to various systems include: the interval of values of the measured quantities; chaotic regimes of dynamics; the dependence on initial conditions and noise.

To study and analyze the measurement results, mathematical tools and methods of the theory of dynamic chaos, open systems, fractal analysis (intervals of values of quantities) are proposed.

The use of the described methods and tools makes it possibly not only to obtain the result of the measurement at a particular time, but also to obtain the basic scientific data in the form of previously unknown mathematical models and to perform their interpretation. The obtained data and models are important for the creation of lasers with a high stabilization of characteristics and practical application of solitons.

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