Hyperbolic Magnetophotonic Crystals with Gyrotropic Layers. Dispersion Characteristics

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***Abstract*—We solved the problem of the Floquet-Bloch waves propagation in one-dimensional magnetophotonic crystal with hyperbolic and gyrotropic layers in the presence of a transverse magnetic field. New fundamental solutions of the Hill equation based on the third boundary-value problem with Cauchy boundary conditions are explicitly obtained in crystal layers. The dispersion equation is obtained in analytical form and its roots are found. The dispersion properties of hyperbolic media and magnetophotonic crystals with hyperbolic and gyrotropic layers are analyzed, and the main features of the propagation of TE and TM waves for two types of hyperbolicity in the presence of gyrotropy of the medium layers are elucidated.**

Keywords—hyperbolic media, gyrotropic media, Floquet theory, magnetophotonic crystal, Hill's equation

I. INTRODUCTION

Materials with unusual electrodynamic properties also known as metamaterials have attracted significant attention of researchers over last some decades [1-3]. Negative values of both material parameters and refraction index are principal features of metamaterials that open up a few interesting physical phenomena. For example negative refraction is positioned as a base of "superlens" developing for novel microwave and optical applications with imaging beyond the diffraction limit [4-6]. In general metamaterials are artificial media and appropriate technologies level is required for their optical applications. At the same time there are possibilities for negative refraction realization without the need for a negative permittivity and permeability. Uniaxially anisotropic media with certain parameters also demonstrate this unusual property. Moreover these media can be not only man-made but also natural with strong structural anisotropy and have only one negative principal material parameter [7, 8]. Opposite signs of diagonal elements in permittivity or permeability tensor result in hyperbolic-type dispersion and associated physical phenomena [9-15]. Hyperbolic media (HM) are characterized by material parameter tensor (e.g. permittivity tensor) with nonzero elements along principal coordinate axes:

$$\overset{\leftrightarrow}{\varepsilon} = \begin{Vmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{Vmatrix},$$

where one of elements ε_{xx}, ε_{yy} and ε_{zz} has opposite sign to other two diagonal components. Same signs of diagonal permittivities components correspond to usual anisotropic medium and lead to elliptical dispersion equation and isofrequencies surfaces in the wavenumber space are bounded and have ellipsoidal form (Fig. 1a). Dispersion equation of hyperbolic medium provides unbounded isofrequency surfaces in the form of two-sheeted (type I HM) hyperboloid or one-sheeted one (type II HM) (Fig. 1b, c).

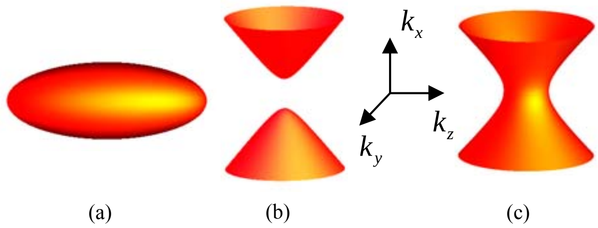

Fig. 1. Isofrequency surfaces for (a) anisotropic medium, (b) Type I hyperbolic medium, (c) Type II hyperbolic medium.

Two basic configuration of HM practical realization are multilayer periodic structures and wire media [16-18]. Layered metal-dielectric structures are used in various optical applications due to physical features of some metals and alloys that provide negative values of permittivity under certain conditions. Such structures allow designing of HM for desired frequency ranges and on the other hand to control their electrodynamic characteristics.

Magnetophotonic crystals (MPhC) are tunable multilayer structures that provide control of electromagnetic radiation by external magnetic field changing [19-22]. Using of hyperbolic properties in these structures opens up new possibilities for manipulation of the bulk and surface waves behavior [14]. In this work, we investigate the dispersion properties of MPhC that contains elements with gyrotropic and hyperbolic properties.

II. FLOQUET-BLOCH WAVES THEORY FOR MPHC

We consider the propagation of TE and TM electromagnetic waves in a 1D two-layer hyperbolic MPhC with discrete layers (Fig. 2). Each of the two MPhC layers with dimensions a and b per period $L = a + b$ is a gyrotropic hyperbolic medium, the dielectric $\vec{\varepsilon}_j(x)$ and magnetic $\vec{\mu}_j(x)$ permeabilities of which are tensor quantities of a standard form [23]:

$$\overset{\leftrightarrow}{\mu}_j(x) = \begin{Vmatrix} \mu_{xx} & -i\mu_a & 0 \\ i\mu_a & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{Vmatrix}, \quad \overset{\leftrightarrow}{\varepsilon}_j(x) = \begin{Vmatrix} \varepsilon_{xx} & -i\varepsilon_a & 0 \\ i\varepsilon_a & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{Vmatrix}. \quad (1)$$

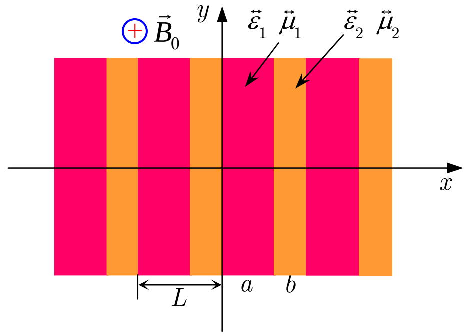

Fig. 2. Schematic of magnetophotonic crystal

We consider the two-dimensional case and H_z -polarization (TE waves) for a hyperbolic gyrotropic MPhC. Then for the component H_z we can write the Helmholtz equation in the form:

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_{yy}(x)}\frac{\partial^2 H_z}{\partial x^2}\right)+\frac{1}{\varepsilon_{xx}(x)}\frac{\partial^2 H_z}{\partial y^2}+k^2\mu_{zz}\varepsilon_{\perp}(x)H_z=0, \quad (2)$$

where

$$\varepsilon_{\perp xy}=\varepsilon_{xx}\left(1-\frac{\varepsilon_a^2}{\varepsilon_{xx}\varepsilon_{yy}}\right)=\varepsilon_{xx}\varepsilon_{\perp},$$

$$\varepsilon_{xx}(x)=\varepsilon_{\infty}\left(1-\frac{\omega_{pj}^2}{\omega^2-\omega_c^2}\right),\ \varepsilon_{yy}(x)=\varepsilon_{\infty}\left(1-\frac{\omega_{pj}^2}{\omega^2}\right),$$

$$\varepsilon_{aj}=-\frac{\omega_c\omega_{pj}^2}{\omega\left(\omega^2-\omega_c^2\right)},\ \varepsilon_{zz}(x)=\varepsilon_{\infty}\left(1-\frac{\omega_{pj}^2}{\omega^2}\right),$$

$$\omega_c=-\frac{e}{m_0}B_0,\ \omega_{pj}=\left(\frac{e^2N_{0j}}{m_0}\right)^{\frac{1}{2}}.$$

Here e and m_0 are negative charge and mass of the electron, ω_c is the gyromagnetic frequency, N_{0j} is electron concentration, ω_{pj} is the plasma frequency of the j^{th} layer.

It should be noted that in the case of E_z -polarization, it is necessary to use the principle of permutation duality to find a solution.

Equation (2) is simplified if we use separation of variables $H_z=X(x)Y(y)$:

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_{yy}}\frac{\partial X(x)}{\partial x}\right)+\left(k^2\mu_z\varepsilon_{\perp}-\frac{k_y^2}{\varepsilon_{xx}}\right)X(x)=0,$$

$$\frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2}=-k_y^2,\ Y(y)=e^{\pm ik_y y}. \quad (3)$$

The tangential component E_y of the electric field is found from Maxwell's equations through the H_z component:

$$E_y(x,y)=\frac{1}{ik\varepsilon_{\perp yx}(x)}\left(\frac{\partial H_z(x)}{\partial x}-k_y\frac{\varepsilon_a(x)}{\varepsilon_x(x)}H_z(x)\right)e^{ik_y y}, \quad (4)$$

Then it is necessary to solve the Hill's equation with periodic coefficients for each layer on the MPhC period:

$$\frac{\partial^2 X(x)}{\partial x^2}+\xi_j^2(x)X(x)=0, \quad (5)$$

where $\xi_j(x)$ is the transverse wave number in the MPhC defined by the expression $\xi_j(x)=\sqrt{k^2\mu_{zz}\varepsilon_{yy}\varepsilon_{\perp}-\frac{\varepsilon_{yy}}{\varepsilon_{xx}}k_y^2}$.
For infinite hyperbolic gyrotropic medium we obtain the dispersion relation:

$$\frac{k_x^2}{\varepsilon_{yy}}+\frac{k_y^2}{\varepsilon_{xx}}=k^2\mu_{zz}\varepsilon_{\perp}=k^2\mu_z\left(1-\frac{\varepsilon_a^2(x)}{\varepsilon_{xx}(x)\varepsilon_{yy}(x)}\right). \quad (6)$$

Any solution of equations (2), (3) and (5) for each layer on the MPhC period can be represented by a linear combination of two fundamental solutions $\psi_1(x)$, $\psi_2(x)$ of the Hill equation [24, 25] with mixed boundary Cauchy conditions [26], namely:

$$X(x)=A\psi_1(x)+B\psi_2(x) \quad (7)$$

Following the method developed in [27, 28] for gyrotropic MPhC, based on the third boundary-value problem with mixed boundary conditions, we can write out analytical expressions for the fundamental solutions $\psi_1(x)$ and $\psi_2(x)$ for two layers on the MPhC period ($a<x<L$):

$$\psi_1(x)=\begin{cases}\cos\xi_1 x+k_y\frac{\varepsilon_{a1}}{\varepsilon_{1xx}}\frac{\sin\xi_1 x}{\xi_1}, & 0<x<a\\ A\cos\xi_2(x-a)+B\frac{\sin\xi_2(x-a)}{\xi_2}, & a<x<L\end{cases} \quad (8)$$

$$\psi_2(x)=\begin{cases}\varepsilon_{\perp 1yx}\frac{\sin\xi_1 x}{\xi_1}, & 0<x<a\\ C\cos\xi_2(x-a)+D\frac{\sin\xi_2(x-a)}{\xi_2}, & a<x<L\end{cases} \quad (9)$$

where

$$A=\left(\cos\xi_1 a+k_y\frac{\varepsilon_{a1}}{\varepsilon_{1xx}}\frac{\sin\xi_1 a}{\xi_1}\right),$$

$$B=\begin{bmatrix}\left(k_y\dfrac{\varepsilon_{a2}}{\varepsilon_{2xx}}\cos\xi_1 a-\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\xi_1\sin\xi_1 a\right)+\\ +k_y^2\dfrac{\varepsilon_{a1}}{\varepsilon_{1xx}}\left(\dfrac{\varepsilon_{a2}}{\varepsilon_{2xx}}-\dfrac{\varepsilon_{a1}}{\varepsilon_{1xx}}\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\right)\dfrac{\sin\xi_1 a}{\xi_1}\end{bmatrix},$$

$$C=\varepsilon_{\perp 1yx}\frac{\sin\xi_1 a}{\xi_1},$$

$$D=\varepsilon_{\perp 1yx}\begin{bmatrix}\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\cos\xi_1 a+\\ +k_y\left(\dfrac{\varepsilon_{a2}}{\varepsilon_{2xx}}-\dfrac{\varepsilon_{a1}}{\varepsilon_{1xx}}\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\right)\dfrac{\sin\xi_1 a}{\xi_1}\end{bmatrix}.$$

To find the characteristic equation that determines the dispersion of waves in the MPhC it is necessary to use the Floquet theorem, namely:

$$\begin{gathered}\rho X_1(0)=X_2(0+L),\\ \rho\frac{1}{\varepsilon_{\perp 1yx}}\left(\frac{\partial X_1(0)}{\partial x}-k_y\frac{\varepsilon_{a1}}{\varepsilon_{1xx}}X_1(0)\right)=\\ =\frac{1}{\varepsilon_{\perp 2yx}}\left(\frac{\partial X_2(0+L)}{\partial x}-k_y\frac{\varepsilon_{a2}}{\varepsilon_{2xx}}X_2(0+L)\right)\end{gathered}. \quad (10)$$

From equations (10) we find the characteristic equation for determining the Floquet factor ρ :

$$\rho+\frac{1}{\rho}=\frac{1}{\varepsilon_{\perp 2yx}}\left[\psi_2'(L)-k_y\frac{\varepsilon_{a2}}{\varepsilon_{2x}}\psi_2(L)\right]+\psi_1(L). \quad (11)$$

The solution of the quadratic equation (11) taking into account the analytical expressions for the fundamental solutions $\psi_1(x)$, $\psi_2(x)$ of the Hill equations (5) leads to the following dispersion equation for TE waves in hyperbolic gyrotropic MPhC:

$$\cos K_{TE}L=\cos\xi_1 a\cos\xi_2 b-\\ -\frac{1}{2}\begin{bmatrix}\dfrac{\varepsilon_{\perp 1yx}}{\varepsilon_{\perp 2yx}}\dfrac{\xi_2}{\xi_1}+\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\dfrac{\xi_1}{\xi_2}+\\ +\dfrac{k_y^{\,2}}{\xi_1\xi_2}\dfrac{\varepsilon_{\perp 1yx}}{\varepsilon_{\perp 2yx}}\left(\dfrac{\varepsilon_{a2}}{\varepsilon_{2xx}}-\dfrac{\varepsilon_{\perp 2yx}}{\varepsilon_{\perp 1yx}}\dfrac{\varepsilon_{a1}}{\varepsilon_{1xx}}\right)^2\end{bmatrix}\sin\xi_1 a\sin\xi_2 b. \quad (12)$$

III. Analysis of Results

An analysis of dispersion equation (12) shows that in the considered hyperbolic gyrotropic MPhC there are two types of solutions in the transmission zones corresponding to bulk and surface waves. However the presence of two properties (hyperbolicity and gyrotropy) in the medium of the MPhC layers introduces certain differences in the existence of these types of waves in comparison with traditional photonic crystals and MPhCs.

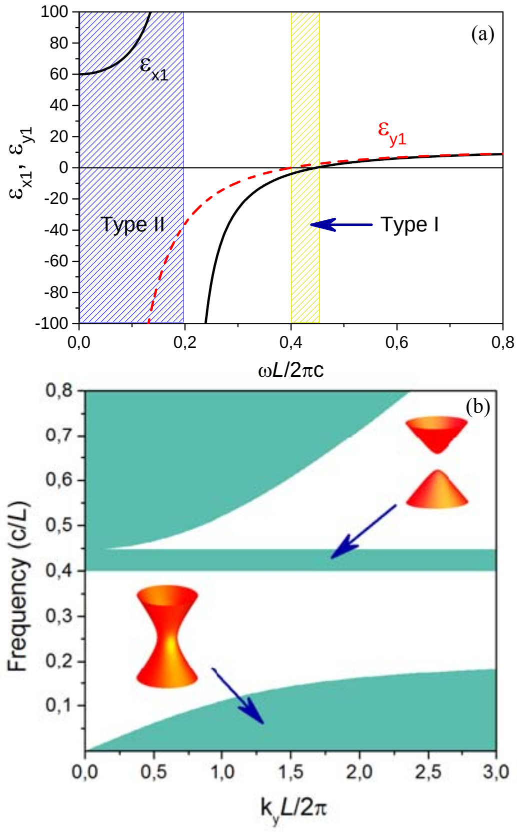

Fig. 3. (a) Permittivity tensor components; (b) dispersion diagram of hyperbolic medium.

The presence of gyrotropy ($\varepsilon_{aj}\neq 0$) leads to nonreciprocal effects. For example, the field structure for the forward and backward waves for the MPhC is different [20, 22, 27]. To clarify the features of the propagation of bulk and surface waves in MPhC, we first consider the dispersion relation for a hyperbolic gyrotropic medium (6) taking into account the frequency dependence of the elements of the dielectric constant tensor $\vec{\varepsilon}$.

Fig. 3a shows components ε_{xx} and ε_{yy} of permittivity tensor versus normalized frequency $\omega L/2\pi c$. Such parameters are used for calculations: $\omega_p=0.4$; $\omega_c=0.2$. Shaded bands correspond to different hyperbolic regimes.

Dispersion diagram for hyperbolic medium is shown in Fig. 3b. Here shaded and unshaded areas indicate transmission and forbidden zones respectively with real and complex values of Bloch wavenumber K_{TE} . It is apparent that bans of hyperbolic regimes correspond to two transmission zones in dispersion diagram. Naturally these zones exist for any values of wavenumber k_y .

Let us further consider the dispersion properties of a hyperbolic gyrotropic medium. Fig. 4a shows non-diagonal element ε_a of permittivity tensor versus normalized frequency. Gyromagnetic frequency ω_c divides the areas of positive and negative values of ε_a . Correspondence dispersion diagram is shown in Fig. 4b. Naturally gyrotropy property results in changing of transmission and forbidden zones configurations on dispersion diagram. Thus by means

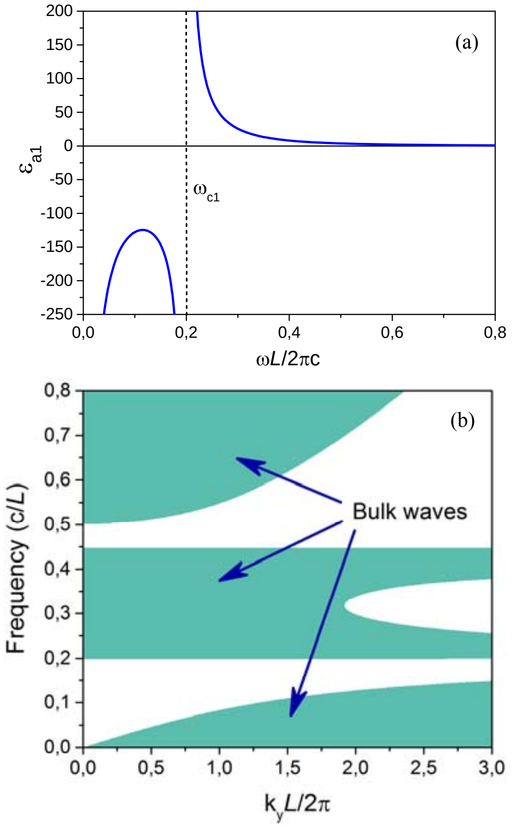

Fig. 4. (a) Nondiagonal permittivity tensor component; (b) dispersion diagram of hyperbolic gyrotropic medium.

of bias magnetic field changing one can control spectral and dispersion properties of hyperbolic gyrotropic medium.

Let us consider MPhC that contains anisotropic (nonhyperbolic) and isotropic dielectric layers on structure period ($\varepsilon_{a1} = \varepsilon_{a2} = 0$). Fig. 5 shows dispersion diagram for such parameters: $a/L = 0.8$; $\varepsilon_{\infty} = 12$; $\omega_{p1} = 0.4$; $\omega_{c1} = 0.2$; $\omega_{p2} = 0$; $\mu_{zz1} = \mu_{zz2} = 1$. Horizontal dashed lines indicate edges of hyperbolic regimes frequency bands. Low-frequency blue line corresponds to Type I HM. High-frequency red lines correspond to Type II HM. It is apparent that MPhC periodicity results in splitting of transmission zones corresponding different hyperbolic regimes ($\varepsilon_{xx}\varepsilon_{yy} < 0$) into a large number of transmission and forbidden zones. The presence of such fine structure in the dispersion diagram is associated with the zeros of the function $\sin\left(a\sqrt{\varepsilon_{y1}\left(k^2\mu_{z1}\varepsilon_{x1}\varepsilon_{\perp 1} - k_y^{\;2}\right)/\varepsilon_{x1}}\right)$ in dispersion equation (12). Surface waves in the transmission zones are observed for the hyperbolic layers when the condition $\varepsilon_{y1}\left(k^2\mu_{z1}\varepsilon_{x1}\varepsilon_{\perp 1} - k_y^{\;2}\right)/\varepsilon_{x1} < 0$ is satisfied. Such narrow zone is marked by arrow in Fig. 5.

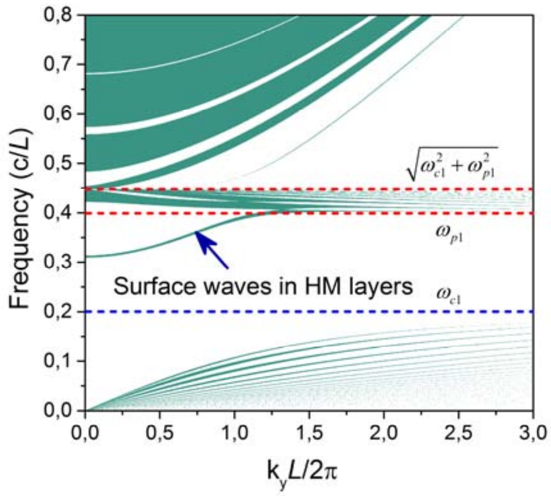

Fig. 5. Dispersion diagram of MPhC with hyperbolic layers.

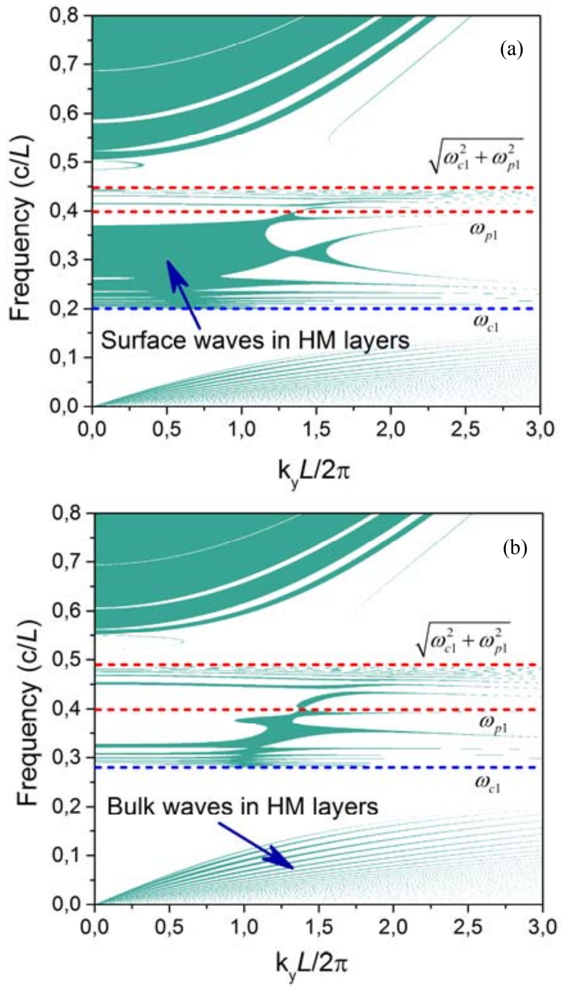

Fig. 6. Dispersion diagrams of MPhC with hyperbolic gyrotropic layers.

Fig. 6a and 6b show dispersion diagrams of hyperbolic gyrotropic MPhC ($\varepsilon_{a1} \neq 0$; $\varepsilon_{a2} = 0$) for two values of gyromagnetic frequency ($\omega_{c1} = 0.2$ and 0.28 respectively). Other parameters are equal to one's associated with Fig. 5. As expected gyrotropy of MPhC layers result in additional splitting of transmission zones in dispersion diagram. Changing of external transversal magnetic field allows controlling dispersion characteristics of hyperbolic gyrotropic MPhC.

IV. Conclusion

We developed an analytic theory of multifunctional systems with hyperbolic and gyrotropic layers based on Floquet-Bloch waves to determine the eigenfunctions and dispersion characteristics for TM and TE waves. We are performed an analysis of the hyperbolic media of the first and second types based on dispersion diagrams and established the regions of existence of bulk and surface waves. The introduction of gyrotropy into a hyperbolic layer leads to a distortion of dispersion diagrams, mainly for a hyperbolic medium of the first type. Passbands and band gaps of MPhC can be controlled by external magnetic field changing. A comparison of hyperbolic MPhC and continuous hyperbolic medium is carried out. It is shown that the dispersion diagrams of the hyperbolic gyrotropic MPhC are split into a number of bands for both hyperbolic regimes. A limited number of bandwidths is associated with hyperbolic layer thickness, upon changing which the bandwidths degenerate into lines. The developed theory of Floquet-Bloch waves for hyperbolic and gyrotropic media and MPhCs allows us to construct a theory of a wide class of magnetic field-controlled Bragg waveguide structures of the terahertz range for various applications [29-31].

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