XXIII IMEKO World Congress "Measurement: sparking tomorrow's smart revolution"
August 30 – September 3, 2021, Yokohama, Japan

DETERMINATION OF THE TIME CONSTANT OF MEASURING TRANSDUCERS

Igor Zakharov [a], Pavel Neyezhmakov [b]

[a] Department of Information and Measurement Technology, Faculty of Infocommunications, Kharkov National University of Radioelectronics, Nauka av. 14, Kharkiv, Ukraine, 61166, email address: newzip@ukr.net
[b] National Scientific Center "Institute of Metrology", Myronosytska str., 42, Kharkiv, Ukraine, 61002, email address: pavel.neyezhmakov@gmail.com

Abstract – The methods for the experimental determination of the time constant of measuring transducers are described. The method is based on determining the parameters of the steady-state response of the measuring transducer using a periodic sequence of rectangular pulses of known time pulse and repetition period as an input signal. Expressions for determining the time constant for various response parameters: maximum, minimum, average, and average rectified values are given. The evaluation of the measurement uncertainty for each of the obtained expressions is made.

The proposed method has high accuracy and low labour intensity in comparison with the known graph-analytical methods.

Keywords: time constant, measuring transducers, rectangular pulses, maximal value, minimal value, average value, average rectified value

1. INTRODUCTION

The dynamic characteristics of the measuring transducers (MT) are one of their normalized metrological characteristics. In metrology, information on the dynamic characteristics of measuring transducers is used to solve the following tasks [1]:

- evaluation of dynamic errors of MT during their operation in dynamic mode;
- organization of the process of measuring changing quantities (selection or design of MT with specified dynamic characteristics);
- performing dynamic measurements of constant quantities;
- reducing the dynamic errors of MT through the use of structural and algorithmic methods for correcting their dynamic characteristics.

The solution of all the problems listed above requires the determination (identification) of the dynamic characteristics of the MT.

In most cases MT can be modelled with an aperiodic link, the transfer function of which is determined by the expression:

$$H(S) = \frac{K}{\tau S + 1}, \qquad (1)$$

where S is the Laplace operator;

τ, K are the time constant and static conversion factor accordingly.

Determination of K (MT calibration) is generally straightforward in practice. To determine the time constant, graphical-analytical methods are usually used [2], which are characterized by low accuracy and high labour intensity, which is associated with the need for complete registration of the time or frequency dynamic characteristics of the MT.

Methods for the experimental determination of the MT time constant τ by the parameters of the steady-state signal $y(t)$ at its output, when a periodic sequence of rectangular pulses $x(t)$ as an input:

$$x(t) = \begin{cases} X, & 0 \le t \le t_p; \\ 0, & t_p < t < T. \end{cases} \qquad (2)$$

where t_p is the known pulse time of pulse signal;

T is repetition period of pulse signal (fig. 1).

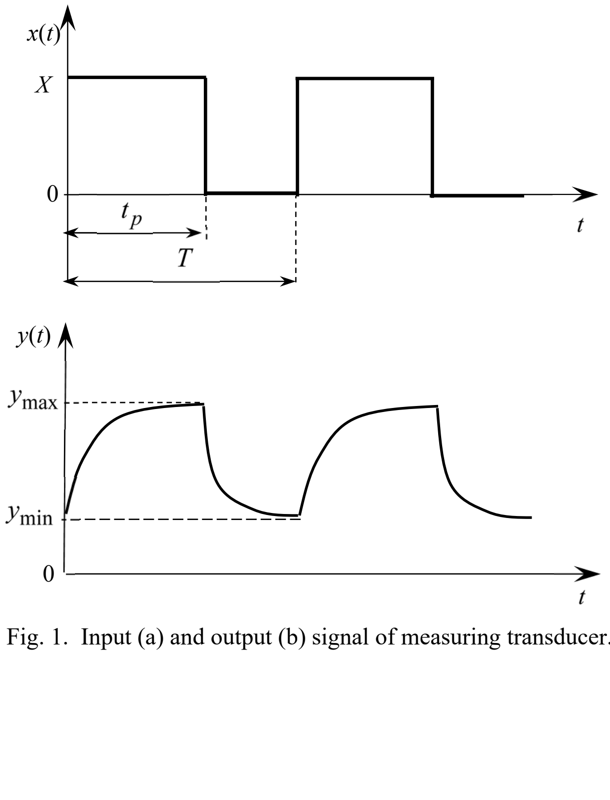

Fig. 1. Input (a) and output (b) signal of measuring transducer.

2. PARAMETERS OF THE OUTPUT SIGNAL OF THE MEASURING TRANSDUCER

The response of the MT with the transfer function (1) to the input signal (2) after the end of the transient processes will be written as an infinite sum:

$$y(t)=\begin{cases} KX\left[1-e^{-\frac{t}{T}}+\left(e^{\frac{t_p}{\tau}}-1\right)e^{-\frac{t}{T}}\sum_{n=1}^{\infty}e^{-\frac{nT}{\tau}}\right],\ 0\le t\le t_p; \\ KX\left(e^{\frac{t_p}{\tau}}-1\right)e^{-\frac{t}{T}}\sum_{n=1}^{\infty}e^{-\frac{nT}{\tau}},\ \le t\le T, \end{cases} \quad (3)$$

which is a decreasing geometric progression with the denominator $e^{-\frac{T}{\tau}}$.

Expression (3) can be rewritten as:

$$y(t)=\begin{cases} KX\left[1-e^{-\frac{t}{\tau}}\dfrac{e^{-\frac{T}{\tau}}-e^{-\frac{t_p}{\tau}}}{e^{\frac{T}{\tau}}-1}\right],\ 0\le t\le t_p; \\ KXe^{-\frac{t}{\tau}}\dfrac{e^{\frac{t_p}{\tau}}-1}{1-e^{-\frac{T}{\tau}}},\ t_p<t<T. \end{cases} \quad (4)$$

From the expression (4), the following MT response parameters can be obtained:

a) maximum value:

$$y_{\max}=y(t_p)=KX\frac{1-e^{-\frac{t_p}{\tau}}}{1-e^{-\frac{T}{\tau}}}; \quad (5)$$

b) minimum value:

$$y_{\min}=y(0)=y(T)=KX\frac{e^{\frac{t_p}{\tau}}-1}{e^{\frac{T}{\tau}}-1}; \quad (6)$$

c) average value (constant component of the response):

$$y_{av}=\frac{1}{T}\int_0^T y(t)dt=KX\frac{t_p}{T} \quad (7)$$

d) rectified average value of the variable component:

$$y_{rav}=\frac{1}{T}\int_0^T\left|y(t)-y_{av}\right|dt=2KX\left\{\frac{t_p}{T}+\left(1-\frac{t_p}{T}\right)\frac{\tau}{T}\ln\left[\left(1-\frac{t_p}{T}\right)\times\right.\right.$$

$$\left.\left.\times\frac{e^{\frac{T}{\tau}}-1}{e^{\frac{T}{\tau}}-e^{\frac{t_p}{\tau}}}\right]+\frac{\tau t_p}{T^2}\ln\left[\frac{t_p}{T}\frac{1-e^{\frac{T}{\tau}}}{e^{\frac{t_p}{\tau}}-1}\right]\right\}. \quad (8)$$

For measuring transducers, the output signals of which are electrical voltages, the values of the response parameters included in the described algorithms for determining the time constant, easily measuring using serial voltmeters.

3. ALGORITHMS FOR DETERMINING THE TIME CONSTANT OF THE MEASURING TRANSDUCERS

Expressions (5)-(7) allow directly writing down algorithms for determining τ:

a) for any ratio of t_p and T of the input signal:

$$\tau=\frac{T-t_p}{\ln\left(\dfrac{y_{\max}}{y_{\min}}\right)}; \quad (9)$$

$$\tau=\frac{-T}{\ln\left(\dfrac{y_{\max 1}}{y_{\max 2}}-1\right)}, \quad (10)$$

where $y_{\max 1}$ and $y_{\max 2}$ are the maximum values of the MT responses to input signal with equal pulse time t_p and repetition periods $T_1=T$ and $T_2=2T$, respectively;

b) for $t_p=\frac{T}{2}$ (meander):

$$\tau=-\frac{T}{2\ln\left(\dfrac{2y_{av}}{y_{\max}}-1\right)}; \quad (11)$$

c) for $t_p\ll T$ (pulse sequence of high intermittency factor):

$$\tau=\frac{T\cdot y_{av}}{y_{\max}-y_{\min}}. \quad (12)$$

The y_{rav}/y_{av} ratio gives the transcendental equation for t_p/T:

a) for $t_p=\frac{T}{2}$:

$$\frac{y_{rav}}{y_{av}}=1+4\frac{\tau}{T}\ln\left[\frac{1+e^{-\frac{T}{2\tau}}}{2}\right]; \quad (13)$$

b) for $t_p\ll T$:

$$\frac{y_{rav}}{y_{av}}=2\left\{\frac{\tau}{T}\ln\left[\frac{\tau}{T}\left(1-e^{-\frac{T}{\tau}}\right)\right]+\frac{1}{1-e^{-\frac{T}{\tau}}}-\frac{\tau}{T}\right\}. \quad (14)$$

The result of solving equations (13), (14) in the form $\tau=T\cdot f(y_{av}/y_{rav})$ with $\tau\ge T$ is well approximated by the dependences:

a) for $t_p=\frac{T}{2}$:

$$\tau=\frac{T}{8}\left(\frac{y_{av}}{y_{rav}}-0,069\right); \quad (15)$$

b) for $t_p\ll T$:

$$\tau = \frac{T}{4}\left(\frac{y_{av}}{y_{rav}} - 0,046\right). \quad (16)$$

The approximation error does not exceed ±0,18% for expression (15) and ±0.24% for expression (16).

The choice of a specific algorithm is determined by the parameters of the pulse generator and the type of voltmeters available to the user and the required accuracy.

4. MEASUREMENT UNCERTAINTY EVALUATION OF THE TIME CONSTANT OF THE MT

For the obtained expressions (9)-(12), (15), (16), in accordance with the recommendations of the Guide to the expression of uncertainty in measurement (GUM) [3], the standard measurement uncertainty of the time constant of the measuring transducer was evaluated.

Standard uncertainty of the time constant using the expression (9) is

$$u(\tau) = T\sqrt{\frac{u^2(T)+u^2(t_p)}{(T-t_p)^2} + \frac{\frac{u^2(y_{\max})}{y_{\max}^2} + \frac{u^2(y_{\min})}{y_{\min}^2}}{\ln^2\left(\frac{y_{\max}}{y_{\min}}\right)}}. \quad (17)$$

where $u^2(T)$, $u^2(t_p)$ are standard uncertainties of the repetition period and time pulse of the MT input signal, accordingly;

$u^2(y_{\max})$, $u^2(y_{\min})$ are the standard uncertainties of the maximum and minimum values of the MT output signal, accordingly.

If $u(T) = u(t_p) = u(t)$ and $u(y_{\max}) = u(y_{\min}) = u(y)$, then from (17) we obtain

$$u(\tau) = \tau\sqrt{\frac{2u^2(t)}{(T-t_p)^2} + \frac{u^2(y)}{\ln^2\left(\frac{y_{\max}}{y_{\min}}\right)}\left[\frac{1}{y_{\max}^2} + \frac{1}{y_{\min}^2}\right]}. \quad (18)$$

When using expression (10), the standard measurement uncertainty of the time constant of the measuring transducer will be equal to

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \frac{\frac{u^2(y_{\max 1})}{y_{\max 1}^2} + \frac{u^2(y_{\max 2})}{y_{\max 2}^2}}{\left[\left(1 - \frac{y_{\max 2}}{y_{\max 1}}\right)\ln\left(\frac{y_{\max 1}}{y_{\max 2}} - 1\right)\right]^2}}, \quad (19)$$

where $u(y_{\max 1})$ and $u(y_{\max 2})$ are the standard uncertainties of the maximum values of the MT response to input signals with equal time pulse and pulse repetition periods T and $2T$, respectively.

If $u(y_{\max 1}) = u(y_{\max 2}) = u(y)$, then from (20) we obtain

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \frac{\left(\frac{1}{y_{\max 2}^2} + \frac{1}{y_{\max 1}^2}\right)u^2(y)}{\left[\left(1 - \frac{y_{\max 2}}{y_{\max 1}}\right)\ln\left(\frac{y_{\max 1}}{y_{\max 2}} - 1\right)\right]^2}}, \quad (20)$$

The standard uncertainty when determination the time constant according to the formula (11) has the form

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \frac{4u^2(y_{av}) + \frac{4y_{av}^2}{y_{\max}^2}u^2(y_{\max})}{\left[y_{\max}\left(\frac{2y_{av}}{y_{\max}} - 1\right)\ln\left(\frac{2y_{av}}{y_{\max}} - 1\right)\right]^2}}, \quad (21)$$

where $u(y_{av})$ is the standard uncertainty of the average value of the MT response.

If $u(y_{av}) = u(y_{\max}) = u(y)$, then from (19) the measurement uncertainty of the time constant of the measuring transducer will be equal to

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \frac{4\left(\frac{y_{av}^2}{y_{\max}^2} + 1\right)u^2(y)}{\left[y_{\max}\left(\frac{2y_{av}}{y_{\max}} - 1\right)\ln\left(\frac{2y_{av}}{y_{\max}} - 1\right)\right]^2}}. \quad (22)$$

In the case of using the formula (12), the standard uncertainty of the time constant the measurement uncertainty of the time constant of the measuring transducer will be equal to

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \frac{u^2(y_{av})}{y_{av}^2} + \frac{u^2(y_{\max}) + u^2(y_{\min})}{(y_{\max} - y_{\min})^2}}. \quad (23)$$

If $u(y_{av}) = u(y_{\max}) = u(y_{\min}) = u(y)$, then from (23) we obtain

$$u(\tau) = \tau\sqrt{\frac{u^2(T)}{T^2} + \left(\frac{1}{y_{av}^2} + \frac{2}{(y_{\max} - y_{\min})^2}\right)u^2(y)}. \quad (24)$$

The standard uncertainty when calculating the time constant according to the formula (15) has the form

$$u(\tau) = \sqrt{\tau^2\frac{u^2(T)}{T^2} + \left(\frac{T}{8y_{rav}}\right)^2 u^2(y_{av}) + \left(\frac{T\cdot y_{av}}{8y_{rav}^2}\right)^2 u^2(y_{rav})}. \quad (25)$$

If $u(y_{av}) = u(y_{rav}) = u(y)$, then from (25) we obtain

$$u(\tau) = \sqrt{\tau^2\frac{u^2(T)}{T^2} + \left(\frac{T}{8y_{rav}}\right)^2\left(1 + \frac{y_{av}}{y_{rav}}\right)^2 u^2(y)} \quad (26)$$

The standard uncertainty when calculating the time constant according to the formula (16) has the form

$$u(\tau) = \sqrt{\tau^2\frac{u^2(T)}{T^2} + \left(\frac{T}{4y_{rav}}\right)^2 u^2(y_{av}) + \left(\frac{T\cdot y_{av}}{4y_{rav}^2}\right)^2 u^2(y_{rav})}. \quad (27)$$

If $u(y_{av}) = u(y_{rav}) = u(y)$, then from (25) we obtain

$$u(\tau) = \sqrt{\tau^2\frac{u^2(T)}{T^2} + \left(\frac{T}{4y_{rav}}\right)^2\left(1 + \frac{y_{av}}{y_{rav}}\right)^2 u^2(y)}. \quad (28)$$

5. CONCLUSIONS

1. The dynamic characteristics of the MP are one of their normalized metrological characteristics and must be determined when the MP is used in a dynamic mode.

2. The proposed method is based on determining the parameters of the steady-state response of the measuring transducer using as an input signal a periodic sequence of rectangular pulses with a known time pulse and repetition period. In contrast to the graphical-analytical methods that are usually used, the proposed method does not require full registration of the dynamic characteristics of the MT, therefore, it has low labour intensity.

3. The advantage of the proposed method is the possibility of its implementation with the help of serial voltmeters, which can measure extreme, average and average rectified values of alternative voltage.

4. The obtained estimates of the measurement uncertainty make it possible to choose the most accurate of the proposed algorithms for the experimental determination of the time constant of the measuring transducer.

ACKNOWLEDGMENTS

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

REFERENCES

[1] V. A. Granovsky, *V.A. Granovsky Dynamic measurements: fundamentals of metrological support.*, Energoatomizdat: Leningad Branch, Leningrad, 1984 (in russ).

[2] I.P. Zakharov, M.P. Sergienko *Metrological identification of the dynamic characteristics of measuring instruments.,* Company SMIT, Kharkiv, 2012 (in russ).

[3] ISO/IEC 2008. GUIDE 98-3. Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement. (GUM:1995).