Stochastic Model and Method of Optimizing the Operating Modes of a Water Network with Hidden Leaks

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Стохастична Модель та Метод Оптимізації Режимів Роботи Водопровідної Мережі із Прихованими Витоками

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***Abstract*—This paper is devoted to the development of a stochastic model and a method for optimizing the operating modes of a water supply network with leaks.**

***Анотація*—Дана робота присвячена розробці стохастичної моделі та методу оптимізації режимів роботи водопровідної мережі з витоками.**

***Keywords*—water supply network; leaks; optimization; stochastic model**

***Ключові слова*—водопровідна мережа; витоки; оптимізація; стохастична модель**

One of the most acute problems in water supply systems is the leakage of water from damaged pipelines and stop valves due to their wear. The most part of leakage are hidden leaks that do not reach the surface of the earth. Fistulas, cracks and minor defects in the pipeline are difficult to identify and even harder to repair without major repairs.

Except water loss, leaks often destroy underground utilities, cause failures of bridge and pavement covers, undermining of building foundations, erosion of cable networks, subsidence of tramways, etc., which often leads to accidents and, in their turn, require costs to repair damage; the electricity costs are increasing to compensate consumers the undelivered volume of water and reagents for its purification.

Another negative effect of the leakage is the drop in pressure in the water supply system. Forced increase in pressure in the pipeline to compensate such losses entails the increase of the electricity consumption. At the same time, the increase in pressure only increases the leakage.

All existing water supply networks have unidentified and irreparable leaks, therefore, the methods of hydraulic calculation of WS are not adequate, since the flow rate is the function of pressure [1].

In mathematical modeling of water supply networks (WS), the difficulties arise due to the huge dimension of real WS, limited information resources and operational data, which does not allow to assess the parameters of the technical equipment and the structure of the WS quite adequately [2].

The real WS is determined by the graph which reflects its structure.

Examine the WS, which operates with *N* PS. To represent the structure of the WS in the form of the orgraph *G(V,E)*, where *V* is the set of vertexes, *E* is the set of arcs (*e*=Card(*E*), *v*=Card(*V*), the real WS is added by the zero vertex and fictitious chords connecting the zero vertex with all the inputs and outputs of the network. For the mathematical formulation of the problem, the following WS coding is performed: the tree of the graph is chosen in such a way that the dummy network sections become the chords. Herewith the actual sections become the chords partly, and partly the branches of the tree. The branches of the tree with the pump has number 1, the rest of the branches – from 2 to *v*-1, the chords of the real sections – from v to $v+\eta_2-1$, dummy ones with given node costs – from $v+\eta_2$ to $v+\eta_2+\xi_1-1$, dummy ones with given pressure – from $v+\eta_2+\xi_1$ to *e*, where η_2 is the number of chords of the real sections, ξ_1 is the number of outputs with given nodal costs.

The stochastic model of quasi-stationary modes of operation of the WS has the form [3]:

$$\Omega : \underset{\omega}{M}\Big(sgn\, q_r(\omega) S_r(q_i(\omega)) q_r^2(\omega) + \sum_{i=1}^{v-1} b_{1ri}\, sgn\, q_i(\omega) S_i(q_i(\omega)) q_i^2(\omega)\Big) = 0,$$

$$(r = v, \ldots, v+\eta_2-1), \tag{1}$$

$$\underset{\omega}{M}\left(h_r^c(\omega) - h_{NS1}(\omega) + \sum_{i=1}^{v-1} b_{1ri}\left(sgn\, q_i(\omega) S_i(q_i(\omega)) q_i^2(\omega) + h_i^g\right)\right) = 0,$$

$$(r = v+\eta_2, \ldots, v+\eta_2+\xi_1-1), \tag{2}$$

$$\underset{\omega}{M}\Big(sgn\, q_r(\omega) S_r\ (q_i(\omega)) q_r^2(\omega) + h_r^g - h_{NSn}(\omega) + h_{NS1}(\omega) + \sum_{i\in M_1} b_{1ri}\left(sgn q_i(\omega) S_i(q_i(\omega)) q_i^2(\omega) + h_i^g\right)\Big) = 0,$$

$$(r = v+\eta_2+\xi_1, \ldots, e; \quad n = 1, \ldots, N), \tag{3}$$

$$\underset{\omega}{M}(q_i(\omega)) = \underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1} b_{1ri} q_r(\omega) + \sum_{r=v+\eta_2}^{e} b_{1ri} q_r(\omega) \right),$$

$$(i = 1, \ldots, v-1), \tag{4}$$

$$P\left(h_i^c(\omega) \ge h_i^+\right) \ge \alpha, \quad (\alpha \cong 1),$$

$$i = (v+\eta_2, \ldots,\ v+\eta_2+\xi_1-1), \tag{5}$$

$$P\left(q_i(\omega) > 0\right) \cong \beta, \quad (\beta \cong 1), \quad i \in N, \tag{6}$$

where random variables characterize: $q_i(\omega)$ – water flow in the *i*-th section of the pipeline; $h_r^c(\omega)$ – free pressure in the *r*-th node of the WS ($r = v+\eta_2, \ldots, v+\eta_2+\xi_1-1$); h_r^+ – the minimum allowable pressure in the *r*-th node of the WS; $S_i(q_i(\omega))$ – value of the hydraulic resistance of the *i*-th pipeline section ($i \in M$); h_i^g – geodetic mark of the *i*-th section of the pipeline ($i \in M$); b_{1ri} – element of the cyclomatic matrix; $h_{NSn}(\omega)$ – pressure at the exit of the *n*-th PS.

In real WS it is almost impossible to carry out operational control of its parameters in all the vertexes and arcs of the graph of the network due to the physical limitations of the means of remote monitoring and control. Therefore, we will control the regime parameters only at the dictating points of the WS, which will be the vertexes of the graph of the equivalent model of the WS. As the dictating points, we will take several nodes of the WS, in which there is maximum excess pressure.

The parameters of the quasi-stationary operating modes of the equivalent water supply network (EWS) must coincide with the actual ones at the dictating points of the WS.

Consider the EWS, the structure of which is represented as an orgraph *G**(*V**, *E**), where: *V** is the set of vertexes (dictating points), *E** is the set of arcs.

In the WS with leaks, the water flow depends on the pressure, i.e., the greater the pressure, the greater the amount of leakage, and therefore the water flow.

While optimizing of the operating modes of the water supply network with hidden leaks, the problem of constructing of the stochastic model of the quasi-stationary operation modes of the WS which includes the identification of the state, parameters and structure of the WS is solved [4].

The initial data for the construction of the stochastic model of the quasi-stationary modes of operation of EWS are:

• graph EWS;

• pressures in the nodes of the EWS (dictating points of the WS);

• costs and pressures at the outputs of PS.

Mathematical formulation of the problem of the construction of the adequate model of the EWS: it is to be minimized at the time interval [0, *T*] the sum of the squares of the deviations of the pressures in the nodes of the EWS and corresponding nodes of the initial WS, the sum of the squares of the deviations of the pressures and costs at the outputs of the pump stations for EWS and initial WS with the limitations Ω.

The problem of the identification of the structure of EWS:

$$\underset{\omega}{M}\left(\sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1} \left(h_j^c(\omega) - h_j^{c*}(\omega)\right)^2 + \sum_{i=1}^{n}\left(h_{NSi}(\omega) - h_{NSi}^*(\omega)\right)^2 + \sum_{i=1}^{n}\left(q_{NSi}(\omega) - q_{NSi}^*(\omega)\right)^2 \right) \to \min_{G(V,E)\in\Omega}, \tag{7}$$

where: $h_j^{c*}(\omega)$ – pressures in the nodes of EWS, $h_{NSi}^*(\omega)$, $q_{NSi}^*(\omega)$ – pressures and costs at the outputs of the pumping stations of the EWS.

The problem of the identification of the parameters of the EWS:

$$\underset{\omega}{M}\left(\sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega)-h_j^{c*}(\omega)\right)^2+\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+\right.$$
$$\left.+\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2\right)\to\min_{S_i^*\in E^*,E^*\in\Omega}. \quad (8)$$

The problem of the identification of the state of the EWS:

$$\underset{\omega}{M}\left(\sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega)-h_j^{c*}(\omega)\right)^2+\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+\right.$$
$$\left.+\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2\right)\to\min_{q_i\in V^*}, \quad (9)$$

where $q_i\in V^*$ – scores of end-consumers of water.

The problem (1) – (7) belongs to the class of nonlinear problems of stochastic programming of *M*-type with statistical (1) – (4) and probabilistic (5), (6) conditions.

The model (1) – (9) can be used for planning of the modes of operation of WS for twenty-four hours:

$$\underset{\omega}{M}\sum_{k=1}^{24}\left(\underset{\omega}{M}\left(\sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega,k)-h_j^{c*}(\omega,k)\right)^2+\right.\right.$$
$$+\sum_{i=1}^{n}\left(h_{NSi}(\omega,k)-h_{NSi}^*(\omega,k)\right)^2+ \quad (10)$$
$$\left.\left.+\sum_{i=1}^{n}\left(q_{NSi}(\omega,k)-q_{NSi}^*(\omega)\right)^2\right)\right)\to\min_{\sum_{i=1}^{n}q_{NSi}\in\Omega}.$$

As a result of solving the problem of load distribution between the pumping stations for the WS with leaks, we obtain pressures and costs at the outputs of PS, at which electric power costs and the sum of the squares of the deviations of free pressures in the nodes of the EWS will be minimal.

The mathematical model of the problem of load distribution between the pumping stations for the WS with leaks is the following:

$$\underset{\omega}{M}\left(\lambda\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+\right.$$
$$\left.(1-\lambda)\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2\right)\to\min, \quad (11)$$

$$\Omega:\underset{\omega}{M}\left(sgn\,q_r(\omega)S_r(q_i(\omega))q_r^2(\omega)\right.+$$
$$\left.+\sum_{i=1}^{v-1}b_{1ri}sgn\,q_i(\omega)S_i(q_i(\omega))q_i^2(\omega)\right)=0,$$
$$(r=v,\ldots,v+\eta_2-1), \quad (12)$$

$$\underset{\omega}{M}\left(h_r^c(\omega)-h_{NS1}(\omega)+\sum_{i=1}^{v-1}b_{1ri}(sgn\,q_i(\omega)S_i(q_i(\omega))q_i^2(\omega)+h_i^g)\right)=0,$$
$$(r=v+\eta_2,\ldots,v+\eta_2+\xi_1-1), \quad (13)$$

$$\underset{\omega}{M}\left(sgn\,q_r(\omega)S_r\;(q_i(\omega))q_r^2(\omega)+h_r^g-h_{NSn}(\omega)+\right.$$
$$\left.+h_{NS1}(\omega)+\sum_{i\in M_1}b_{1ri}\left(sgnq_i(\omega)S_i(q_i(\omega))q_i^2(\omega)+h_i^g\right)\right)=0,$$
$$(r=v+\eta_2+\xi_1,\ldots,e;\quad n=1,\ldots,N), \quad (14)$$

$$\underset{\omega}{M}(q_i(\omega))=\underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1}b_{1ri}q_r(\omega)+\sum_{r=v+\eta_2}^{e}b_{1ri}q_r(\omega)\right),$$
$$\left(i=1,\ldots,v-1\right), \quad (15)$$

$$P\left(h_i^c(\omega)\ge h_i^+\right)\ge\alpha,\quad(\alpha\cong1),$$
$$i=(v+\eta_2,\ldots,\;v+\eta_2+\xi_1-1), \quad (16)$$

$$P\left(q_i(\omega)>0\right)\cong\beta,\quad(\beta\cong1),\quad i\in N, \quad (17)$$

$$\underset{\omega}{M}\left(q_{i\,ut}(\omega)-\frac{\sqrt{2gh_i^c(\omega)}\cdot\pi d_{i\,ut}^2}{4}\right)=0,$$
$$(i=v+\eta_2,\ldots,\;v+\eta_2+\xi_1-1), \quad (18)$$

$$\underset{\omega}{M}\left(q_i(\omega)-q_{i\,ut}(\omega)-q_{i0}(\omega)\right)=0,$$
$$(i=v+\eta_2,\ldots,\;v+\eta_2+\xi_1-1), \quad (19)$$

where: $q_{i\,ut}(\omega)$ is the leakage value in the *i*-th node of the WS, $q_{i0}(\omega)$ is the rate of water flow without leakage in the *i*-th node of the WS, $d_{i\,ut}$ is the diameter of the leakage in the *i*-th node of the WS, coefficient $\lambda\in[0,1]$.

For the first time, the method of the optimization of the operation modes of the WS with leaks has been offered. At the moment, this approach allows to describe the WS with leaks more adequately. There are the results of solving the problems with leaks in this report.

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