OPTIMIZATION OF LINEAR FUNCTIONS ON CYCLIC PERMUTATIONS

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The problem of minimizing a linear function on the set of cyclic permutations is considered: $L(x) = \sum_{i=1}^{n} c_i x_i$, where $c_1 ... c_n$ are the given real coefficients, and $x_1, x_2 ..., x_n$ take the values of all possible cyclic permutations of generating elements $a_1 < a_2 < ... < a_n$. The report proposes an exact and heuristic solution for this problem.

The report introduces the concept of partial permutation, analyses the combinatorial properties of these permutations associated with the formation of new cycles by adding a new element to the partial permutation. Corresponding propositions are formulated and proved.

For the exact solution of the problem branch and bound algorithm is used. At every step of the algorithm each of the generating elements not included in the current partial permutation joins it. It is the technique of branching.

For each partial permutation the estimate of the objective function is calculated. A partial permutation with the better estimate undergoes further branching. The full cyclic permutation with the best objective function value is an intermediate solution. Branching continues until there is no partial permutation with the better estimate than the intermediate solution.

At the beginning of the heuristic solution all the possible partial permutations of length q are constructed. After this a certain fraction of permutations having the best evaluation of the objective function is selected. Next, other generating elements are joined to each of the selected permutations. Then the “best” permutations are selected again. The process continues until the length of the permutations reaches n.

The results of computer experiments on the exact and heuristic solution of the problem are described and analyzed.