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MULTIPROBE MICROWAVE MULTIMETER ERROR DEFINITION ON ITS SENSOR ERROR BASE

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Abstract There was proposed a new method of estimation of multiprobe microwave multimeter algorithms precision by mean of accumulation of partial error. The least square solution is used for variance and covariance matrix definition. The weighed coefficient is obtained from algorithms derivatives with respect to intermediate variable. Substituting the expression for standard deviation (variance) end weighed coefficient in formula for partial error accumulation we can compare different algorithms, study frequency properties of algorithms and make conclusion about its applicability in multiprobe microwave multimeter for passing power and reflection coefficient definition. It was proved that solutions (i.e.algorithms) are the same for analytic and numerical (least square) method. So it is possible apply least square solution for three equation with three unknown variable.

Keywords: multiprobe microwave multimeter, least squares solution, weighted coefficient, accumulation of partial error

1. INTRODUCTION

The multiprobe microwave multimeter is a new measurement device, designed to determine incident, reflected, passing into termination power, wavelength and the complex reflection coefficient of termination. Its principle of action is based on standing wave in the transmission line restoration on the ground of reading of discreet sensor that located definite manner along the transmittion line. The sensors can be thermometers, thermistors, bolometers, diodes and other measurement transformator. The sensors signals, passing through an amplifying and normalization channel expose on microprocessor system bus, it determines the meaning of measured signal and tract parameters.

The real sensors differ from their idealistic models by nonlinearity and deviation of technological characteristics, non point shape, dependence of transformation coefficient on frequency, in spite of preliminary estimation of other errors and correction making. The normalizing channel adds errors too. It causes that final result has errors.

The multiprobe powermeter and circuit analyzer error calculation methods known until now have its disadvantages: the partial derivative calculation on sensor signal base result in cumbersome and non simple result [3], if it was used Jacobian for estimation a priori of equation system, than the solution not tied directly to calculation algorithm [4,5] ; when dispersion ellipsoid or variance and covariance matrix was used, the calculation is concerned to a reflection coefficient, not to a passing power.

There was described two stage esrtimation procedure in the work [1,2]: at the first stage the primary transformator error is determined, at the second stage, from the first stage results the indirect measurement errors are determined. This report purpose is a passing power, a modulus and phase of reflection coefficient error determination on the base of two stage method.

2. MULTIMETER ALGORITHMS DEFINITION

Lets consider three probes multimeter for fixed frequency. There are three sensors disposed on $\lambda/8$ distance; the second sensor is origin of coordinates. The sensor signals is described by following equation system under condition that their frequency and amplitude characteristics are identical

$$\begin{cases} P_1 = P_{пад}\left(1+\Gamma^2+2\Gamma\cos(\varphi-\theta)\right), \\ P_2 = P_{пад}\left(1+\Gamma^2+2\Gamma\cos\varphi\right), \\ P_3 = P_{пад}\left(1+\Gamma^2+2\Gamma\cos(\varphi+\theta)\right) \end{cases} \qquad (1)$$

For nonlinear equation system the solution has some difficulties. For simplifying purpose the linearization was made by means of intermediate variable and trigonometric transformation was performed.

$P = P_{пад}(1+\Gamma^2)$, $\qquad \Delta P = 2\Gamma$

The equation system (1) can be written in matrix form

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & \cos\theta & \sin\theta \\ 1 & 1 & 0 \\ 1 & \cos\theta & -\sin\theta \end{bmatrix} \begin{bmatrix} P \\ \Delta P\cos\varphi \\ \Delta P\sin\varphi \end{bmatrix}$$

The equation system analytic solution method is grounded on inverse matrix calculation. To invert matrix was performs actions: algebraic complement for matrix element calculation, than matrix of algebraic complement was transposed, after that transposed matrix elements were divided on system determinant. The unknown variables was found by inverse matrix multiplication on column of free terms.

There is given equation system A

$$A = \begin{bmatrix} 1 & \cos\theta & \sin\theta \\ 1 & 1 & 0 \\ 1 & \cos\theta & -\sin\theta \end{bmatrix}.$$

Let's calculate confactor and determinant of A

$$A_{11} = -\sin\theta \quad A_{12} = (-1)\sin\theta \quad A_{13} = \cos\theta - 1$$

$$A_{21} = (-1)(\cos\theta(-\sin\theta) - \cos\theta\sin\theta) = 2\cos\theta\sin\theta$$

$$A_{22} = -\sin\theta - \sin\theta = -2\sin\theta \qquad A_{23} = (-1)(\cos\theta - \cos\theta) = 0$$

$$A_{31} = -\sin\theta \quad A_{32} = (-1)(-\sin\theta) = \sin\theta \quad A_{33} = 1 - \cos\theta$$

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = -\sin\theta + \cos\theta\sin\theta$$

$$+ (\cos\theta - 1)\sin\theta = -\sin\theta + \cos\theta\sin\theta + \cos\theta\sin\theta - \sin\theta =$$

$$= 2\sin\theta(\cos\theta - 1)$$

The inverse matrix is

$$A^{-1} = \begin{bmatrix} -\frac{1}{2(\cos\theta-1)} & \frac{\cos\theta}{\cos\theta-1} & -\frac{1}{2(\cos\theta-1)} \\ \frac{1}{2(\cos\theta-1)} & \frac{-1}{2(\cos\theta-1)} & \frac{1}{2(\cos\theta-1)} \\ -\frac{1}{2\sin\theta} & 0 & \frac{1}{2\sin\theta} \end{bmatrix}$$

The equation system solution is

$$\begin{bmatrix} P \\ \Delta P\cos\varphi \\ \Delta P\sin\varphi \end{bmatrix} = \begin{bmatrix} -\frac{1}{2(\cos\theta-1)} & \frac{\cos\theta}{\cos\theta-1} & -\frac{1}{2(\cos\theta-1)} \\ \frac{1}{2(\cos\theta-1)} & \frac{-1}{2(\cos\theta-1)} & \frac{1}{2(\cos\theta-1)} \\ -\frac{1}{2\sin\theta} & 0 & \frac{1}{2\sin\theta} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

The formulas for intermediate variable (2) were obtained from equation system (1)

$$P = \frac{2P_2\cos\theta - P_1 - P_3}{2(\cos\theta - 1)},$$

$$\Delta P\cos\varphi = \frac{P_1 + P_2 - 2P_3}{2(\cos\theta - 1)},$$

$$\Delta P\sin\varphi = \frac{P_3 - P_1}{2\sin\theta}.$$

(2)

Than sought after expression for passing power, modulus and phase of complex reflection coefficient, connect to intermediate variable looks like this

$$P_{np} = \sqrt{P^2 - (\Delta P\cos\varphi)^2 - (\Delta P\sin\varphi)^2},$$

$$\Gamma = \sqrt{\frac{P_{omp}}{P_{nad}}} = \sqrt{\frac{P - P_{np}}{P + P_{np}}} = \frac{P - \sqrt{P^2 - (\Delta P\cos\varphi)^2 - (\Delta P\sin\varphi)^2}}{\sqrt{(\Delta P\cos\varphi)^2 + (\Delta P\sin\varphi)^2}},$$

$$\varphi = arctg\frac{\Delta P\sin\varphi}{\Delta P\cos\varphi}.$$

(3)

After substituting (2) into (3) it was obtained

$$P_{np} = \sqrt{P_1\frac{P_1 + P_3 - P_1(1 + \cos\theta)}{1 - \cos\theta} - \frac{(P_1 - P_3)^2}{4\sin^2\theta}}.$$

(4)

$$P_{nad} = \frac{P + P_{omp}}{2},\ P_{omp} = \frac{P - P_{omp}}{2}, |\Gamma| = \sqrt{\frac{P_{omp}}{P_{nad}}},$$

$$\varphi = arctg\frac{P_3 - P_1}{P_1 + P_2 - 2P_3}.$$

3.ACCUMULATION OF PARTIAL ERROR

The general error of measurement parameters can be determined by the law of accumulation of private errors with using of weighted coefficient and sensors standard deviation. So for passing power an reflection coefficient can be obtained following formulas

$$\sigma_{np} = \sqrt{W_1\sigma_P^2 + W_2\sigma_{\Delta P\cos\varphi}^2 + W_2\sigma_{\Delta P\sin\varphi}^2 + 2W_1W_2\,\mathrm{cov}_{P,\Delta P\cos\varphi}}$$

$$\sigma_\Gamma = \sqrt{W_4\sigma_P^2 + W_5\sigma_{\Delta P\cos\varphi}^2 + W_6\sigma_{\Delta P\sin\varphi}^2 + 2W_4W_5\,\mathrm{cov}_{P,\Delta P\cos\varphi}}$$

(5)

$$\sigma_\varphi = \sqrt{W_7\sigma_P^2 + W_8\sigma_{\Delta P\cos\varphi}^2 + W_9\sigma_{\Delta P\sin\varphi}^2 + 2W_8W_9\,\mathrm{cov}_{P,\Delta P\cos\varphi}}$$

To use this formulas it is necessary to solve two tasks: first, to determine weighed coefficient, second, to determine variance and covariance.

3.1 WEIGTED COEFFICIENT DEFINITION

From expression (3) were found weighed coefficient as derivatives with respect to intermediate variable for indirect measurement error. For passing power and other parameters weighted coefficients are

$$W_1 = \frac{-\Gamma}{P_{nad}(1 - \Gamma^2)}, \quad W_2 = \frac{\cos\varphi(1 + \Gamma^2)}{2P_{nad}(\Gamma^2 - 1)}, \quad W_3 = \frac{\sin\varphi(1 + \Gamma^2)}{2P_{nad}(\Gamma^2 - 1)}$$

$$W_4 = \frac{1 + \Gamma^2}{1 - \Gamma^2}, \quad W_5 = \frac{-2\Gamma\cos\varphi}{1 - \Gamma^2}, \quad W_6 = \frac{-2\Gamma\sin\varphi}{1 - \Gamma^2}$$

$$W_7 = 0 \qquad W_8 = -\frac{\sin\varphi}{2\Gamma P_{nad}} \qquad W_9 = \frac{\cos\varphi}{2\Gamma P_{nad}}$$

3.2. VARIANCE AND COVARIANCE DEFINITION

The standard deviation definition is a more complex task. At first we have sensor errors either known from sensor passport or calculated on first stage of two stage procedure. Then it is necessary to transform them into intermediate variable error. Lvov and other [7] show plausibility of the least square solution application for multiprobe system, which method is usually used when equation quantity is more than unknown quantity. Further it will be shown, that the least square solution can be used in particular case when equation quantity is equal to unknown variance quantity. The least square solution suppose following order of action:

1) to calculate Fisher matrix as product of equation system matrix and its transpose matrix

$$\left(\begin{bmatrix} 1 & 1 & 1 \\ \cos\varphi & 1 & \cos\varphi \\ \sin\varphi & 0 & -\sin\varphi \end{bmatrix}\begin{bmatrix} 1 & \cos\varphi & \sin\varphi \\ 1 & 1 & 0 \\ 1 & \cos\varphi & -\sin\varphi \end{bmatrix}\right)=\begin{bmatrix} 3 & 2\cos\varphi & 0 \\ 2\cos\varphi & 2\cos^2\varphi & 0 \\ 0 & 0 & 2\sin\varphi \end{bmatrix}$$

2) to normalize column of free term by means of multiplication on transposed matrix from the right side

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ \cos\varphi & 1 & \cos\varphi \\ \sin\varphi & 0 & -\sin\varphi \end{bmatrix}=\begin{bmatrix} P_1+P_2+P_3 \\ (P_1+P_3)\cos\varphi+P_2 \\ (P_1-P_3)\sin\varphi \end{bmatrix}$$

3) to perform Fisher matrix inversion and to obtain variation and covariation matrix

$$\begin{bmatrix} 3 & 2\cos\varphi & 0 \\ 2\cos\varphi & 2\cos^2\varphi & 0 \\ 0 & 0 & 2\sin\varphi \end{bmatrix}^{-1}=\begin{bmatrix} \frac{2\cos^2\varphi+1}{2(\cos\varphi-1)^2} & \frac{-(2\cos\varphi+1)}{2(\cos\varphi-1)^2} & 0 \\ \frac{-(2\cos\varphi+1)}{2(\cos\varphi-1)^2} & \frac{3}{2(\cos\varphi-1)^2} & 0 \\ 0 & 0 & \frac{1}{2\sin^2\varphi} \end{bmatrix}.$$

4) to perform inverse matrix multiplication on normalized column of free terms

$$\begin{bmatrix} P^{\cdot} \\ \Delta P\cos\varphi \\ \Delta P\sin\varphi \end{bmatrix}=\begin{bmatrix} \frac{2\cos^2\varphi+1}{2(\cos\varphi-1)^2} & \frac{-(2\cos\varphi+1)}{2(\cos\varphi-1)^2} & 0 \\ \frac{-(2\cos\varphi+1)}{2(\cos\varphi-1)^2} & \frac{3}{2(\cos\varphi-1)^2} & 0 \\ 0 & 0 & \frac{1}{2\sin^2\varphi} \end{bmatrix}\begin{bmatrix} P_1+P_2+P_3 \\ (P_1+P_3)\cos\varphi+P_2 \\ (P_1-P_3)\sin\varphi \end{bmatrix}$$

5) to write down and to simplify solution by uniting similar terms

$$P=\frac{(P_1+P_2+P_3)(2\cos^2\varphi+1)-(2\cos\varphi+1)((P_1+P_2)\cos\varphi+P_2)}{2(\cos\varphi-1)^2}$$

$$\Delta P\cos\varphi=-\frac{(2\cos\varphi+1)(P_1+P_2+P_3)+3((P_1+P_3)\cos\varphi+P_2)}{2(\cos\varphi-1)^2}$$

$$\Delta P\sin\varphi=\frac{(P_1-P_3)\sin\varphi}{2\sin^2\varphi}.$$

6) the solution analysis shows that this solution is equal the solution obtained analytically.

4. CONCLUSION

Conclusions: 1) the least square solution can be applied for three equation with three unknown; 2) the variance and covariance matrix for insertion in expression for error accumulation was obtained so all elements for expression (5) are found.

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