ELECTROMAGNETIC WAVES DIFFRACTION ON A SYSTEM OF OPEN CONES IN TIME DOMAIN

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Abstract – **A model problem of electromagnetic waves diffraction on an equalperiodical multilayered perfectly conducting conical grating is considered. The solution method is based on using the Meler-Fock integral transforms and the method of the coupling problem. This allows both analytical and numerical solutions obtaining. The graphs representing the electric field component dependencies on the time parameter are provided for the case of the single cone with a longitudinal slot.**

I. INTRODUCTION

Periodical structures including the diffraction gratings are widespread in different branches of science and technology. Many works are devoted to the problems of the electromagnetic waves diffraction on the gratings [1, 2]. Nowadays the interest to the conical structures scattering properties is raised due to the broadband features of such structures and their ability to form the signals in time domain.

The numerical methods for the non-sinusoidal waves diffraction problems are the most widespread in mathematical modeling. They allow the electromagnetic waves distribution studying in space and time. However, these methods can not be applied in case of the open structures since in this case it is necessary to set so-called absorption conditions [3, 4] those are not always possible to perform. It is either necessary to make a physical experiment or compare the solution results in specific cases with the ones derived by means of analytical or analytically-numerical methods that is problematic.

In this work the rigorously stated model problem of the electromagnetic waves diffraction on a complex conical grating is investigated. The problem is solved by means of the analytically-numerical method based on the integral transforms applying without the absorption conditions introducing.

II. PROBLEM STATEMENT. SOLUTION METHOD

The conical structure Σ contains M open thin semi-infinite perfectly conducting cones Σ_1, $\Sigma_2, \ldots,$ Σ_M, with shared apex and axis ($\Sigma = \Sigma_1 \cup \Sigma_2 \cup \ldots \cup \Sigma_M$). Each of the cones has N slots periodically cut along the rulings (fig.1). Let $2\gamma_j$ be the aperture angle of the cone Σ_j and d_j is the angular slot width, $j = 1, 2, \ldots, M$, $l = 2\pi/N$ is the structure period. In the spherical coordinate system the cone Σ_j is defined by the $\theta = \gamma_j$ equation. The source of the electromagnetic waves is considered to be an electric ($\chi = 1$) or a magnetic ($\chi = 2$) radial dipole placed into the point M_0.

The Debye's potential $\upsilon^{(\chi)}(\vec{r}, t)$, $\chi = 1, 2$, corresponding to the $\vec{E}$, $\vec{H}$ field surrounding the conical surface and the source, satisfy the following conditions for every moment of time:

1) three-dimensional wave equation

$$\left(\Delta - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right)\upsilon^{(\chi)}(\vec{r}, t) = -\hat{F}^{(\chi)}(\vec{r}, t), \ \vec{r} \notin \Sigma, \tag{1}$$

$$\hat{F}^{(\chi)}(\vec{r}, t) = \frac{1}{\varepsilon^{2-\chi}\mu^{\chi-1} r} M_r^{(\chi)} \delta(\vec{r} - \vec{r}_0) f(t - t_0), \ \varepsilon\mu = \frac{1}{a^2};$$

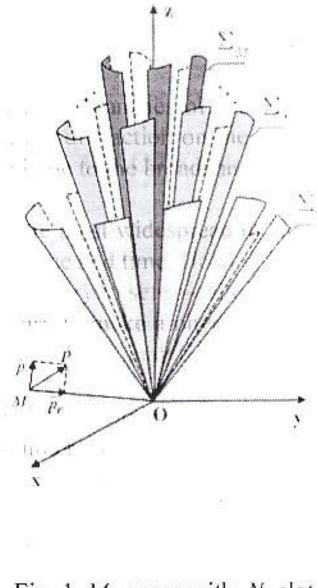

Fig. 1 M cones with N slots

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2) initial and boundary conditions

$$\upsilon^{(\chi)} \equiv 0 \equiv \frac{\partial \upsilon^{(\chi)}}{\partial t}, \ t \le t_0; \quad \frac{\partial^{\chi-1}}{\partial n^{\chi-1}}\left(\frac{\partial \upsilon^{(\chi)}}{\partial t}\right)\Bigg|_{\Sigma} = 0 \tag{3}$$

4) finite stored energy condition.

The boundary problem of mathematical physics (1)–(3) stated in such a way has the unique solution.

In order to find the function $\upsilon^{(\chi)}(\vec{r},t)$ one can use Green's function $G^{(\chi)}(\vec{r},t)$ apparatus. This function satisfies the three-dimensional wave equation, initial and boundary conditions and finite stored energy condition. In this case the potential $\upsilon^{(\chi)}(\vec{r},t)$ has the following form:

$$\upsilon^{(\chi)}(\vec{r},t) = \frac{M_r^{(\chi)}}{r_0 \varepsilon^{2-\chi} \mu^{\chi-1}} \int_0^{t-t_0} G^{(\chi)}(\vec{r}-\vec{r}_0, z) f(t-t_0-z)\, dz, \tag{4}$$

The sought function $G^{(\chi)}(\vec{r},t)$ can be represented according to the below formula:

$$G^{(\chi)}(\vec{r},t) = G_0(\vec{r},t) + G_1^{(\chi)}(\vec{r},t), \quad G_0(\vec{r},t) = \frac{\delta[t-t_0-R/a]}{4\pi R} \tag{5}$$

where $G_0(\vec{r},t)$ is the Green's function for the free space [5], and $G_1(\vec{r},t)$ corresponds to the scattered by the conical system field.

The stated electromagnetic problem solution is reduced to the Green's function $G_1^{(\chi)}(\vec{r},t)$ finding for the complex conical structure Σ.

In order to find the Green's function one can use the Meler-Fock integral transforms and represent the sought function $G_1^{(\chi)}(\vec{r},t)$ from (5) in the integral form

$$G_1^{(\chi)}(\vec{r},t) = \int_0^{+\infty} th\pi\tau \hat{G}_1^{(\chi)} P_{-1/2+i\tau}(chb)\, d\tau,$$

$$\hat{G}_1^{(\chi)} = -\frac{a}{r}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\sum_{m=-\infty}^{+\infty} \hat{a}_{m\tau}\hat{b}_{m\tau}^{(\chi),p}(\gamma_p,\theta_0)\, U_{m,i\tau}^{(\chi)}(\theta,\varphi),$$

where $\hat{a}_{m\tau}$, $\hat{b}_{m\tau}^{(\chi),p}$ are known coefficients,

$$U_{m\tau}^{(\chi)}(\theta,\varphi) = \begin{cases}
\sum\limits_{n=-\infty}^{\infty} \alpha_{m,n}^{(\chi,1)} P_{-1/2+i\tau}^{m+nN}(\cos\theta), & 0<\theta<\gamma_1, \\
\sum\limits_{n=-\infty}^{\infty} \left[\alpha_{m,n}^{(\chi,2)} P_{-1/2+i\tau}^{m+nN}(\cos\theta) + \beta_{m,n}^{(\chi,2)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta)\right] e^{i(m+nN)\varphi}, & \gamma_1<\theta<\gamma_2, \\
\sum\limits_{n=-\infty}^{\infty} \left[\alpha_{m,n}^{(\chi,3)} P_{-1/2+i\tau}^{m+nN}(\cos\theta) + \beta_{m,n}^{(\chi,3)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta)\right] e^{i(m+nN)\varphi}, & \gamma_2<\theta<\gamma_3, \\
\dots & \dots \\
\sum\limits_{n=-\infty}^{\infty} \left[\alpha_{m,n}^{(\chi,p)} P_{-1/2+i\tau}^{m+nN}(\cos\theta) + \beta_{m,n}^{(\chi,p)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta)\right] e^{i(m+nN)\varphi}, & \gamma_{p-1}<\theta<\gamma_p, \\
\dots & \dots \\
\sum\limits_{n=-\infty}^{\infty} \left[\alpha_{m,n}^{(\chi,M)} P_{-1/2+i\tau}^{m+nN}(\cos\theta) + \beta_{m,n}^{(\chi,M)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta)\right] e^{i(m+nN)\varphi}, & \gamma_{M-1}<\theta<\gamma_M, \\
\sum\limits_{n=-\infty}^{\infty} \beta_{m,n}^{(\chi,M+1)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta) e^{i(m+nN)\varphi}, & \gamma_M<\theta<\pi,
\end{cases}$$

where $\alpha_{m,n}^{(\chi,p)}$, $\beta_{m,n}^{(\chi,p)}$ are sought coefficients, $p = 1,2,\ldots,M+1$.

In the case of the two cones system ($M = 2$)

$$U_{m\tau}^{(\chi)}(\theta,\varphi) = \begin{cases} \sum\limits_{n=-\infty}^{\infty} \alpha_{m,n}^{(\chi)} P_{-1/2+i\tau}^{m+nN}(\cos\theta), & 0<\theta<\gamma_1, \\ \sum\limits_{n=-\infty}^{\infty} \left[\beta_{m,n}^{(\chi)} P_{-1/2+i\tau}^{m+nN}(\cos\theta) + \gamma_{m,n}^{(\chi)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta)\right] e^{i(m+nN)\varphi}, & \gamma_1<\theta<\gamma_2, \\ \sum\limits_{n=-\infty}^{\infty} \xi_{m,n}^{(\chi,M+1)} P_{-1/2+i\tau}^{m+nN}(-\cos\theta) e^{i(m+nN)\varphi}, & \gamma_2<\theta<\pi. \end{cases}$$

For finding the unknown coefficients, the boundary conditions on the conical stripes and the condition of the Green's function and its partial derivatives continuity inside the slots are used In this case the system of dual series equations has the following form:

$$\sum_{n=-\infty}^{+\infty} z_{m,n}^{(\chi),p} e^{inN\varphi} = g_{p,i\tau}^{(\chi),m}(\gamma_\kappa) e^{im_0N\varphi}, \quad \text{stripes of } \Sigma_p,\ p=1,2, \tag{6}$$

$$\sum_{n=-\infty}^{+\infty} \left[N(n+\nu)\right]^{\bar\rho(\chi)} \frac{|n|}{n}(1-\bar\varepsilon_{n,\kappa}^{(\chi)}) \left\{ z_{m,n}^{(\chi),1}\left[\hat h_{i\tau}^{(\chi),(n+\nu)N}(\pi-\gamma_2,\pi-\gamma_1)\right]^{p-1} - \right.$$

$$\left. - z_{m,n}^{(\chi),2}\left[\hat h_{i\tau}^{(\chi),(n+\nu)N}(\gamma_1,\gamma_2)\right]^{2-p} \right\} e^{inN\varphi} = 0, \text{ slots of } \Sigma_p, \tag{7}$$

where
$$z_{m,n}^{(\chi),\kappa} = \delta_p^1 \tilde\alpha_{m,n}^{(\chi)} \frac{d^{\chi-1}}{d\gamma_1^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(\cos\gamma_1) + \delta_p^2 \tilde\eta_{m,n}^{(\chi)} \frac{d^{\chi-1}}{d\gamma_2^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(-\cos\gamma_2)$$

$$\left[N(n+\nu)\right]^{\bar\rho(\chi)} \frac{|n|}{n}(1-\bar\varepsilon_{n,p}^{(\chi)}) = \frac{(-1)^{(n+\nu)N+\chi-1} ch\pi\tau}{\pi(\sin\gamma_p)^{1-\bar\rho(\chi)}} \frac{\Gamma(1/2+i\tau+(n+\nu)N)}{\Gamma(1/2+i\tau-(n+\nu)N)} \times$$

$$\frac{1}{\dfrac{d^{\chi-1}}{d\gamma_p^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(\cos\gamma_p) \dfrac{d^{\chi-1}}{d\gamma_p^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(-\cos\gamma_p)} \frac{1}{1-C_{i\tau}^{(\chi),(n+\nu)N}(\gamma_1,\gamma_2)}, \tag{8}$$

In order to investigate the physical processes appearing during the electromagnetic waves diffraction, one should study a basic problem for a single cone with a longitudinal slot. In this case the system of dual series equations (6)-(8) is of the form:

$$\sum_{n=-\infty}^{+\infty} x_{m,n}^{(1)} e^{inN\varphi} = e^{im_0N\varphi}, \text{ sectors of } \Sigma_2,$$

$$\sum_{n=-\infty}^{+\infty} \left[N(n+\nu)\right] \frac{|n|}{n}(1-\hat\varepsilon_n^{(1)}) x_{m,n}^{(1)} e^{inN\varphi} = 0, \text{ slots of } \Sigma_2,$$

$$\left[N(n+\nu)\right] \frac{|n|}{n}(1-\hat\varepsilon_n^{(1)}) = \frac{(-1)^{(n+\nu)/N} ch\pi\tau}{\pi} \frac{\Gamma(1/2+i\tau+(n+\nu)N)}{\Gamma(1/2+i\tau-(n+\nu)N)} \times$$

$$\times \frac{1}{P_{-1/2+i\tau}^{(n+\nu)N}(\cos\gamma_2) P_{-1/2+i\tau}^{(n+\nu)N}(-\cos\gamma_2)},$$

For finding the unknown coefficients $x_{m,n}^{(1)}$, the system of functional equations is reduced to the system of linear algebraic equations with compact matrix operator by means of the method of the coupling problem [1].

III. NUMERICAL RESULTS ANALYSIS

The electric field component E_θ behavior in time domain is studied depending on the fixed distance between the source and the cone apex r_0, on the distance between the cone apex and the observation point r, on the cone aperture angle γ, on the θ angle of the observation point provided that $\theta > 2\gamma$ and $t > (r + r_0)/a$, where a is the speed of the signal spreading in the medium. The graphs illustrating the numerical results (fig. 2, 3) include the dimensionless quantity $\xi = at/(r + r_0)$, ($\xi > 1$). The calculations of the dependencies mentioned above have been made according to the formula for E_θ.

The curves illustrating the electric field component E_θ (6) dependence on the ξ for different fixed values of the slot width and aperture angle are shown in the fig. 2, 3.

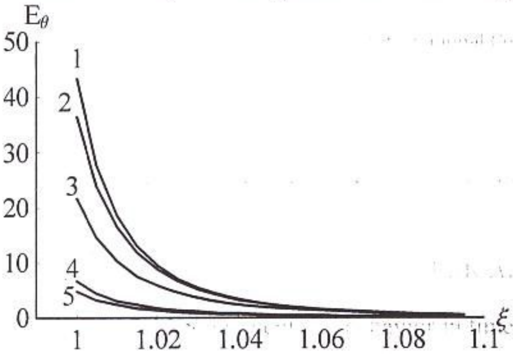

Fig. 2 – E_θ dependence on the slot width d.

$1 - d = 0$, $2 - d = \dfrac{\pi}{6}$, $3 - d = \dfrac{\pi}{2}$, $4 - d = \dfrac{3\pi}{2}$, $5 - d = \dfrac{5\pi}{3}$

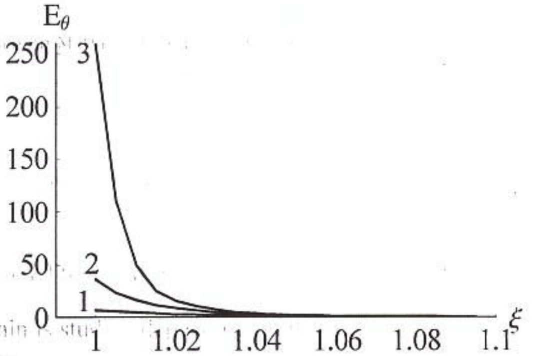

Fig. 3. – E_θ dependence on the cone aperture angle γ.

$1 - \gamma = \dfrac{\pi}{8}$, $2 - \gamma = \dfrac{\pi}{6}$, $3 - \gamma = \dfrac{\pi}{5}$

It can be seen in the fig. 2 that the less the scattering surface is (the wider the slot is), the scattered field observed in front of the slot middle decays. If the aperture angle of the cone increases, the scattered field increases as well (fig. 3).

IV. CONCLUSION

It is shown that the model electromagnetic waves diffraction problem on the multilayered conical gratings is equivalent to the system of linear algebraic equations for the Fourier coefficients. These coefficients are decomposition elements of the sought scalar function by the azimuth variable in the spherical coordinate system after the integral transforms and the method of the coupling problem applying. The field distribution in presence of a radial impulse dipole and a cone with longitudinal slot is studied in time domain.

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