Extraordinary Reflection from Photonic Crystal with Metamaterials

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***Abstract* — We predict an extraordinary perfect reflection of a plane wave from a finite-thickness photonic crystal made of dielectrics and metamaterials. The physical nature of this phenomenon is revealed. We show that this complete reflection is related to a resonance of surface plasmon-polariton wave along the crystal thickness.**

Keywords—photonic crystals; extraordinary reflection; metamaterials, plasmon-polariton waves

I. Introduction

The photonic-crystal structures made of metamaterials attract a special attention mastering the terahertz and sub-terahertz frequency ranges [1-3]. The periodic boundaries between the dielectric and metamaterial layers produce the plasmon-polariton surface waves propagating along these interfaces. These waves lead to new properties of the photonic crystals. It is well known that practically all energy of a long-wave range plain electromagnetic wave transmits through a one-dimensional finite-thickness photonic crystal made of two dielectric layers. If a crystal represents a periodic sequence of two layers, one of which is a metamaterial with negative refractive index, such a crystal possess extraordinary properties for certain frequencies. The perfect reflection of a plain wave from the crystal boundary in a very narrow frequency range can be observed. This phenomenon can be used to construct narrow-band filters based on photonic crystals made of metamaterials.

In the present work we, basing on the rigorous numerical solution of the diffraction problem, focus the main attention on the extraordinary perfect reflection of a plain wave from a finite-thickness photonic crystal made of metamaterials. The diffraction problem is solved for the rigorous mathematic formulation and arbitrary relations between the wave length and geometric sizes of the crystal elements. We consider different combinations of two media of two-layer photonic crystal structure. Besides, we study the phase grating, for which the wave resistances of two media made from magnetodielectric and metamaterial are the same in absolute values but different in the sign.

II. Main Part

We study the scattering of a plain E_x-polarized wave on the photonic crystal made of metamaterial with given arbitrary values of the material parameters ε_j, μ_j ($j = 1, 2$) for each segment of the crystal on its period l. Here $d \times h$ are the sizes in cross-section of one type of bars of the crystal, $(l-d) \times h$ are the sizes in cross-section of the other type of

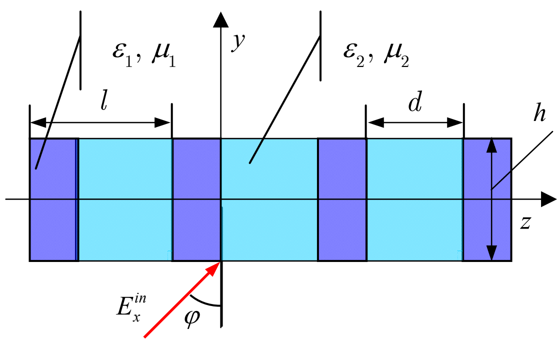

Fig. 1. The model of photonic crystal

bars, φ is the incident angle of the wave (see Fig. 1). To find the scattered field, one should solve the uniform Helmholtz equation for E_x-component of electric field:

$$\Delta E_x + k^2 \varepsilon(z)\mu(z) E_x = 0\,, \qquad (1)$$

$$\varepsilon(z)\mu(z) = \begin{cases} \varepsilon_1\mu_1 & z \in \left[\frac{d}{2} - l, -\frac{d}{2}\right) \\ \varepsilon_2\mu_2, & z \in \left[-\frac{d}{2}, \frac{d}{2}\right) \end{cases}.$$

We can present the solution for three spatial regions of the crystal in the following form:

$$E_x(z,\,y) = \begin{cases} e^{ik_y(y+\frac{h}{2})+ik\alpha z} + \sum\limits_{m=-\infty}^{\infty} A_{\lambda_m} e^{-i\gamma_m(y+\frac{h}{2})} e^{-i\alpha_m z}; & y < \frac{h}{2} \\ \sum\limits_n Y_{\beta_n}(y) Z_{\beta_n}(z); & |y| \le \frac{h}{2} \\ \sum\limits_{m=-\infty}^{\infty} B_{\lambda_m} e^{i\gamma_m(y+\frac{h}{2})} e^{-i\alpha_m z}; & y > \frac{h}{2} \end{cases}, \quad (2)$$

where $\gamma_m = \sqrt{k^2 - \alpha_m^2}$, $\alpha_m = k\alpha + \frac{2\pi}{l} m$, $\alpha = \sin\varphi$, $k_y = k\cos\varphi$, $k = \frac{2\pi}{\lambda}$ is wave number, A_{λ_m}, B_{λ_m} are unknown coefficients of Fourier expansion. The solution (2) in the region filled with photonic crystal is written as an expansion in Fourier series over the eigenfunctions Z_{β_n} of the Sturm-Liouville's problem, which can be found from the equation:

$$\ddot{Z}_{\beta_n} + \varsigma^2 Z_{\beta_n} = 0\,, \qquad (3)$$

$$Z_{\beta_n}(z) = \begin{cases} A_{\varsigma_n^{\mathrm{I}}} \cos\varsigma_n^{\mathrm{I}}(z + \frac{d}{2}) + B_{\varsigma_n^{\mathrm{I}}} \sin\varsigma_n^{\mathrm{I}}(z + \frac{d}{2}), \; z \in [\frac{d}{2} - l, -\frac{d}{2}) \\ A_{\varsigma_n^{\mathrm{II}}} \cos\varsigma_n^{\mathrm{II}}(z + \frac{d}{2}) + B_{\varsigma_n^{\mathrm{II}}} \sin\varsigma_n^{\mathrm{II}}(z + \frac{d}{2}), \; z \in [-\frac{d}{2}, \frac{d}{2}) \end{cases}$$

where $\varsigma_n^{\mathrm{I}} = \sqrt{k^2\varepsilon_1\mu_1 + \beta_n^2}$, $\varsigma_n^{\mathrm{II}} = \sqrt{k^2\varepsilon_2\mu_2 + \beta_n^2}$, $A_{\varsigma^{\mathrm{I}}}$, $A_{\varsigma^{\mathrm{II}}}$, $B_{\varsigma^{\mathrm{I}}}$, $B_{\varsigma^{\mathrm{II}}}$ are unknown coefficients. Function $Y_{\beta_n}(y) = C_{\rho_n} e^{\beta_n y} + D_{\rho_n} e^{-\beta_n y}$ in (2) represents the general solution of the uniform differential equation $\ddot{Y} + \rho_n^2 Y = 0$ with the unknown coefficients C_{ρ_n}, D_{ρ_n}.

The characteristic equation for finding the eigenvalues β_n of the Sturm-Liouville's problem has the following form:

$$\cos k\alpha l = \cos\varsigma^{\mathrm{II}} d \cos\varsigma^{\mathrm{I}}(d-l) + \frac{1}{2}\left(\eta\frac{\mu_2}{\mu_1} + \frac{1}{\eta}\frac{\mu_1}{\mu_2}\right)\sin\varsigma^{\mathrm{II}} d \sin\varsigma^{\mathrm{I}}(d-l)\,, \; (4)$$

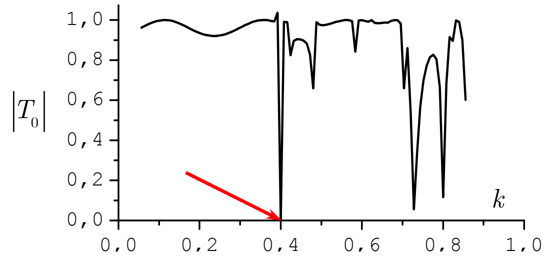

Fig. 2. Amplitude-frequency characteristic

with $\eta = \varsigma^{\mathrm{I}} / \varsigma^{\mathrm{II}}$. The solution of this equation enables us to find eigenvalues β_n and corresponding eigenfunctions Z_{ς_n} in the region of the photonic crystal. Note one peculiarity of the solution of the characteristic equation (4). If the parameter α is spectral, then one can find an analytic solution of Eq. (4). In our case, α is not a spectral parameter since it is defined by the conditions of the initial problem. The spectral parameter is β_n. The value of β_n was found numerically. The analysis of Eq. (4) shows, that the value of β can be either real or purely imaginary for real ε_j, μ_j. In the first case, the waves decay along Oy axis, while in the second case they propagate without decaying. The structure of this waves in Oz direction is defined by the values ς_n^{I}, $\varsigma_n^{\mathrm{II}}$. If $\left(\varsigma_n^{\mathrm{I}}\right)^2 < 0$ and $\left(\varsigma_n^{\mathrm{II}}\right)^2 < 0$, then, at certain conditions, the surface plasmon-polaritons can exist in the photonic crystal.

III. Discussion

As an example, we consider the case of photonic crystal that consists of two segments. One of them is a metamaterial with parameters $\varepsilon_1 = -1.5$, $\mu_1 = -1$, while the other is a magnetodielectric with parameters $\varepsilon_2 = 3.5$ and $\mu_2 = 1$. Fig. 2 shows the amplitude-frequency characteristics (AFC, the dependence of the transmission coefficient $|T_0|$ on the wave number k) for $l = \pi$, $d = \pi/3$, $\varphi = 0$.

The analysis of AFC in Fig. 2 shows that, in the long-wave part of the wavelength spectrum ($k = 0.4$), an extraordinary phenomenon can be observed – the transmission coefficient

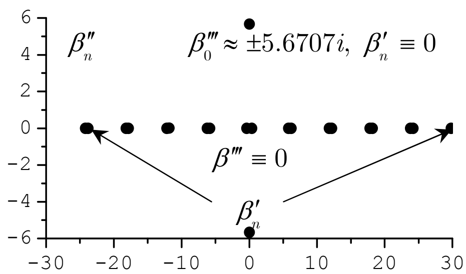

Fig. 3. Roots of the dispersion equation

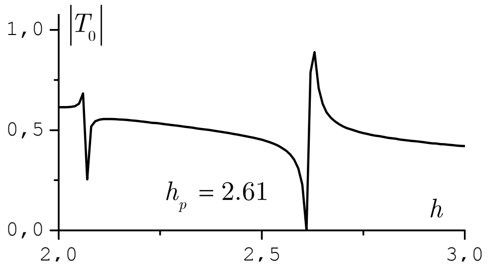

Fig. 4. The transmission coefficient versus crystal thickness

becomes zero, i.e. the perfect wave reflection occurs. Such a reflection cannot be obtained for a photonic crystal, which elements are usual magnetodielectrics. The nature of such phenomenon is apparently caused by a surface plasmon-polariton wave propagating along the interfaces of two media of the crystal (one of which is a metamaterial with $\varepsilon_1 = -1.5$, $\mu_1 = -1$) [4]. In this case, a high-Q resonance over the sample thickness occurs, which leads to the perfect reflection of the incident wave from a finite-thickness crystal.

Fig. 3 shows numerical results for the solution of characteristic equation (4) for the resonance case (Fig. 2, $k = 0.4$) and for the normal incidence of the plain wave on the crystal. There are two types of the roots, imaginary and real. The number of imaginary roots β_n'' is always limited for the given values of the problem parameters, and it is always even since the waves can propagate in two opposite directions along the Oy axis. The real values of the roots of the dispersion equation define the waves decaying along the Oy axis.

Fig. 4 shows the dependence of the transmission coefficient on the crystal thickness h for the resonance value of the frequency parameter $k = 0.4$. It should be noted that the perfect reflection occurs also for other crystal thicknesses. The discussed phenomenon can be observed if the crystal thickness matches the integer number of wavelengths of the surface plasmon-polariton.

Fig. 5 and Fig. 6 present the amplitude distribution of the electric field of the plasmon-polariton wave over the crystal thickness for $h_p = 2.61$. As one can notice from the field distribution in Fig. 5, there is no field behind the crystal ($y > \frac{h}{2}$), while the oscillations of the field occur over the crystal thickness. With the decreasing of the crystal thickness,

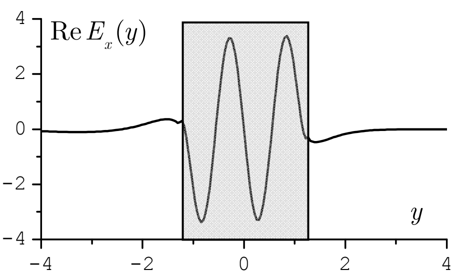

Fig. 5. Distribution of $\operatorname{Re} E_x(y)$ in resonance $k = 0.4$

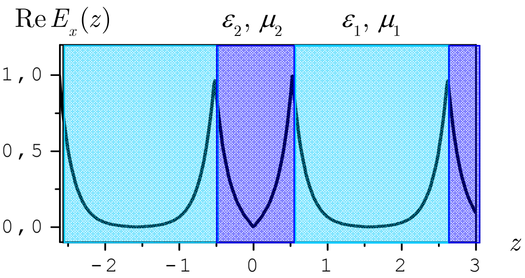

Fig. 6. Distribution of $\operatorname{Re} E_x(z)$ in resonance $k = 0.4$

the number of resonances decreases. An additional analysis of the field distribution under conditions, when the transmission coefficient is close to one, shows that there is no resonance phenomenon and the wave transmits in this case through the crystal almost completely. Fig. 6 shows the spatial amplitude distribution of the electric field of the surface plasmon-polariton wave over periodicity of the crystal. Maximum of the field amplitude $\operatorname{Re} E_x(z)$ observed on the boundary of section of media with metamaterial and dielectric.

IV. Conclusion

The incidence of a plain wave on the photonic crystal made of magnetodielectric and metamaterial layers is studied. We predict the perfect reflection in the long-wave range of the spectrum. We show that this extraordinary phenomenon is caused by the spatial resonance of the surface plasmon-polariton waves over the crystal thickness.

References

[1]. S..A. Masalov, Ya.T. Repa, V.P. Shestopalov. “The diffraction of a plain electromagnetic wave on a dielectric grating.” Republic Interdepartmental thematic scientific-technical. Collected articles «Radio-engineering ». 1969. No. 10. pp. 15-24.

[2]. V.P. Shestopalov, L.N. Litvinenko, S.A. Masalov, V.G. Sologub. The diffraction of the waves on the gratings: teaching aid, Kh.: Kharkiv university named by Gorky, 1973. pp. 289.

[3]. A.V. Kazanko, E.N. Odarenko, A.A. Shmat'ko. “Interaction of a plain electromagnetic wave with the diffraction grating from metamaterial”, Vistnyk KhNU named by Karazin. Series. Radiophysics and electronics, – 2012. No. 1010. pp. 57–65.

[4]. H. Raether. Surface Plazmon. Shpringer-Verlag, 1986, pp. 134.