

PLASMONIC MODES OF COUPLED PLASMA COLUMNS CLUSTER WITH TRIANGULAR OR SQUARE CONFIGURATIONS

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Abstract – Theoretical investigation of the plasmonic resonances of coupled cylindrical plasma columns is presented. Mechanism of plasmonic mode coupling in a cluster with triangular or square configurations that can be considered as bonding and antibonding combinations of isolated column plasmons is investigated.

I. INTRODUCTION

The possibility of using plasmons to create an effective optical nanoantenna [1], subwavelength resonators [2], spacer [3] and to increase the sensitivity of biosensors [4] are widely discussed recently. Surface and localized plasmons have been explored for their potential in single molecule detection [5], biomolecular interaction studies and early stage cancer detection [6]. For these applications an accurate modeling that provides a fundamental understanding of physical phenomena is of great importance.

II. MATHEMATICAL BACKGROUND

In this paper we solve the eigenvalue problem for clusters with triangular or square configurations of coupled cylindrical plasma columns. Radius of each column is a , separation distance between them is d , the surrounding space is a vacuum, the time dependence is $e^{i\omega t}$. Figures 1 and 2 represent schematic diagrams of the structures with triangular and square configurations respectively. Plasma is described by the permittivity ε_p that is given by the Drude model

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \quad (1)$$

here ω_p represents the plasma frequency, γ is the material absorption. Sub-wavelength resonances are possible when $\varepsilon(\omega) < 0$ (or equivalently $\omega_p > \omega$), they are called plasmon resonances or surface plasmons. H-polarized fields are considered. The main goal of our paper is to study surface plasmons in terms of their eigenfrequencies and quality (Q) factors.

To describe the fields we introduce N ($N=3$ or $N=4$) cylindrical coordinate systems associated with each infinite column. The solution is presented in the form of a series of the Bessel functions inside each plasma column and the second-order Hankel functions in outer space

$$H^{in}(\rho_n, \varphi_n) = \sum_{s=-\infty}^{+\infty} A_s^{(n)} J_s(k_p \rho_n) e^{is\varphi_n}, \quad (2)$$

$$H^{out}(\rho_n, \varphi_n) = \sum_{n=1}^N \sum_{s=-\infty}^{+\infty} \bar{A}_s^{(n)} H_s^{(2)}(k \rho_n) e^{is\varphi_n}, \quad (3)$$

here $k = \omega \cdot c^{-1}$, $k_p = n_p \omega c^{-1}$, $n_p = \sqrt{\varepsilon_p(\omega)}$, c is light velocity in a vacuum.

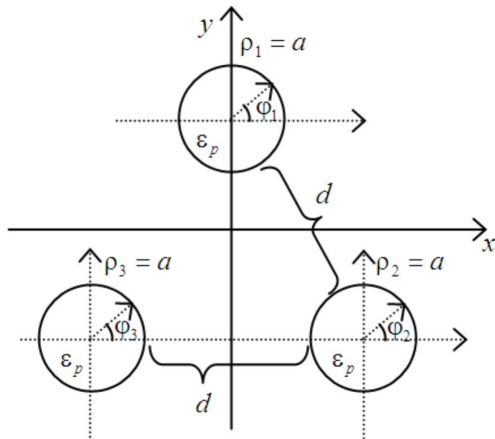


Fig. 1. Schematic diagram of the structure: a cluster of triangular configuration.

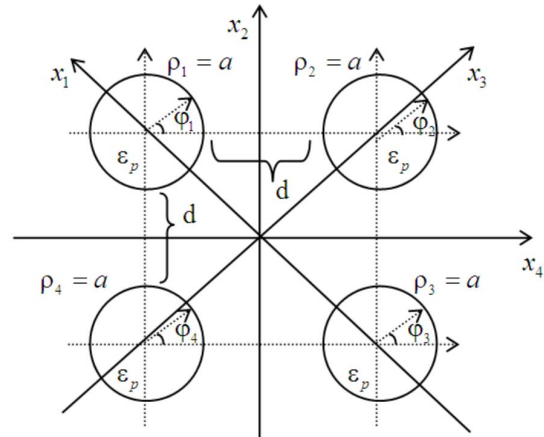


Fig. 2. Schematic diagram of the structure: a cluster of square configuration.

Unknown coefficients A_s and \bar{A}_s are found from the boundary conditions, requiring the continuity of the tangential components of the total electric and magnetic fields at each cylindrical column's surface. Using the addition theorem for the Bessel functions we arrive to an infinite system of algebraic equations that can be truncated in order to provide a controlled numerical precision. We have to mention that all eigenfrequencies are complex $\omega = \omega' + i\omega''$, where $\omega'' > 0$ represents damping and ω' is associated with the eigen oscillation frequencies. Q-factor of plasmons can be evaluated through the formula $Q = \omega' / 2\omega''$.

III. RESULTS AND DISCUSSION

For the case of cluster of triangular configuration shown in Fig. 1 the structure has three symmetry axes that causes four classes of excited plasmons with different symmetry: EEE (x_1, x_2, x_3 - even), OOO (x_1, x_2, x_3 - odd), OEE (x_1 - odd, x_2, x_3 - even), EOO (x_1 - even, x_2, x_3 - odd) (see Fig. 3 (a)). Axes of symmetry pass through the centre of each column and a midpoint of the opposite side of a triangle. For the case of square cluster shown in Fig. 2 the structure has four symmetry axes associated with horizontal, vertical, and oblique axes that causes six classes of excited plasmons with different symmetry: EEEE (x_1, x_2, x_3, x_4 - even), OOOO (x_1, x_2, x_3, x_4 - odd), OEOE (x_1, x_3 - odd, x_2, x_4 - even), EOEO (x_1, x_3 - even, x_2, x_4 - odd), EEEO (x_1, x_2, x_3 - even, x_4 - odd), OOOE (x_1, x_2, x_3 - odd, x_4 - even). (see Fig. 3 (b)). Similar symmetry classes exist in the photonic molecules of coupled microdisk resonators [7, 8].

Figure 4 illustrates the value of the real part of plasmon eigenfrequency versus normalized frequency (ka) for different values of $w_p = \omega_p ac^{-1}$ that we will call further a normalized plasma frequency for coupled plasma columns of triangular (see Fig. 4 (right panel)) and square configurations (see Fig. 4 (left panel)) for $s = 2$. Here s indicates the number of angular field variations of corresponding plasmonic mode. For cluster of square configuration with the decreasing of the separation distance between the plasma cylinders we see decreasing of the resonant frequency for EEEE, OEOE, EEEO and OOOE plasmons and increasing for the OOOO and EOEO plasmons. The eigenfrequency of isolated cylinder is shown in black dotted line of Fig. 4. For the case of triangular configuration it is clearly seen that for distant cylinders eigenfrequencies are nearly identical for all four symmetry. As separation distance d becomes smaller, the frequency shift of the coupled plasmons of the

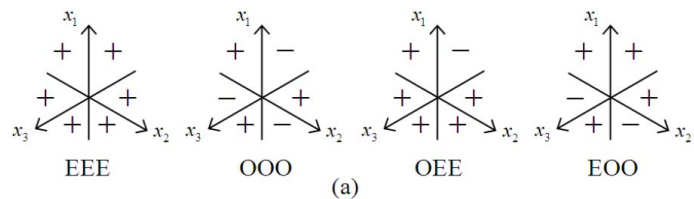


Fig. 3 (a). Four classes of symmetry of the field for the cluster with triangular configuration.

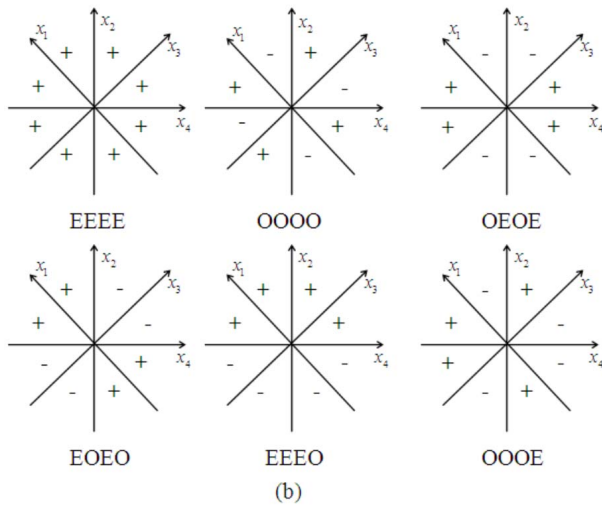


Fig. 3 (b). Six classes of symmetry of the field for the cluster for four coupled cylinders.

for distant columns: in a cluster of triangular configuration it is about 3.5 and 4.5 wavelength, in a cluster of square configuration it is nearly 5 wavelength. The enhancement of Q is seen for square cluster. Maximum value of Q has plasmon with odd symmetry with respect to all axes.

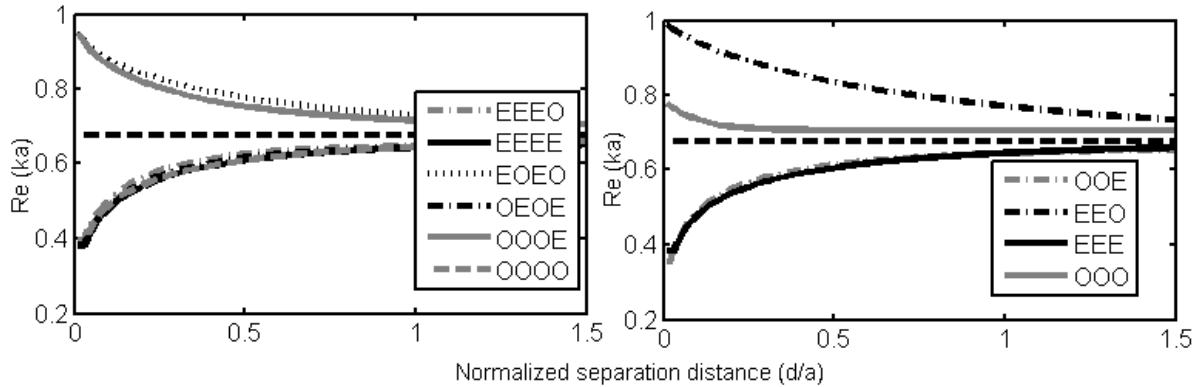


Fig. 4. The normalized frequency versus the normalized separation distance between the coupled plasma columns in a cluster with square (left panel) and triangular (right panel) configurations for different plasmons ($s = 2$).

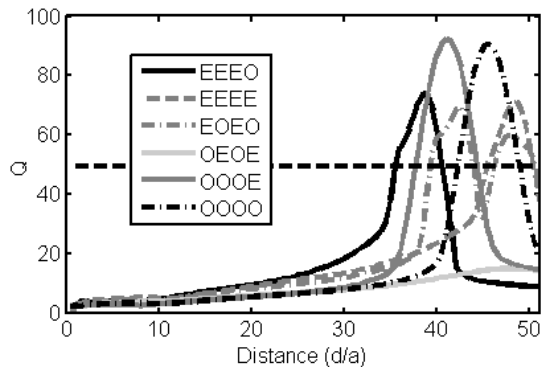


Fig. 5. Q-factor of plasmons in a cluster with square configuration ($s = 2$).

cluster becomes much stronger. We see decreasing of the resonant frequency for EEE and OOE plasmons and increasing for the OOO and EEO plasmons.

It is seen that for large separation distances between cylinders eigenfrequencies of all modes tend to the eigenfrequency of plasmon of an isolated plasma cylinder.

The Q-factor of plasmons of cluster of three coupled plasma columns ($s = 2$) is shown in Fig. 6. The Q-factors of isolated cylinder are shown in black dotted line of Figs. 5 and 6. Maximum peak of Q-factor is seen for OOE plasmon.

Figure 5 presents the Q-factor of four coupled plasma cylinders ($s = 2$). Maximum peak of Q-factor is seen for OOOE plasmon.

We see that for closely placed plasma columns Q of coupled plasmonic modes is low that Q of isolated column. The increasing of Q is observable

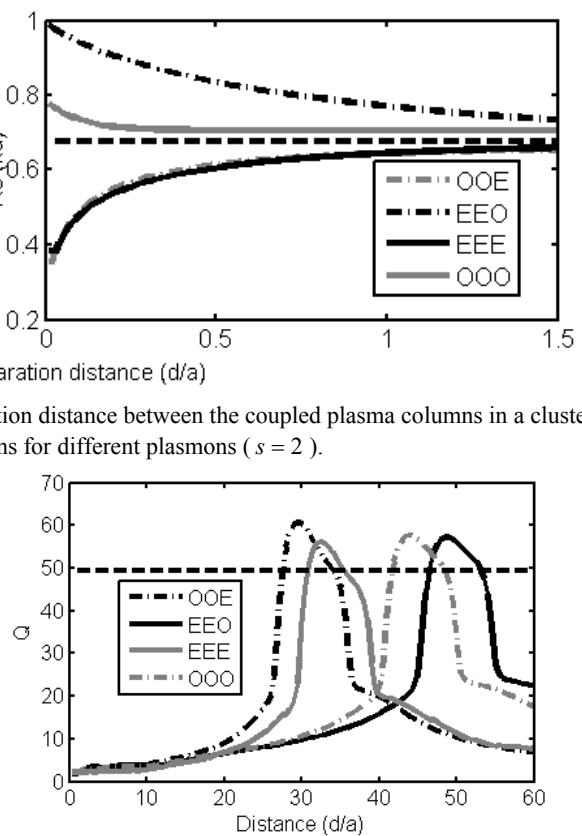


Fig. 6. Q-factor of plasmons in a cluster with triangular configuration ($s = 2$).

Figure 7 demonstrate the near-field portraits of different plasmons of cluster with square configuration for $s = 2$ for $d/a = 1.5$.

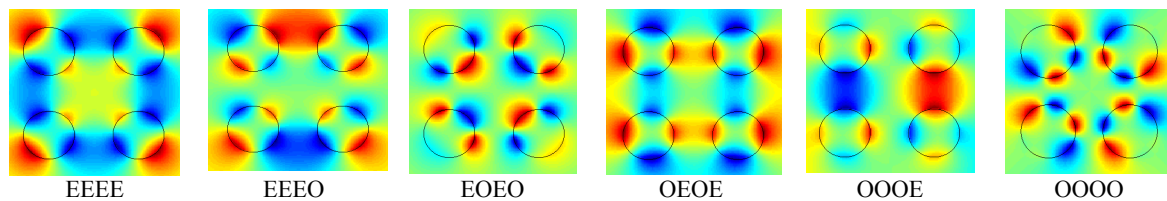


Fig. 7. The near-field distributions of different plasmons of cluster with square configuration for $s = 2$ ($d/a = 1.5$).

VI. CONCLUSION

The eigenfrequencies of the coupled cylindrical columns with triangular and square configurations filled with negative permittivity plasma have been analyzed. It has been shown that individual plasmons of isolated column interact and form bonding and antibonding plasmonic coupled modes of different types. Frequency characteristics plasmonic modes in a cluster of triangular and square configurations have been studied.

ACKNOWLEDGEMENT

Ms. N. Stognii is thankful to the German Academic Exchange Service (DAAD), she also acknowledges the support of this research by IEEE AP-Society through Doctoral Award.

REFERENCES

- [1.] P. Muhlschlegel, H. Eisler, O. Martin, B. Hecht, D. Pohl, "Resonant optical antennas," *Science*, vol. 308, pp. 1607-1609, 2005.
- [2.] J. Li, N. Engheta, "Ultracompact sub-wavelength plasmonic cavity resonator on a nanowire," *Phys. Rev. B*, vol. 74, pp. 115125, 2006.
- [3.] M. Noginov, G. Zhu, A. Belgrave, R. Bakker, V. Shalaev, E. Narimanov, S. Stout, E. Herz, T. Suteewong, and Wiesner, "Demonstration of a spaser-based nanolaser," *Nature*, vol. 460, pp. 1110-1113, 2009.
- [4.] K. Kim, S.J. Yoon, D. Kim, "Nanowire-based enhancement of localized surface plasmon resonance for highly sensitive detection: a theoretical study," *Optics Express*, vol. 14, no. 25, pp. 12419-12431, 2006.
- [5.] K. Kneipp, Y. Wang, H. Kneipp, L. Perelman, I. Itzkan, R. Dasari, and M. Feld, "Single molecule detection using surface-enhanced Raman scattering (SERS)", *Phys. Rev. Lett.*, vol. 78, pp. 1667-1670, 1997.
- [6.] S. Mallidi, T. Larson, J. Tam, P. Joshi, A. Karpouk, K. Sokolov, S. Emelianov, "Multiwavelength photoacoustic imaging and plamon resonance coupling of gold nanoparticles for selective detection of cancer," *Nano Lett.*, vol. 9(8), pp. 2825-2831, 2009.
- [7.] E.I. Smotrova, A.I. Nosich, T.M. Benson, P. Sewell, "Ultralow lasing thresholds of the pi-type supermodes in cyclic photonic molecules composed of sub-micron disks with monopole and dipole modes," *IEEE Photonics Technology Letters*, vol. 18, no. 19, pp. 1993-1995, 2006.
- [8.] S.V. Boriskina, "Symmetry, degeneracy and optical confinement of modes in coupled microdisk resonators and photonic crystal cavities," *IEEE J. Selected Topics in Quantum Electronics*, vol. 12, no. 6, pp. 1175-1182, 2006.