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MATHEMATICAL FOUNDATIONS OF MEASUREMENT IN NONLINEAR DYNAMICS SYSTEMS

Yu. P. Machehin

Yu. P. Machehin, Doctor of Technical Science, Professor, Chief of Physical Foundations of Electronic Engineering Department,
Kharkov National University of Radio Electronics, Kharkov

МАТЕМАТИЧНІ ОСНОВИ ВИМІРЮВАНЬ В НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМАХ

Ю. П. Мачехін, доктор технічних наук, професор, завідувач кафедри фізичних основ електронної техніки,
Харківський національний університет радіоелектроніки, м. Харків

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Ю. П. Мачехин, доктор технических наук, профессор, заведующий кафедрой физических основ електронной техники, Харьковский национальный университет радиоэлектроники, г. Харьков

In the present work the theory of measurement in nonlinear dynamic systems are developed. We study the radically new conditions measuring tasks associated with the peculiarities of the behavior of nonlinear dynamic systems. It show that between qualitative theory of differential equations and methods for estimating uncertainty of measurement there is a link that provides an assessment of the conditions of measurement.

Keywords: *measurement, nonlinear dynamic systems, uncertainty.*

У статті одержала розвиток теорія вимірювання у нелінійних динамічних системах. Досліджено принципово нові умови вимірювальної задачі, пов'язані з особливостями поведінки нелінійних динамічних систем. Показано, що між якісною теорією диференціальних рівнянь і методами оцінювання похибки вимірювання є зв'язок, що визначається оцінкою умов вимірювання.

Ключові слова: *вимірювання, нелінійні динамічні системи, невизначеність.*

В настоящей работе получила развитие теория измерений в нелинейных динамических системах. Исследованы принципиально новые условия измерительной задачи, связанные с особенностями поведения нелинейных динамических систем. Показано, что между качественной теорией дифференциальных уравнений и методами оценки погрешности измерения есть связь, которая определяется оценкой условий измерения.

Ключевые слова: *измерение, нелинейные динамические системы, неопределенность.*

INTRODUCTION

The realities of modern development of science, technology, medicine, economics and many other areas of social development, require the use of different measurement processes as the primary method of obtaining objective information. Methods of analysis of measurement results [1] based on tried and tested in practice, the physical models and their corresponding mathematical methods. In particular, the basis of modern mathematical methods of measurement theory, are experimentally established facts of classical mechanics [2].

At one time, a variety of tasks led to the formation of several directions in mechanics. Newtonian mechanics, studying the motion of material points in three-dimensional Euclidean space. Lagrangian mechanics describes the motion of mechanical systems using configuration space, and

Hamiltonian mechanics, allows one to investigate such problems as the approximate methods of perturbation theory of celestial mechanics, optics and quantum mechanics. Newtonian mechanics is a special case of Lagrangian mechanics in Euclidean space, and the Lagrangian represents a special case of Hamiltonian mechanics.

Our space is three dimensional and Euclidean, and the time is one-dimensional. All the laws of nature at all times are the same in all inertial coordinate systems. However, the main condition, which always holds in classical mechanics, is that the initial state of the mechanical system uniquely determines its movement through space and time. The totality of these facts has allowed formulating the principle of determinism of Newton-Laplace, according to which the behavior of the system in time and space is described, in general, the system of ordinary differential equations. Following Cauchy-Kovalevskaya existence and uniqueness of solutions of ordinary differential equations of a physical quantity described by these equations has a unique value.

If we consider the process of measuring the experiment from the standpoint of classical mechanics, two provisions determine the measurement results. First, the measured value has a unique value [1]. Secondly, the precision with which set to the measured value is determined by measuring the initial conditions of the experiment. Further development of measurement theory, of course, follows the development of classical mechanics and mathematical methods for describing the behavior of not only linear but also nonlinear dynamic systems, including those that belong to the class of dissipative dynamical systems [3], for which the compression phase volume take place.

However, mechanical or classical determinism in the conventional sense is possible only in an abstract mathematical space. In this phase space, position of the point corresponding to the initial condition of the state, of the studied system can determined by the actual number with any desired accuracy. This means that the initial conditions differ from each other, if the values of the first N characters coincide; while in N+1 mark is registered differences, even if N is a very large number, for example, 10^{20}.

It is clear that in describing the real physical system, use that form of determinism is impossible, because it implies that the state of physical objects can consider different, even though no actual measurements cannot distinguished. Consequently, such a representation is necessary notions of determinism that it was not only physically meaningful, but also could be used in the measurement experiments.

Each state is a physical object is described with a small, but always the ultimate uncertainty and the state of a physical object, is never defined by one real number, but only through a probability distribution (Gaussian, uniform, etc.) possible values of physical quantities. In this regard, the task of analyzing the measurement results presented as a problem of solving the equation of measurements, given the probability distribution of the initial parameters of the last [4].

The main feature of this approach is that the measurement equation, transforming the original probability distribution in the probability distribution of outcome measurement does not change it in time and it remains stable [4]. It should note that the test system must be fixed and stable over time. This means that, based on the equation of measurement uncertainty of the outcome of indirect measurements set by the uncertainty of all parameters of the equation as the initial conditions, given a probabilistic way. An important feature of this approach is the fact that the initial state of a dynamic system is established using a probabilistic description of system parameters. The dynamics of studied system described by motion equations. Probability distribution of random values of the parameters used by external or internal processes and noise. For measuring tasks, it is important that the developed mathematical tools needed in dealing with statistical problems, based on the theory of Brownian motion and processes of diffusion type.

This unit is the theory of Markov processes. An important special case of Markov processes are Gaussian processes with exponentially decaying correlation function. Basic property of Gaussian processes determined by the central limit theorem. According to which the impact of a large number of uncorrelated random variables, provides properties of a random Gaussian process. It is essential that the ordinary linear differential equations with Gaussian fluctuations in the parameters describing the Markov properties of the fluctuations of motion of dynamical systems. If the non-linear equations, then, in general, there is a complicated transformation of Gaussian fluctuations, the values of parameters of the equation of measurements, leading to non-Gaussian law of distribution of values of the measured value. Thus, any attempt to consider the behavior of nonlinear systems under the influence of Gaussian noise, is facing a fundamental difficulty, since the temporal evolution of such a system is no longer Markov and lost opportunity to use powerful methods of the theory of Markov processes.

To solve the problem of determining the transformation of uncertainty parameters of dynamic and static systems in the uncertainty of the outcome measurements, recently used numerical methods of Monte Carlo [5]. With the help of numerical simulation can find a form of probability distribution of measurement results. The existing theory of measurement, the impact behavior of the studied dynamical system on the results of the measurements investigated insufficiently.

All the main theoretical results obtained under the assumption that the real, the states of a dynamical system in phase space correspond to the only stable singular point or a stable closed trajectory. Otherwise, only those parameters that correspond to the states 1.a (Fig. 1) and 2.a (Fig. 2) motion of a dynamical system, formed near the stable states.

Steady state has an area of attraction, so that if the system due to external influence leaves this state, then within a finite time, it comes back to him. Since under the influence of fluctuations (typically ergodic) steady state in time is «smeared» in a certain region in phase space, the results of measurements performed within a finite time interval will belong to this area, which you can use statistical methods for estimating measurements.

Mathematical characteristics of stable states — a node or focus, are the roots of the characteristic equation [6]. In cases for which there are negative real parts of the roots of the characteristic equation in dynamic systems, there is a stable state and can be realized the problem of measurement. The results of measurements in a dynamic system under conditions not only stable point or limit cycle, and stable dynamic regimes, such as dynamic chaos, will have an independent interpretation, while not regarded as one of the main problems of measurement theory applicable to dynamical systems. In this regard, we have further developed the theory of measurement, created in recent years [7—10], to ensure the correct assessment of the results of measurements in nonlinear dynamic systems. It shown that between qualitative theory of differential equations and methods for estimating uncertainty of measurement there is a link, that provides an assessment of the conditions of measurement. In this context, the aim of this work was the study of mathematical methods and physical conditions, which may be applicable for the analysis of measurement results in dynamical systems in simple and strange attractors [11].

Fig. 1a

Fig. 1b

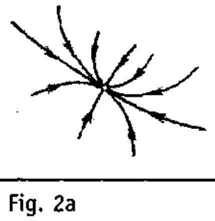

Fig. 2a

Fig. 2b

Mathematical Foundations of Measurement Theory in Linear Systems

Applying the principle of classical Laplacian determinism in measurement theory is the basic condition for the evaluation of measurement results. Better placed measuring experiment, namely, the lower the value of uncertainty of the initial parameters, the less uncertainty set, the measurement result [12] To describe dynamical systems typically used a system of differential equations:

$$\frac{\partial X}{\partial t} = f(X, Y_1, ..., Y_N). \quad (1)$$

The condition of the stationary state of a dynamical system described by the equation:

$$\frac{\partial X}{\partial t} = 0, \quad (2)$$

or equation:

$$f(X, Y_1, ..., Y_N) = 0. \quad (3)$$

The same equation, rewritten in a simpler form, is analyzed in the Manual of uncertainty [1], and represents the measurement equation:

$$X = f(Y_1, ..., Y_N). \quad (4)$$

When the value is determined based on indirect measurements, ie determined by measuring the values $Y_1, ..., Y_K$, then the measuring process for determining the value X can roughly described as follows: the mathematical expression:

$$X = \lim_{n,m \to \infty} \frac{1}{n} \sum_{i=1}^{n} f^{(m)}(Y_i^{(m)}). \quad (5)$$

here $Y_i^{(m)}$ — the result of i — the observation for m — initial condition.

The condition $m \to \infty$ requires consistent refinement of the initial conditions of measurement. Each successive refinement of the initial conditions of the measuring experiment reduces the uncertainty of the parameters characterizing the system being studied, and, consequently, to reduce the uncertainty of the measured value X. I.e. the index m, formally indicating the accuracy of the initial conditions, always has a finite value. This means that this condition is consistent formulation of physical problems in classical mechanics, when the initial conditions given in the form of intervals of possible values of the original values. An example of this can be a steady improvement in the conditions of incubation, which determine the stability of measured values. As a result of the measurement uncertainty — $u_c(x)$ is expressed through the uncertainty of individual parameters $u(y_i)$:

$$u_c^2(x) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial y_i}\right)^2 u^2(y_i). \quad (6)$$

In a real physical experiment to determine the limit (5), which characterizes the result of measurement, is not possible (at least by virtue of the finiteness of time available for making measurements). Therefore, based on indirect measurements, even very many, is determined by the value of variable, which is within the limits of uncertainty is considered as the actual value of the desired size.

The initial conditions for any physical deterministic system installed with a small, but always the ultimate uncertainty. Stochastic processes that affect the parameters of the system have different characteristics, but in the measurement theory is that they are ergodic, ie allowing the use of either the averaging time, or an average over the phase space of possible values. The scatter of measurements is considered as an influence on the measured value and a means for measuring fluctuations in the external environment and internal, random processes. It is the property of ergodicity of random processes provides a statistical processing of measurement results, as a result of which is determined by the actual value of the measured value and set the uncertainty which was obtained this result. A mathematical description of the ergodicity of the random process can be based on different characteristics, but most of them are of interest to the theory of measurement equivalence is the result of averaging in time random process and the result of averaging over an ensemble of all possible states implemented a random process subject. For metrology ergodicity of random variations of the measured values of physical quantities, is the most satisfactory, if not the sole justification simultaneous applications, such as averaging time, and averaging over the probability law of distribution of possible states. This means that if the average time the function defined as:

$$\overline{X} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} X(n,\tau), \tag{7}$$

here N — number of observations; τ — the time between observations, and the average over the ensemble of possible realizations:

$$\langle X \rangle = \int_{-\infty}^{\infty} XP(X)dX, \tag{8}$$

here $P(X)$ — density of probability, then the ergodicity of the process means that there is a solution of the equation in the ergodic:

$$\langle X \rangle = \overline{X}. \tag{9}$$

A rigorous solution of equation (9) represents only the value, which is a required value of the measured value. However, in real conditions of the measuring experiment, there always exist finite limits of summation and integration. Therefore, only (7) and (8) restrictions on the amounts and limits of integration, once there is a situation in which equation (9) can satisfy the numerical values of the measured physical quantity from a finite range of values. As a result, a solution of equation (9) is a value (real) of the measured value is set with some uncertainty. Crucially in this case is that during the process of measurement is set to a physical quantity corresponding to the stable, equilibrium state of the system studied.

Mathematical Foundations of Measurement Theory in Nonlinear Dynamics Systems

The mathematical basis of methods of analysis of measurement results based on the mathematical apparatus of the qualitative theory of differential equations, which allows us to study the solution in phase space, near the stable points or trajectories. Therefore, considering the dynamic system as an objective measurement problem, it should be noted that as a result of measurements set the value of the unknown quantities, corresponding to a stable, sustainable state, or deterministic transition time. Due to the fact that measurements in dynamic systems in a steady state, the greatest interest, they were considered in the work. Let the dynamic system, whose parameters will be measured, is described by n - dimensional differential equation:

$$\frac{\partial}{\partial t}\vec{X} = \vec{f}(\vec{X}), \tag{10}$$

here: $\vec{X} = (X_1, X_2 .. X_n)$, $\vec{f} = (f_1, f_2 ... f_n)$.

An important property of these equations is that they can explore how to use rigorous analytical solutions, and with the help of qualitative methods in differential equations for assessing the behavior of the system near the singular points of stable states. There is a classification of singular points, near which the system can behave in a regular fashion. Mathematically, the stable equilibria: the roots of the characteristic equation [11] determine a node or focus. The roots are real and negative (positive) correspond to stable (unstable) node. Complex roots with negative (positive) real parts, provide a screw motion in the focal region. Thus, only in those states for which there are negative real parts of the roots of the characteristic equation in dynamical systems can be realized the problem of measurement.

When analyzing the results of measurements must consider as a dynamic behavior of the system near the stationary point of attracting and perturbations of the system due to external noise influences. However, the feature measurements in dynamical systems near singular points have not been the subject of deep theoretical studies. For the theory of measurement, two important properties of these singular points. First, these points correspond to stationary solutions, which satisfy the condition:

$$T^t X^{(0)} = X^{(0)}. \tag{11}$$

In which the point $X^{(0)}$ exists at all t. Secondly, this stationary solution is still attractive, if:

$$\lim_{t\to\infty} TX = X^{(0)}, \quad (12)$$

for all X, enough to near to $X^{(0)}$. To determine whether the steady-state solution attracting, use the method of linear stability analysis is attractive, provided that all eigenvalues:

$$A_{ij} = \frac{\partial f_i X^0}{\partial X_j}, \quad (13)$$

have strictly negative real parts. These stationary solutions called an additive attracting. When performing measurements in an additive attracting an additive but the decision should take into account that small random perturbations of the system do not destroy this decision. It should be noted that the qualitative Letters of Credit differential equations and is also exploring the processes of bifurcation, ie conditions under which the stability of some points lost, and born DIT stable states, ie by an additive but one state to another.

However, measurement problems, these properties do not play an additive tion value, since the performance measurement carried out only near the stable and steady states. Using equation (10), to describe the magnitude of deviation $\eta = X - X^{(0)} << X^{(0)}$, it is possible to take into account influence of dynamical system behavior additive noise near a stable point on the measurement results, taking into account an. On the basis of a linear Langevin equation:

$$\frac{\partial}{\partial t}\eta = -\lambda\eta + \varphi(t), \quad (14)$$

with initial conditions $\eta(0) = 0$ and $\varphi(0) = 0$ will be to demonstrate the influence of the behavior of a dynamical system on the measurement results. Fluctuations of the value η under action of random forces $\varphi(t)$ described by a solution:

$$\eta(t) = \int_0^t d\tau\varphi(\tau)\exp\{-\lambda(t-\tau)\}. \quad (15)$$

Knowing the time realization and, therefore, the statistical characteristics of $\varphi(t)$, we can find the statistical characteristics of the process $\eta(t)$. It is clear that the temporary nature of the behavior of the process $\eta(t)$ will also determined by the rate of return to a stable state, i.e.value λ.

The spectral characteristics of the process $\varphi(t)$ determine the rate and magnitude of deviations from steady state. If we consider the random forces in the form of a sequence of delta functions, wich can take out system of the state of stable equilibrium, then the expression $\eta(t)$ to be simplified:

$$\eta(t) = \int_0^t d\tau \sum_{m=1} \delta(\tau - \tau_m)\exp\{-\lambda(t-\tau)\} = = \sum_{m=1}\exp\{-\lambda(t-\tau_m)\} = \sum_{m=1}\exp\{-\lambda\Delta t_m\}. \quad (16)$$

Hence characteristics of behavior linear dynamic system influenced on the results of measurements. This example represents the simplest situation when at the same time with random processes must take into account the behavior of dynamic systems. Thus, if the task of measuring the quantity $X^{(0)}$, the results of measurements X may used for of the area can be set (set) of values that can attributed, study size. The statistical variation, the measured values of magnitude, is set X_i for $1<i<N$. The average value of measured values is:

$$\langle X\rangle = \frac{1}{N}\sum_{i=1}^{N} X_i = \frac{1}{N}\sum_{i=1}^{N}\eta_i + X^{(0)},\ \eta_i = \eta(t_i). \quad (17)$$

The average value of the measured quantities differs from the values of the steady state by the average deviation from the steady state. The average deviation from the stationary value during the observation time T is determined:

$$\langle\eta\rangle = \frac{1}{T}\int_0^T d\eta(t) = \frac{1}{T}\int_0^T dt\int_0^t d\tau\varphi(\tau)\exp\{-\lambda(t-\tau)\}$$

standard uncertainty of type A is determined:

$$u = \sqrt{\frac{1}{T}\int_0^T (\eta(t) - \langle\eta\rangle)^2 dt}. \quad (18)$$

When the results of time-series measurements is determined by the average deviation, in this case the integrals replaced by sums. Noted, that experts in the field of automatic control systems know the consideration of the average deviation. The principle of laser automatic frequency stabilization based on Langevin equation. Therefore, (17) to evaluate measurement uncertainty of type A and type B uncertainty is associated with a shift of the stable point.

Further development of the fundamental foundations of the theory of measurement associated with an increase in the dimension of phase space in which the studied dynamical system has a stable limit set. In the case where the topological dimension of phase space study of a dynamic system for more than two, then the stable states may represent not only the specific stable point (in space), but stable, marginal area. By analogy with the stable, attracting singular points (attractor), these areas are also attracting. Some of these areas in modern mathematics and physics have called strange attractors (strange attractor). Features of these areas described in detail in numerous articles and monographs [13—15]. Edward Lorenz [16] for numerical simulation of convective effects in the atmosphere, as the basis of meteorological processes, faced with chaotic solutions of the system of equations, derived from the three-dimensional Navier-Stokes equations. An important result of this work was the fact of discovery of fundamentally new types of stationary states in nonlinear, dissipative, dynamical systems. The motion of this dynamical system described by first order differential equations (10) if clearly $\vec{f}$ does not ▶

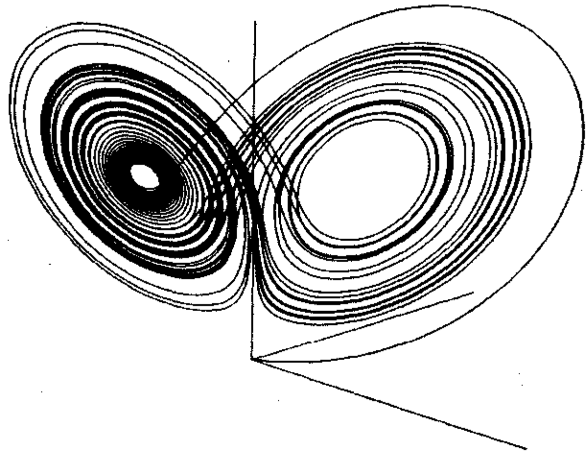

Fig. 3. The Lorenz attractor

depend on the time and represents a smooth function. In this case, the solution $X = X(X_0,t)$ exists at all t. Any point of the phase space uniquely determines the state of the system described by equations (10). In the case where the system under consideration dissipative, then randomly selected in the phase space volume element w bounded by the surface s, is compressed. Therefore, dissipative systems are dynamic systems, for which the condition of phase volume — $dw/dt < 0$.

The most important case for the theory of measurement is one in which the trajectory generated by equation (10) is a limited region of phase space. Because of limited space of two-dimensional chaotic flow cannot exist (the Poincare-Bendixson [17]), then the only possible attractors in a limited area of two-dimensional space are the limit cycles and fixed points. In the three-dimensional set of points, which attracted the trajectory of a dynamical system, is in such a way that the dimension of this set is fractional. The fractal dimension of the Lorenz attractor has a value of 2.06. In three-dimensional phase space a typical picture of a strange attractor shown in Fig. 3 [18].

The stability of the set combined with the instability of each individual trajectory, so with the light hand of Ruelle and Takens [19] have called strange attractors. For strange attractors characterized by the following characteristics that significantly affect the measurement process. First, the attractor is a region in phase space within which moves a dynamic system. The trajectory of the system passes through this limited region of phase space, and fills it very difficult to read. Second, the attractor is strange, because of sensitivity to initial conditions. Arbitrarily close initial points in phase space, through a sufficiently large but finite time, lead to the fact that the trajectory of the system diverge at a finite distance, which leads to positive Kolmogorov entropy. It is this entanglement leads to a complicated irregular movement. Poincare section looks like a strange attractor, as shown in Fig. 4.

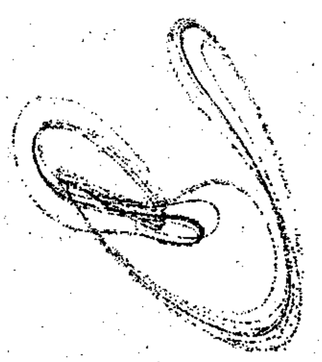

Fig. 4. The strange attractor

Culiarity of the behavior of dynamic systems, in this case is the fact that the trajectory at which the dynamical system moves in phase space never intersects with itself, but it is in the closed area of finite size phase space. Third, an important property of strange attractors is the structural stability and genericity. This property has a significant impact on the correctness of the analysis of measurement result, since just such attractors characterize physical systems. Small changes in the system parameters change the structure of the attractor in a continuous manner. Nevertheless, the size of the region of attraction remained virtually unchanged, therefore, to assess the uncertainty of measurement results can used by the maximum size of the attractor. It remains an open question how to address the internal structure of the attractor, which can vary. The set of points of intersection with the trajectory of the system selected in the phase space plane, obtained for a very long time. Fractal structure of strange attractors manifested in the fact that each conditional line in Fig. 4 has a repeating structure. Depending on the increase, in which the trajectory is considered, the structure of the strange attractor repeated, which leads to a multi-modal distribution function on the plane of Poincare [20].

If we assume that each measurement is performed at a time when the trajectory of a dynamical system crosses the selected plane, and the measurement results will be applied on this plane, depending on the nature of the behavior of dynamic systems measurement results can be placed in a certain area of finite size. Thus, for successive n measurements of physical quantities obtained time series of values that will be randomly different from each other. Nevertheless, all these records the measurement results correspond to the real state of a dynamical system. Therefore, because of the measuring experiment recorded a random sequence of measured values, each of which corresponds to the system at the time of measurement. As in the general case ergodicity strange attractor has proved, then use the classical methods for the statistical analysis of the set of measurements correctly. Therefore, the problem of assessing the real value of the measured value in this case should be resolved based on the properties of the strange attractor. The only thing that can be considered in this case correctly is that the definition of the size of the area in the Poincare section, within which the changing state of the system. It certainly would not have been implemented single measurement, the scatter of measurement results characterized by dynamic behavior of the system. Therefore, assessment of uncertainty of measurement result determined not by the size and nature of external, random perturbations, and dynamic behavior of the system studied. To obtain information about the size of the strange attractor must study to observe the system for a long time. Length of time series should enable registers not only those areas in which there is compression, but also those areas in which there is stretching of the phase volume. To experimentally investigation the steady-state multi-dimensional dynamical system, and more so to estimate the structure of a strange attractor, a method developed of the correlation integral [21]. The used expression is:

$$C_m(\varepsilon) = \lim_{N\to\infty} \sum_{i,j=1}^{N} \frac{1}{N^2} \Theta(\varepsilon - |y_i^{(m)} - y_j^{(m)}|), \qquad (19)$$

here Θ — the Heaviside function.

Ruelle and Takens invited to construct the trajectory of a dynamical system in phase space dimension m, described by vectors $y_i^{(m)} = \{x_i, x_{i+1}, \ldots x_{i+m-1}\}$. Vectors Belt to the results of measurements of one of the strap x values of a dynamical system x_i, if $1 \le i \le N$ and $x_i = x(i\tau)$. According to the results of measurements can strap the correlation sum based on the original belt ranks. As the belt ranks measurements always have finite length, ie $N<<\infty$, Then (19) is somewhat simplified:

$$C(\varepsilon) = \sum_{i,j=1}^{N} \frac{1}{N^2} \Theta(\varepsilon - |y_i - y_j|). \qquad (20)$$

With the help of that fixed vector dimension m, a measure, which does not exceed the value ε. For the strange attractors always exists a maximum value ε, above which does not increase the value $C(\varepsilon)$ [22]. The correlation sum can use as a means of distinguishing the chaotic behavior of dynamical systems and external, white noise. In addition, we can estimate the value ε of the maximum linear dimension of the volume in phase space, which occupies a strange attractor. Evaluation of the quality of measurement results in this case must made according to the level ε corresponding to the maximum size of the region in phase space, which occupies a strange attractor. When calculating the correlation sum value ε is set, so if in the process of sorting values ε of the variable is set its maximum value, thus fixed the maximum value of measurement uncertainty in a dynamic system which is in the mode of a strange attractor. The randomness of the measurement results is of fundamental nature — can not get rid of her, gathering more information.

Mathematical Foundations Evaluation of Measurement Results in a Strange Attractor

In theory, the measurement is no restriction on the dimension of the studied dynamical systems. Therefore, the measurements can made in systems, a complete description of which is possible in three-dimensional phase space. However, in this case must take into account the possibility of implementing a dynamic system state strange attractor and the influence of the latter on the results of measurements and their uncertainty. Sustainable attracting set, which is a strange attractor, has a significant influence on the measurement results. If all the above cases, it was shown how to take into account the deterministic behavior of a dynamical system in the evaluation of measurement uncertainty, then in the strange attractor must consider the impact of the irregular, chaotic regime of motion of a dynamical system. Since the theoretical studies of strange attractors are relatively recent, it is not all more, the study of the conditions of influence of the chaotic regime of the measurements, until recently, have studied in detail. When observing a dynamic system for a very long time (much longer than the characteristic period of motion of the system) time series of measurements can be studied using methods of spectral analysis. General Principles of the probabilistic descriptions of strange attractors are not currently exists, as there is no probabilistic description of strange attractors with $t_i \to \infty$. In general, in experimental studies of dynamic systems, the main result of the latter is the time series of measurements. It can represented as follows:

$$X = X_1, \ldots, X_N, X_i = X(t_i), t_i = t_{i-1} + \tau. \qquad (21)$$

In these series are recorded as a random variation of the measured quantity of WMD influence Compact burner dimension of the environment, and random changes in the dynamic behavior of the system. Since we are interested in the volume chaotic motion of the system, then we will assume that the random perturbations are negligible. In this case, spatial-temporal properties of the strange Compact burner dimension will define the irregular results of measurements. To study the features of this influence Compact burner dimension using the mathematical method, this adequately describes the measurement results and the structure of strange Compact burner dimension.

We measured physical quantities of linear and nonlinear systems are in steady state, there is a single value. The magnitude of the measured parameter of a nonlinear dynamical system in a strange mode Compact burner dimension, has an infinite number of possible values, the region determined by the structure of the Compact burner dimension, more precisely, ▶

its projection on the axis of the space measured. Methods for processing the measurement results are based in this case, the properties of strange attractor as the Hausdorff set. Hausdorff set has two principal features that used to analyze the set of measurements. These features include:

- Metric Hausdorff space [23], which ompakt the metric of Euclidean space;
- Hausdorff dimension of sets in phase space can be fractional and not necessarily with the topological dimension [22].

The metric Hausdorff space allows us to introduce the concept of limit of a sequence of sets. For example, if a sequence $E_n, n = 1,2,3,...$ Compact burner dimension sets, nested:

$$E_1 \supset E_2 \supset E_3 \supset ..., \quad (22)$$

and E-represents the intersection of sets E_n, then:

$$E = \bigcap_{n=1}^{\infty} E_n. \quad (23)$$

Otherwise, the sequence of sets E_n converges to E in the Hausdorff metric:

$$\lim_{n\to\infty} E_n = E. \quad (24)$$

However, in the Hausdorff metric distance $H(E_n, E)$, as well as in the Euclidean metric, between the two sets satisfy the limit passage:

$$\lim_{n\to\infty} H(E_n, E) = 0. \quad (25)$$

Thus, if we know the nested set E_n, you can define the limit set E. However, ichislit conditions known only to the original set E_1, which represents the results of the measurements. Therefore, in accordance with the Hausdorff metric properties of the original set of measurements can calculate the limit set E, satisfying the equation:

$$T(E) = E. \quad (26)$$

Thus, the limit set of measurements is a set of strange attractor, which is stable and attracting set. As a result, the motion of a dynamic system within the domain of attraction of an attractor, the system returns to a stable state with complex behavior. Therefore, the size of uncertainty can limited by the size limit set E. The projection of the diameter of the limit set on the axis of the measured parameter to evaluate the uncertainty of measurement results. However, in this case to evaluate the diameter of the limit set requires additional information associated with the structure of the attractor. To estimate the size of the limit set, you can use the results of measurements and calculations of the fractal dimension d. Assuming that the measure of the set X, denoted B_d, can be calculated using the classical method of coating dimensional volume, we use the dimensional size, defined as the maximum difference between two measurements $|x_i - x_j|$. The maximum amount of uncertainty can be calculate using the value of the fractal dimension:

$$U_d = \max |x_i - x_j|^d. \quad (27)$$

In fact, the maximum diameter of this set can be regard as the primary assessment of uncertainty of measurement results. The main feature of the calculation is to preserve the dimension of the uncertainty value. In expression (27) remains equal to the dimension of the measured value. Increasing the interval of uncertainty due to the dimension due to possible condition that the trajectory of a dynamical system on large time intervals.

CONCLUSION

We considered with one-voice analysis methods of measuring results in linear and nonlinear dynamic systems. The peculiarities of the influence the behavior of a dynamical system on the measurement results were definded.

Main feature of the results is the fact that they are applicable for measuring parameters of a dynamic system in a steady state. If steady state had described by a limit cycle or a stable focus the evaluation of the measurement results based on attracting properties of stable states. In the case when the stable state is a strange attractor, an analysis of the measuring experiment must be performe taking into account the characteristics of this attractor. However, the measurements of one of perametrov dynamic system, the uncertainty of these measurements had caused by the projection of the diameter of the set on the axis of the investigated parameter.

In view of the fractal dimension of the investigated attractor, the uncertainty had calculated as a projection on the axis of action, power dimension. Becouse a result of the research it became clear that in the process of analyzing the measurement results must take into account the statistical properties of measurements and properties of the set of outcomes associated with stochastic dynamics behavior of the most dynamic system.

The peculiarity of such an evaluation of measurement results is that, in multivariate dynamic systems such as biological, medical and economic not interest the magnitude, but range of possible values, which in certain conditions can make the quantity under investigation.

Performance measurement systems have led to the fact that research can described in phase spaces with dimension higher than two can. The measurements determine the values of parameters corresponding to a stable attracting point or limit cycle, or attracting set.

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