

Development of the theoretical basis of magnetic measurement uncertainty evaluation

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Abstract — The theoretical foundations of measurement uncertainty evaluation magnetic moment of radio electronic means using point methods are considered. A sequence and implementation of the uncertainty assessment procedure is proposed. A model equation based on the multipole description of the object's magnetic field is obtained. An analysis of the equation is made. The uncertainty budget has been drawn up. Found standard and extended uncertainty. The results are in demand for control procedures and tests in the area of technical magnetism. The results contribute to the standardization of the quality assessment of magnetic measurements.

Keywords — external magnetic field, dipole magnetic moment, measurement uncertainty, induction sensor

I. INTRODUCTION

The area of application of magnetic measurements is constantly expanding. These measurements are in demand in many areas of science, technology and industry, such as electric power, electromagnetic compatibility, etc.

Electrical machines and radio devices are the main sources of electromagnetic interference. This interference interferes with the normal functioning of various objects of radio electronics, and has a negative effect on people in the zone of influence. This situation requires control of the electromagnetic environment in the areas of industrial and energy facilities. To solve the problem of electromagnetic compatibility, international organizations develop recommendations and standards for the application in European countries of permissible and limiting levels of external magnetic field strength in production and non-production conditions. Based on these recommendations, national standards are being developed. The external magnetic field levels used in different countries differ in magnitude.

At distances comparable to the overall dimensions of the source of an external magnetic field, its external magnetic field in the surrounding space is of a dipole nature. Such a field is proportional to the equivalent dipole magnetic moment. The dipole magnetic moment ($M, A \cdot m^2$) is a constant value characterizing the magnetic field of an object. Its value is governed by standards for certain types of electrical and electronic equipment. In contrast to the magnetic field strength, this value does not depend on the coordinates of the observation points. This allows you to determine the magnetic field strength at specified points in external space from measured values of the components of the dipole magnetic moment of the source, to get an idea of the spatial configuration of the external magnetic field [1, 2].

Currently, point and integral methods are used to measure the dipole magnetic moment of electrical, radio-electronic objects. Point methods involve performing

measurements at certain points in the external space using - the number of induction sensors. Induction method involves the use of contour measuring windings. These methods have their advantages, disadvantages, spheres of application.

Modern tasks in the field of magnetic measurements require the improvement of metrological assurance of measurements of the dipole magnetic moment. Improving the metrological assurance of measuring the dipole magnetic moment includes the development of more accurate measurement methods, the development of methods for estimating the uncertainty of measurements, the elaboration of the questions of validation of measurement methods.

Currently, the regulatory framework for assessing the uncertainty of measurements in the field of measuring magnetic quantities needs to be improved. In scientific publications [3, 4] the general algorithm for estimating uncertainty is described, the general algorithm for carrying out and processing the results of indirect measurements is described. There is no method for estimating the uncertainty of measurements of the dipole magnetic moment. A scientific publication [5] proposes a method for estimating the methodological error of point measurement methods. There are no estimates of uncertainty. The research [6] considers actual questions of magnetic measurements, but do not contains recommendations on estimation of measurement uncertainty. The scientific publication [7] do not takes into account the categories of measurement uncertainty.

Based on the above, work aimed at improving the metrological assurance of measurements of magnetic quantities is relevant. This involves the development of the theoretical foundations of the model and empirical approaches to the assessment of the uncertainty of magnetic measurements and the elaboration of the questions of validation of measurement methods.

II. METHODOLOGY DEVELOPMENT OF MEASUREMENT UNCERTAINTY EVALUATION OF THE DIPOLE MAGNETIC MOMENT

The processes of international standardization of quality assessment of measurements and the presentation of their results involves the introduction of the concept of uncertainty in the practice of measurement, control and testing.

Processing the uncertainty of the results of any measurements performed by the general algorithm. However, the structure of this algorithm and the implementation of its blocks are specific for a particular type of measurement. This applies to measurements of magnetic quantities and measurements of the dipole magnetic moment in particular.

The main point in estimating the uncertainty of any measurements is the compilation of a model equation and its analysis. Therefore, we consider the theoretical aspects of the compilation of the model equation for measuring the dipole magnetic moment using point-based measurement methods.

Any source of the external magnetic field can be represented as a collection of a finite number of elementary magnetic dipoles, each of which is in the centre of a small volume element. According to the principle of superposition, the sum of elements of elementary dipoles is equal to the equivalent dipole moment [2]. In external space, the magnetic field is characterized by a scalar magnetic potential U , A . By expanding the function U in a power series with respect to the radius of the observation point, we obtain a multipole model of the external magnetic field. Such a model corresponds to partial solutions of the Laplace equation. In the spherical coordinate system, this model has the form

$$U = R \sum_{m=0}^n \left(\frac{R}{r} \right)^{n+1} \cdot (g_{nm} \cos m\varphi + h_{nm} \sin m\varphi) P_{nm}(\cos \theta), \quad (1)$$

where r, θ, φ – the coordinates of the observation point; g_{nm} , h_{nm} , – coefficients of spatial harmonics; R – the radius of the base surface on which the coefficients are determined g_{nm} , h_{nm} ; $P_n^m(\cos \theta)$ – Legendre polynomials. The resulting series corresponds to the provisions of the multipole theory of the external magnetic field proposed by Gauss.

To find the magnetic field, it is necessary to differentiate the function U according to the corresponding coordinate. For the radial component, the expression is

$$H_R = -\frac{\partial U}{\partial R} = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{n+1}{R^{n+2}} \cdot \sum_{m=0}^n (g_{nm} \cos m\varphi + h_{nm} \sin m\varphi) \cdot P_n^m(\cos \theta). \quad (2)$$

The magnetic field strength of the first spatial harmonic is described by the expression (3)

$$H_{R1} = \frac{1}{4\pi R^3} \cdot [g_{10} \cos \theta + (g_{11} \cos \varphi + h_{11} \sin \varphi) \sin \theta]. \quad (3)$$

The coefficients g_{nm} , h_{nm} in expressions (1-3) have a certain physical meaning. They are equal to the magnetic moments of the elementary multipoles of the n – order of the spatial m – harmonic. The coefficients g_{10} , g_{11} , h_{11} , are respectively equal to the components of the dipole magnetic moment $g_{11} = M_x$, $h_{11} = M_y$, $g_{10} = M_z$.

In accordance with the developed point method, magnetic field strength H_{Ri} measurements are performed in eight external spaces. Coordinates of points $\phi_i = (i-1)90^\circ$, $\phi_k = (k-5)90^\circ$, where $i = 1 \dots 4$, $k = 5 \dots 8$.

As a result of a series of transformations, the resulting signal E_x, V is described by the following expression

$$E_x = \frac{12g_{11}}{k_f R^3} + \frac{1}{k_f} \sum_{n=5}^{\infty} \frac{1-(-1)^n}{R^{n+2}} \cdot \frac{(R_2/R_1)^{3-n} - 1}{R_2^2/R_1^2 - 1} \sum_{m=0}^n M_{xnm}.$$

Then the magnitude of the radial dipole moment with the methodological error is described by the expression (4)

$$M_x = g_{11} = E_x k_f R^3 / 12. \quad (4)$$

This expression (4) is a model equation for further estimating uncertainty.

Perform assessment of the measured value M_x , Am^2 and the associated standard and extended uncertainty. The source of measurements is the source of an external magnetic field of the “black box” type. To find the measured value, direct repeated observations $n = 20$ of the resulting signal E_x, V and distance were made (the object of measurement is the sensor) R, m . Rough errors were excluded from the number of observations. Corrected for systematic errors.

Specification of measurement data: measurements were performed under normal laboratory conditions. The resulting signal E_x, V was recorded with a universal digital voltmeter. The distance was measured with a steel ruler. An inductive sensor $k_f = 6 \pm 10^{-3} A/m \cdot V$ was used as a converter of the magnetic field strength into an electrical signal. Input values do not have a correlation.

Calculate the arithmetic average of multiple observations of the resulting signal

$$\bar{E}_{ind} = \frac{1}{20} \sum_{i=1}^{20} \hat{E}_{ind i} = 10.65 mV.$$

The standard deviation of the observation of the resulting signal:

$$S(\hat{E}_{ind}) = 0.05 mV.$$

Standard uncertainty of evaluation of the resultant measurement result

$$S(\bar{E}_{ind}) = \frac{0.05}{\sqrt{20}} = 0.011 mV.$$

Calculate the arithmetic average of multiple observations of the distance object – sensor

$$\bar{R}_{ind} = \frac{1}{20} \sum_{i=1}^{20} \hat{R}_{ind i} = 500 mm.$$

The standard deviation of the observation object distance – sensor:

$$S(\hat{R}_{ind}) = 0.01 mm.$$

Standard uncertainty of evaluation of the result of measuring the distance object – sensor

$$S(\bar{R}_{ind}) = \frac{0.01}{\sqrt{20}} = 0.002 \text{ mm}.$$

Standard Type B uncertainty estimates of input quantities that have a uniform law:

$$u(k_f) = \frac{0.001}{\sqrt{3}} = 5.7 \cdot 10^{-4} \frac{A}{m \cdot V}.$$

Find the sensitivity coefficients using the model equation (4)

$$c_i = \frac{\partial f}{\partial x_i} = \frac{\partial f}{\partial X_i} \Big|_{x_1, x_2, \dots, x_m};$$

$$c(\bar{E}_{ind}) = 0.083 \cdot \hat{k}_f \bar{R}^3 = 0.066 \frac{Am^2}{V};$$

$$c(\bar{R}_{ind}) = 0.25 \cdot \hat{k}_f \bar{E} \bar{R}^2 = 0.004 Am;$$

$$c(\hat{k}_f) = \frac{1}{12} \bar{R}^3 \bar{V} = 1.8 \cdot 10^{-5} Vm^3.$$

Further make the uncertainty budget (Table 1)

The model (4) is nonlinear. It is necessary to use the Taylor formula to find the total measurement uncertainty of the radial magnetic moment.

$$u_c(M_x) = \left(\left(\sum_{i=1}^3 c_i^2 u^2(x_i) + \sum_{i,j=1}^3 \left(\frac{1}{2} \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right)^2 + \frac{\partial f}{\partial x_i} \frac{\partial^3 f}{\partial x_i \partial x_j^2} \right) \cdot u^2(x_i) u^2(x_j) \right)^2 \right)^{1/2} =$$

$$= \left[c^2(\bar{V}_{ind}) u^2(\bar{V}_{ind}) + c^2(\bar{R}_{ind}) u^2(\bar{R}_{ind}) + c^2(\hat{k}_f) u^2(\hat{k}_f) + \frac{1}{2} \left(\left(\frac{\partial^2 M_x}{\partial U^2} \right)^2 + \left(\frac{\partial^2 M_x}{\partial R^2} \right)^2 + \left(\frac{\partial^2 M_x}{\partial k_f^2} \right)^2 + \dots \right) \right]^{1/2} =$$

$$= 1.1 \cdot 10^{-7} Am^2.$$

The evaluation of measurement result of the radial magnetic moment is determined by the expression

$$\bar{M}_x = \frac{1}{n} \sum_{k=1}^n f(E_{1,k}, R_{1,k}) = \frac{1}{20} \sum_{k=1}^{20} f(E_{1,k}, R_{1,k}) = 7 \cdot 10^{-4} A \cdot m^2. \quad (5)$$

According to [8] coverage ratio

$$k = t_{0.95}(v_{eff}).$$

Find effective number of freedom degrees

$$v_{eff} = \frac{u_c^4}{\sum_{i=1}^m \frac{u^4(y_i)}{v_i}} =$$

$$= \frac{u_c^4}{\left(\frac{(u(\bar{U}) \cdot c(\bar{U}))^4}{v_i} + \frac{(u(\bar{R}) \cdot c(\bar{R}))^4}{v_i} \right) + \frac{(u(\hat{k}_f) \cdot c(\hat{k}_f))^4}{\infty}} =$$

$$= \frac{(1.1 \cdot 10^{-7})^4}{(u(\bar{U}) \cdot c(\bar{U}))^4 + (u(\bar{R}) \cdot c(\bar{R}))^4} \approx 23.$$

Then get the extended uncertainty.

$$U = k u_c(\hat{M}_x) = t_{0.95}(23) \cdot 1.1 \cdot 10^{-7} = 2.07 \cdot 1.1 \cdot 10^{-7} = 2.27 \cdot 10^{-7} Am^2.$$

As a result, the assessment of the measured value, taking into account the expanded uncertainty, is

$$M_x = (0.700 \pm 0.002) mA \cdot m^2, \quad p = 0.95.$$

In order to study the measurements carried out by the developed point magnetometric methods and measuring instruments, for repeatability and reproducibility, experimental studies were carried out using an exemplary source of magnetic field.

Five observation groups were obtained, each of which contained eight, nine, ten, eight, and seven replications, respectively.

TABLE 1. THE UNCERTAINTY BUDGET OF MAGNETIC MOMENT MEASUREMENTS

Input value	Estimated input value	Standard uncertainty	Number of degrees of freedom	Probability distribution function of the input values	Sensitivity coefficient	Contribution of uncertainty, $A \cdot m^2$
E_{ind}	10.65 mV	0,011 mV	19	Normal distribution	$0,066 \frac{A \cdot m^2}{V}$	$7,3 \cdot 10^{-7}$
R_{ind}	500 mm	0,002 mm	19	Normal distribution	$0.004 A \cdot m$	$0,8 \cdot 10^{-5}$
k_f	$6 \frac{A}{mV}$	$5.7 \cdot 10^{-4} \frac{A}{mV}$	∞	uniform distribution	$1.8 \cdot 10^{-5} Vm^3$	$0.1 \cdot 10^{-7}$
M_x	$7 \cdot 10^{-4} A \cdot m^2$	$1.1 \cdot 10^{-7} Am^2$	–	–	–	–

Observations were obtained by measuring the dipole magnetic moment with a device with sensors located in the equatorial plane.

To check the homogeneity of the observations, the following mathematical methods were used:

1) the Bartlett criterion to test the admissibility of differences between estimates of variances;

2) Fisher's method for checking the admissibility of differences between arithmetic averages.

These methods are used in the analysis of several groups of observations.

For groups of observations $L=5$ and repeated observations n_i in each group, we obtain

$$M = 37 \ln \left(\frac{1}{37} \sum_{i=1}^5 k_i S_i^2 \right) - \sum_{i=1}^5 k_i \ln S_i^2, \quad \chi_1^2 = \frac{37}{54} M.$$

The level of significance is assumed to be 0.05, which corresponds to the probability $P = 0.95$. Find the tabulated value $\chi_q^2 = \chi_{0,05}^2 = 3,325$ and the values of the unbiased estimates of the variances of each of the five groups, $(Am^2)^2$: $S_1^2 = 5.7 \cdot 10^{-4}$; $S_2^2 = 11.5 \cdot 10^{-4}$; $S_3^2 = 8.2 \cdot 10^{-4}$; $S_4^2 = 17.3 \cdot 10^{-4}$; $S_5^2 = 23.1 \cdot 10^{-4}$.

From the known values of the dispersion estimates $S_1^2, \dots, S_5^2$, we find the value $\chi_1^2 = 2.687$.

Since the value χ_1^2 determined from the experimental data is less than the tabulated value χ_q^2 (condition $\chi_1^2 < \chi_q^2$ is satisfied), the differences between the estimates of the variances are valid.

Thus, as a result of a study on the convergence of measurements, it was established that the measurements are stable and the dispersions are homogeneous. Now, to answer the question about the reproducibility of measurements, we find the variance of reproducibility. Since the number of repeated observations in groups is different, the dispersion of reproducibility is determined by the formula:

$$S^2 = \frac{\sum_{i=1}^L k_i S_i^2}{\sum_{i=1}^L k_i}.$$

Value $S^2 = 11.1 \cdot 10^{-4} (Am^2)^2$, which confirms the reproducibility of measurements.

Similar studies have been conducted for devices with sensors located on spherical and cylindrical surfaces. For them, the values of the estimates of the variances of reproducibility $8.3 \cdot 10^{-4}$; $7,6 \cdot 10^{-4} (Am^2)^2$ are obtained.

Using the Fisher criterion, we perform an analysis of the admissibility of differences between arithmetic averages of groups whose values are equal, Am^2 : $\bar{M}_1 = 5,467$;

$\bar{M}_2 = 5,464$; $\bar{M}_3 = 5,466$; $\bar{M}_4 = 5,469$; $\bar{M}_5 = 5,462$. Then the values of the variance estimates S_L^2 , $\bar{S}^2$ are: $S_L^2 = 5.7 \cdot 10^{-5} (Am^2)^2$, $\bar{S}^2 = 6.6 \cdot 10^{-5} (Am^2)^2$.

Thus, the analysis showed that measurements carried out using the developed point magnetometric methods and measuring instruments have the qualities of convergence and reproducibility.

CONCLUSION

The proposed approach can be used in estimating the uncertainty of all point methods for measuring the magnetic moment. This can be used for appropriate control and test procedures to provide and compare results.

Based on the multipole model for describing an external magnetic field of an object, a model equation is obtained for estimating the uncertainty of measurements of the radial magnetic moment by the point method. A budget of uncertainty has been drawn up. Standard and extended uncertainties were obtained.

Weaknesses of this study occur when the object of measurement of large longitudinal form. It is also important to note the high requirements for the accuracy of the installation of sensors according to the proposed scheme of their location, which may affect the accuracy of measurements. This circumstance imposes special requirements for the qualification of operators.

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