Excitation of Surface Plasmons by Localized Transient Sources

Nadiia P. Stognii [1,2] , Nataliya K. Sakhnenko [1]

[1] Department of Higher Mathematics, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine

[2] Laboratory of Micro and Nano Optics, Institute of Radio Physics and Electronics NASU Kharkiv, Ukraine

e-mail: nstognii@gmail.com

Abstract—**Theoretically investigate the excitation of plasmon resonances of metallic nanowires and nanoshell by external transient pulses is presented. To solve the problem, we used a accurate mathematical approach based on the Laplace transform, which allowed us to obtain an analytic representation of the solution. Finding the inverse transformation is based on the assessment of residues at singular points that correspond to the plasmon resonances of our structures**.

Keywords— complex source point; plasmon resonances; localized surface plasmons

I. INTRODUCTION

Amazing progress in invention have spawned a extensive interest in nanotechnology which includes a large diapason of disciplines. Particularly, metal nanoobjects are the topic of measureless concernment in last years due to the potential of a strong light localization beyond the diffraction limit via the stimulation of surface and localized plasmons [1]. For example in the paper of M. Stockman [2] resumes new achievements in nanoplasmonics. This field of investigation has newly exhibited the practice demonstration of many new and effective conceptions and appeared as an extremely budding development with several basic regions of application: information developments [3, 4], life sciences [5, 6] and security.

The interaction of metallic nanoobjects with light leads to the excitation of localized and delocalized surface plasmons (SPs) with different mode frequencies in the form of scattering electromagnetic waves or oscillations that initiated by collective oscillations of the electrons. Many metals have a negative permittivity at optical frequency as the plasma frequency of the conductance electron gas lies in this domain. The resonances properties of nanoobjects with dimensions down to 2 nm can be considered using classical Maxwell's theory [7]. When the frequency illumination of passes nearby the plasma frequency of the metal the real part of the dielectric constant becomes negative and resonances of the SPs can be excited. The resonances modes of coupled nanostructures can be investigated as bonding and antibonding combinations of SPs of single structure with different eigenfrequencies and field patterns [8-13].

We investigate the excitation of SPs on metal wire and shell by a non-stationary pulsed source. The goal of this investigation is to gain a fundamental understanding and to describe theoretically details of the transient dynamics of the plasmon excitation on nanowire and nanoshell by a complex source point. The excited plasmon dynamics in the metallic nanowires and generated fields will be characterized by means of accurate mathematical tool that based on the analytic solution in the Laplace transformation range and its rigorous inversion into time domain by virtue of residues evaluation at singular points that coincides with the eigenfrequency of excited localized plasmons of the our structure. This problem has applications in the most growing areas of nanophotonics and nanotechnologies which demand devices that can generate coherent plasmonic fields in subwavelength scale. This approach, represented by C. Baum in the 1970-s, has been felicitously applied by authors to a diversity of 1-3D time domain problems with non-stationary media [14-16].

II. STATEMENT OF THE PROBLEM AND SOLUTION

In this article, we consider problem of the excitation of localized SPs on metal wire and metal shell by transient external beam. The complex source point is used to model an incident non-stationary beam. The model is based on the idea of analytic continuation of the functions of real point source in a complex space [15]. H-polarized fields will be considered.

In the first, assume that the source is at a point with the real coordinates represented by a radius vector $\vec{\rho}_s$. Using the expression for the 2D Green's function in the time domain

$$g(t,t',\vec{\rho},\vec{\rho}_s)=\frac{1}{2\pi}\frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}_s|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}_s|^2/c^2}}, \qquad (1)$$

here $\theta(..)$ is the unit Heaviside function, we can obtain the formula for the magnetic field in free space

$$h_0(t,\vec{\rho})=\frac{1}{2\pi}\int_0^{\infty}dt'\int_0^{\infty}\rho' d\rho'\int_0^{2\pi}d\varphi'\frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}'|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}'|^2/c^2}}\varepsilon_0\frac{\partial}{\partial t'}\hat{j}(t',\vec{\rho}') \quad (2)$$

If the source is located in the real point of the space $\hat{j}(t,\vec{\rho})=j(t)\delta(\vec{\rho}-\vec{\rho}_s)/|\vec{\rho}-\vec{\rho}_s|$, here $\delta(..)$ is the Dirac delta function, that

978-1-4673-8841-2/16/$31.00 ©2016 IEEE

$$h_0(t,\vec{\rho}) = \frac{\varepsilon_0}{2\pi}\int_0^{\infty} dt' \frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}_s|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}_s|^2/c^2}} \frac{\partial}{\partial t'} j(t'). \quad (3)$$

Laplace transform of the equation (3) has the form

$$H_0(p,\vec{\rho}) = \frac{\varepsilon_0}{2\pi} K_0\left(\frac{p}{c}|\vec{\rho}-\vec{\rho}_s|\right) pJ(p) \quad (4)$$

here $J(p)$ is image of the function $j(t)$, $H_0(p,\vec{\rho})$ is image of the function $h_0(t,\vec{\rho})$, $K_0(..)$ is modified Bessel function of the 2-th kind. Let us further assume that the vector $\vec{\rho}_s$ is a complex $\vec{\rho}_s = \vec{\rho}_{cs}$, which $\vec{\rho}_{cs}$ is given by

$$\begin{cases} x_{cs} = x_0 + ib\cos\beta \\ y_{cs} = y_0 + ib\sin\beta, \end{cases} \quad (5)$$

x_0, y_0, b, β are real numbers. For this problem, the separable distance between the point source and the observation point is complex too $|\vec{\rho}-\vec{\rho}_{cs}| = \sqrt{(x-x_{cs})^2+(y-y_{cs})^2}$.

Let us consider the transient dynamics of localized SPs excited by pulsed complex source point on the metal wire (Fig. 1 (a)).

Dispersive medium is characterized by the Drude model

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega - i\gamma_e)}, \quad (6)$$

here ω_{pe} is frequency of plasma, γ_e is the absorption of material. The susceptibility of a plasma in the frequency domain is of the form

$$\chi(\omega) = -\frac{\omega_{pe}^2}{\omega(\omega - i\gamma_e)} \quad (7)$$

The susceptibility of the medium in the time domain can be obtain using the inverse Fourier transform

$$\chi(t) = -\frac{\omega_{pe}^2}{\gamma_e}(1-e^{-\gamma_e t})\Theta(t) \quad (8)$$

Find a solution to the problem for the metal wire in the following form:

$$H = \sum_{m=-\infty}^{\infty} A_m I_m(\bar{n}_p q\rho) e^{im(\varphi-\varphi_{cs})}, \ \rho < a, \quad (9)$$

$$H = \sum_{m=-\infty}^{\infty} B_m K_m(q\rho) e^{im(\varphi-\varphi_{cs})}, \ \rho > a,$$

here $I_k(..)$, $K_k(..)$ are modified Bessel functions of the 1-th and 2-th kind, respectively. Representation of the external field in the form of expression $K_k(..)$ provides the radiation conditions at infinity.

Applying the boundary conditions, which represent continuity of the tangential field component to the cylindrical boundary $\rho = a$, come up to the system (provided that the source is located outside, $\rho_0 > a$) to determine the unknown coefficients A_m and B_m

$$A_m I_m(\bar{n}_p qa) - B_m K_m(qa) = I_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \quad (10)$$

$$A_m I'_m(\bar{n}_p qa) - \bar{n}_p B_m K'_m(qa) = \bar{n}_p I'_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \quad (11)$$

For metal nanoshell (Fig. 1 (b)) the solution can be found in the form

- if $\rho < b$: $H^{(1)} = \sum_{m=-\infty}^{\infty} A_m^{(1)} I_m(q\rho) e^{im(\varphi-\varphi_{cs})}$, (12)
- if $b < \rho < a$,

$$H^{(2)} = \sum_{m=-\infty}^{\infty} \left(A_m^{(2)} I_m(n_p q\rho) + B_m^{(2)} K_m(n_p q\rho)\right) e^{im(\varphi-\varphi_{cs})}, \quad (13)$$

- if $a < \rho$: $H^{(3)} = \sum_{m=-\infty}^{\infty} B_m^{(3)} K_m(q\rho) e^{im(\varphi-\varphi_{cs})}$. (14)

Similarly, after the application of the boundary conditions, we determine the unknown coefficients of the system

$$A_m^{(1)} I_k(qb) - A_m^{(2)} I_m(n_p qb) - B_m^{(2)} K_m(n_p qb) = 0 \quad (15)$$

$$n_p A_m^{(1)} I'_k(qb) - A_m^{(2)} I'_m(n_p qb) - B_m^{(2)} K'_m(n_p qb) = 0 \quad (16)$$

$$A_m^{(2)} I_m(n_p qa) + B_m^{(2)} K_m(n_p qa) - B_m^{(3)} K_m(qa) = I_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \quad (17)$$

$$A_m^{(2)} I'_m(n_p qa) + B_m^{(2)} K'_m(n_p qa) - n_p B_m^{(3)} K'_m(qa) = n_p I'_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \quad (18)$$

The coefficients of expansions have poles corresponding to plasmon resonance and the branch point. The eigenfrequencies of the excited plasmon resonances are complex $\omega = \omega' + i\omega''$. The dependence of the time of the transient pulse source is

$$j(t) = e^{i\omega_0 t}\left[\Theta(t) - \Theta(t-\tau)\right] \quad (19)$$

III. Numerical Results and Disscusion

In this paper, we consider normalized plasma frequency $w_p = \omega_p a/c$, $w_p = 1$, $\gamma = 10^{-3}\cdot w_p$, $b/a = 0.5$. Fig. 2 shows the scattering cross section (SCS) of metallic nanowire (dashed line) and metallic nanoshell (solid line). In the spectrum of the metal wire is present dipole ($\mathrm{Re}(ka) = 0.63$) and quadrupole ($\mathrm{Re}(ka) = 0.675$) plasmons (see Fig. 3 ((a) and (b))). The directions of dipole moments are present.

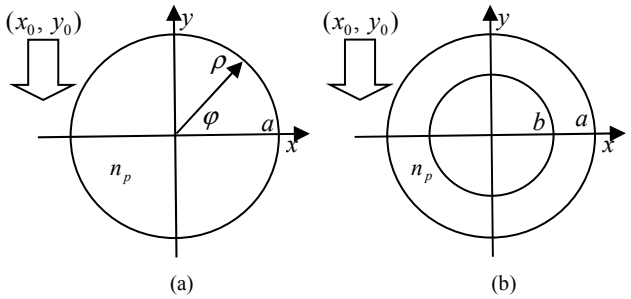

Fig. 1. Schematic diagram of the investigated phenomenon.

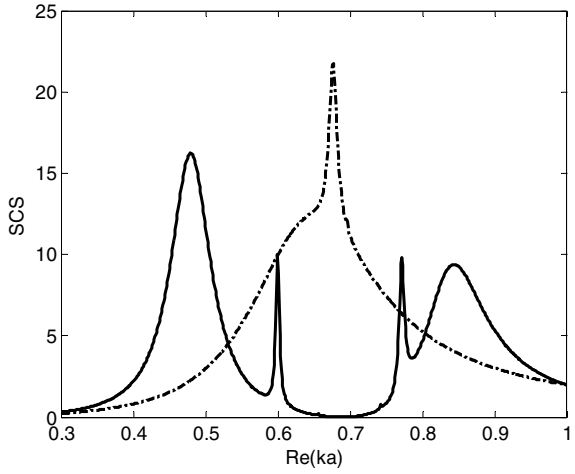

Fig. 2. SCS of the metal nanowire (dashed line) and metal nanoshell (solid line).

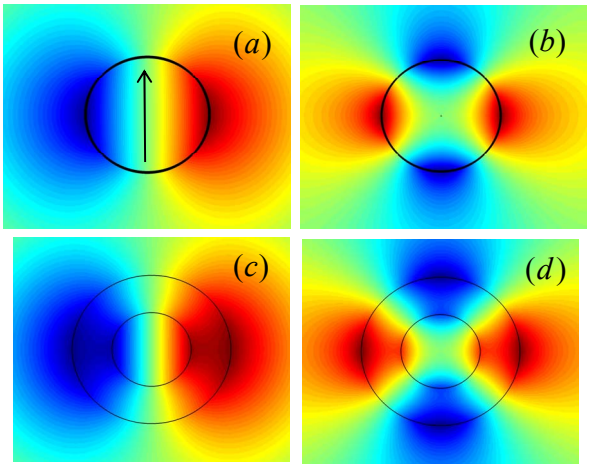

Fig. 3. The near-field distributions of metal wire and metal shell: (a) dipole SP; (b) quadrupole SP; (c) even dipole SP; (d) odd dipole SP.

In the spectrum of metal nanoshell two types plasmons are present: even and odd (see Fig. 3 (c) and (d))). Odd plasmon resonances are shifted to lower frequencies ($\mathrm{Re}(ka) = 0.48$ and $\mathrm{Re}(ka) = 0.6$) and even plasmon resonances are shifted to the region of higher frequencies ($\mathrm{Re}(ka) = 0.77$ and $\mathrm{Re}(ka) = 0.83$).

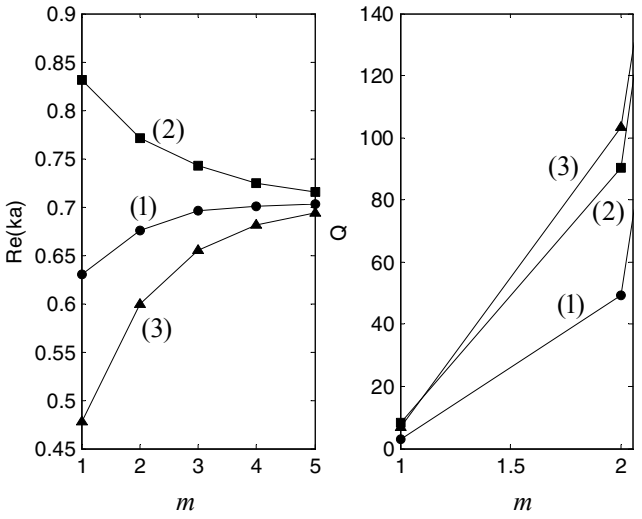

Fig. 4. Eigenfrequency and Q-factor of metal wire (1) and metal shell ((2) for even SPs and (3) for odd SPs).

Fig. 4 describes the real parts of eigenfrequencies and quality factors of the coupled dipole SPs of metal wire (1) and metal shell ((2) for even dipole SPs and (3) for odd SPs) for different values of angular variations of the field (m). Maximal values of quality factor is seen for localized SPs of metallic nanoshell.

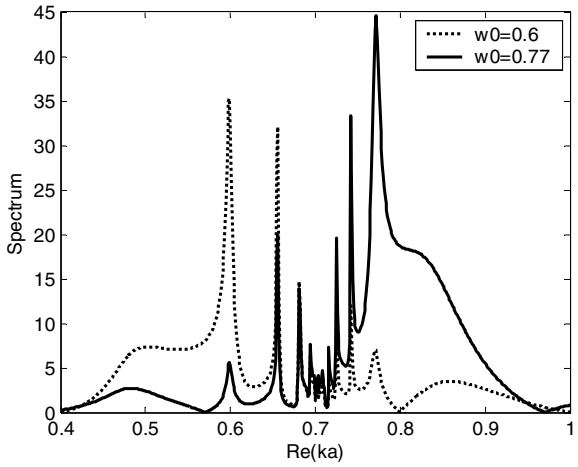

Fig. 5. Spectral density of the field in metal nanoshell ($w_p = 1$, $\tau = 2\pi a/c$).

Fig. 5 presents the spectrum density of the field in the metal shell at its excitation complex source point. We are using the normalized frequency of the source $w_0 = \omega_0 c/a$. The solid line corresponds to the case when the source's frequency coincides with the real part of the frequency of even dipole plasmon and the dashed line corresponds to odd dipole plasmon. The duration of the pulse is $\tau = 2\pi a/c$. In this case, unlike the case of the incident harmonic wave (Fig. 2), in the spectrum multiple peaks that associated with higher plasmons are present.

Fig. 6 shows snapshots of the absolute value of the field of the non-stationary excited localized SPs travelling about the metal nanoshell. In this case eigenfrequency of the external pulsed beam is $\omega_0 = 0.83$. We see that pulse of of the simultaneously localized plasmons gives growth to asymmetrical running field distribution. The field distribution on the surface of the metal shell is more complicated because in this case excited both even and odd plasmons with different quality factors and distribution fields.

Fig. 7 presents the transient dynamic of the absolute value of the magnetic field of the nanoshell for two values of the pulse duration. We see that with increasing duration of the pulse the intensity of the field increases.

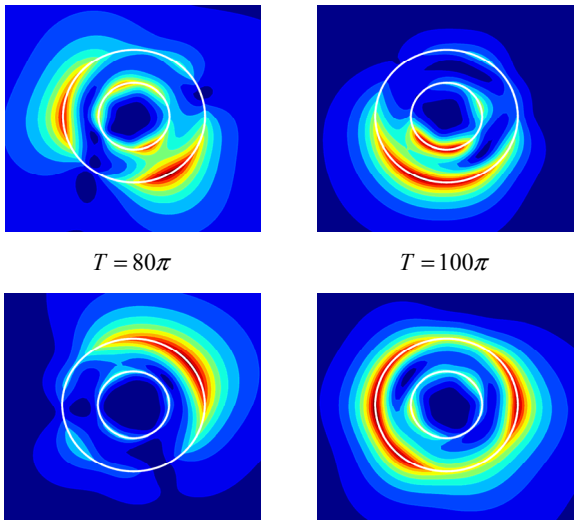

Fig. 6. Snapshots of the dynamic SP propagation along the metal shell for $\omega_0 = 0.83$ and different values of normalized time, T ($T = tc/a$, where t is real time, a is radius of nanowire).

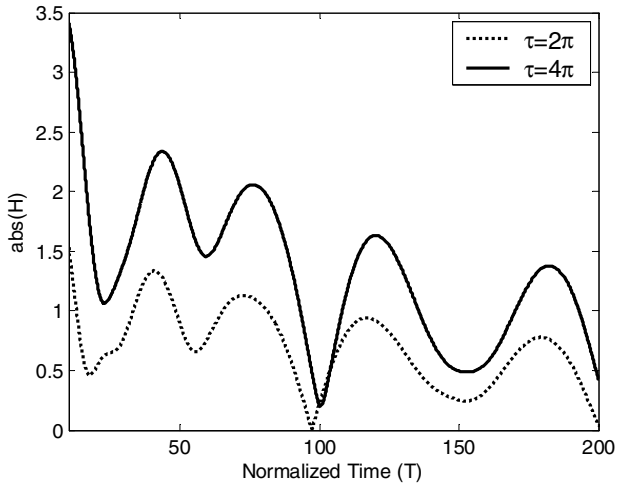

Fig. 7. The evolution in time of the magnetic field of the metal nanoshell for $w_0 = 0.8318$.

CONCLUSIONS

In this paper we constructed an analytic solution, in the format of the Laplace transformation, of the problem of transient plasmon excitation to complex pulse source. For the simulation of transient external beam we used the concept of complex source point. The model is based on the idea of analytic continuation of the functions of an real source point in a complex space. This kind of source is a very useful model for describing the excitation of non-stationary waves. It is shown that pulse of the simultaneously localized plasmons gives growth to asymmetrical running field distribution.

REFERENCES

[1] J.K. Kitur, G. Zhu, Yu.A. Barnakov and M.A. Noginov, "Stimulated emission of surface plasmon polaritons on smooth and corrugated silver surfaces," J. Opt., vol. 14, no. 7, 114015, 2012.

[2] R. Yan, D. Gargas and P. Yang, "Nanowire photonics," Nature photonics, vol. 3, pp. 569 – 577, 2009.

[3] M.L. Brongersma, "Nanoscale photonics: nanoshells: gifts in a gold wrapper," Nature Materials, vol. 2, pp. 296 – 297, 2003.

[4] E. Prodan, C. Radloff, Naomi J. Halas and P. Nordlander, "A hybridization model for the plasmon response of complex nanostructures, Science," vol. 302, no. 5644, pp. 419 – 422, 2003.

[5] W.L. Barnes, A. Dereux and T.W. Ebbesen, "Surface plasmon subwavelength optics," Nature, vol. 424, no. 6950, pp. 824 – 830, 2003.

[6] J. Aizpurua, P. Hanarp, D.S. Sutherland, M. Kall, G.W. Bryant and F.J. Garcia de Abajo, "Optical properties of gold nanorings," Phys. Rev. Lett., vol. 90, no. 5, 057401, 2003.

[7] Y. Sun and Y. Xia, "Shape-controlled synthesis of gold and silver nanoparticles," Science, vol. 298, no. 5601, pp. 2176 – 2179, 2002.

[8] R. Jin, Y.C. Cao, E. Hao, G.S. Metraux, G.C. Schatz and C.A. Mirkin, "Controlling anisotropic nanoparticle growth through plasmon excitation," Nature, vol. 425, pp. 487–490, 2003.

[9] U. Kreibig and M. Vollmer, "Optical properties of metal clusters, Springer," New York, 1995.

[10] P. Scholz, S. Schwieger, B. Ashall, D. Zerulla and E.Runge, "The influence of wire shape on surface plasmon mode distribution," Appl. Phys. B., vol. 93, Spec. Issue, 111, 2008.

[11] S.A. Maier, M.L. Brongersma, P.G. Kik, S. Meltzer, A.A.G. Requicha and H.A. Atwater, "Plasmonics – a route to nanoscale optical devices," Adv. Mater., vol. 13, pp. 1501 – 1505, 2001.

[12] M.Y. Ng and W.C. Liu, "Controlling surface plasmon excitation of pair arrays of metallic nanocylinders," Applied Physics A: Materials Science and Processing, vol. 89, no. 2, pp. 391-395, 2007.

[13] M.K. Kinnan, S. Kachan, C.K. Simmons and G. Chumanov, "Plasmon coupling in two-dimensional arrays of silver nanoparticles: I. Effect of the Dielectric Medium," J. Phys. Chem. *C,* vol. 113, no. 17, pp. 7079-7084, 2009.

[14] N. Sakhnenko and A. Nerukh, "Rigorous analysis of whispering gallery mode frequency conversion due to time variation of refractive index in a spherical resonator," J. Opt. Soc. Am. A, vol. 29, pp. 99-104, 2012.

[15] E.V. Bekker, A. Vukovic, P. Sewell, T. M. Benson, N. K. Sakhnenko, and A. G. Nerukh, "An assessment of coherent coupling through radiation fields in time varying slab waveguides," Opt. Quant. Electron., vol. 39, no. 7, pp. 533-551, 2007.

[16] N. Sakhnenko, A. Nerukh, T. Benson, and P. Sewell, "Whispering gallery mode transformation in a switched microcavity with concentric ring geometry," Opt. Quant. Electron., vol. 40, no. 11-12, pp. 818-82, 2008.