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METHOD OF ROBUST ALGORITHM SYNTHESIS FOR SEPARATION SIGNALS AND INTERFERENCE IN ADAPTIVE ANTENNA ARRAYS

Problem formulation

It is known that an adaptive antenna array (AAA) may be applied to separation of the desired signals, or desired signals and interference, arrived from different directions.[1, 2].

To form an AAA-separator directivity characteristic the algorithms of a projective type are used [2, 3]

$$\vec{W}_j = \hat{\mathrm{P}}\hat{\vec{V}}_j, \quad j = \overline{1, L}, \tag{1}$$

where $\vec{W}_j$ is the N – dimensional weighting coefficient vector (WCV) of the j-th channel of an AAA; $\hat{\mathrm{P}}_j = \hat{\mathrm{B}}_j\left(\hat{\mathrm{B}}_j^H\hat{\mathrm{B}}_j\right)^{-1}\hat{\mathrm{B}}_j^H$; $\hat{\mathrm{B}}_j = \left[\hat{\vec{V}}_1\ \hat{\vec{V}}_2 \ldots \hat{\vec{V}}_{j-1}\ \hat{\vec{V}}_{j+1} \ldots \hat{\vec{V}}_L\right]$; $\hat{\vec{V}}_i, \quad i = \overline{1, L}$, is an estimation of vector $\vec{V}_i$, characterized spatial structure of the i-th signal; L is the number of signal sources; H is a Hermit conjugation sign.

In the case when vector $\vec{V}_i$ allows a scalar parameterization of $\vec{V}(\Theta_i)$ type for obtaining $\vec{V}(\hat{\Theta}_i)$ estimations, the traditional methods of spectral analysis may be used (MUSIC, method of entropy maximum and so on). If such a parameterization is not allowed, the so called "blind" algorithms are used [3, 4]. At the same time in both cases estimations $\hat{\vec{V}}_i$ are never accurately equal to the corresponding true values of $\vec{V}_i$. Therefore, the situations are possible when at some AAA output the signal/(interference+noise) ratio (SINR) becomes intolerably small.

The purpose of the work is the synthesis of the algorithms providing formation of AAA directivity characteristics satisfying the condition: "deviation of SINR ($\eta(\vec{W})$) from a potentially achievable (η_0) does not exceed a given value ($\Delta\eta$)".

Problem solution

Let us assume that the estimation accuracy of vectors $\vec{V}_i, \quad i = \overline{1, L}$, is characterized by

$$\mathrm{R}_{ii} = \vec{V}_i\vec{V}_i^H \in G_{yi};\ G_{yi} = \left\{\mathrm{R}_{ii} \mid \max_i\left\|\hat{\mathrm{R}}_{ii} - \mathrm{R}_{ii}\right\| \le \delta_0\right\}, i = \overline{1, L}, \tag{2}$$

where $\hat{\mathrm{R}}_{ii} = \hat{\vec{V}}_i\hat{\vec{V}}_i^H$; $\delta_0 \in R_+$; $\|\cdot\|$ is a designation of Frobenius norm matrix.

Analyzing the projective WCV (1), we come to the statement:

<u>Statement 1.</u> Projective WCL (1) coincides, with an accuracy of a constant coefficient, with an eigenvector (EV), corresponding to a maximum eigennumber (EN) of a matrix pencil $L_j(\lambda) = \hat{\mathrm{R}}_{jj} - \lambda\hat{\mathrm{R}}_\Sigma$,

where $\hat{\mathrm{R}}_\Sigma = \sum\limits_{i=1,\, i\ne j}^{L}\hat{\mathrm{R}}_{ii}$.

Direct use of statement 1 is possible only in the case when pencil $L_j(\lambda)$ is not singular, i.e. with $L \ge N$. At the same time, in practice, the most interesting case is just the case with $L < N$. Therefore, let us introduce a corresponding $L_j(\lambda)$ regularized pencil $L_j'(\lambda) = \hat{\mathrm{R}}_{jj} - \lambda\left(\hat{\mathrm{R}}_\Sigma + \alpha\mathrm{I}\right)$, where I is a unit matrix, $\alpha \in R_+$, and write a limiting relation

$$\lim_{\alpha\to 0}\vec{Q}\left(\lambda_{\max}\left(\mathrm{R}_{jj} - \lambda_N\left(\hat{\mathrm{R}}_\Sigma + \alpha\mathrm{I}\right)\right)\right) = \vec{W}_j, \tag{3}$$

re λ_N is a maximum EN of a regularized pencil $L_j'(\lambda)$; $\vec{Q}(\lambda_{max}(B))$ is an EV, corresponding to a ximum EN of matrix B.

A limiting relation (3) gives us a possibility to take advantage of the results of [5] and to represent a robust WCV in the form of $\vec{W}_{pj} = \vec{Q}(\lambda_{max}(F\{\hat{R}_{jj} - \lambda\hat{R}_{\Sigma}\}))$, where F{.} is some contracting operator. Substituting the problem of F{.} synthesis for the problem of this operator image construction, by some not complex reasoning, we come to the conclusion that the algorithms for the forming the AAA directivity characteristics meeting the condition $\eta(\vec{W}_p) \geq \eta_0 - \Delta\eta \; \forall \; R_{ii} \in G_{yi}$ may be presented in the form of

$$\vec{W}_{pj} = \vec{Q}\left(\lambda_{max}\left(F\left\{\lambda_j A_j - \sum_{i=1,\, i\neq j}^{L} \lambda_i A_i\right\}\right)\right), \quad j = \overline{1, L}, \tag{4}$$

where A_i is a center-symmetric matrix, the elements of which take the form of $(A_i)_{kk} = \left(\hat{\vec{V}}_i \hat{\vec{V}}_i^H\right)_{kk}$, $(A_i)_{kl} = \rho_{kl}\left(\hat{\vec{V}}_i \hat{\vec{V}}_i^H\right)_{kl}$, $k = \overline{1, N}$, $l = \overline{1, N}$, $\rho_{kl} = \rho_{lk} \in]0 \div 1]$; $\lambda_i \in R_+$, $i = \overline{1, L}$. For calculation of $\vec{Q}(\lambda_{max}(.))$ the traditional methods of linear algebra may be used (method of Givens, Jacobi and so on). At the same time algorithms (4) may be presented also in the form of globally converging recurrent procedures of the form

$$\vec{W}_{pj}(k+1) = \Pr\left\{\vec{W}_{pj}(k) + \mu_k\left(\lambda_j A_j - \sum_{i=1,\, i\neq j}^{L} \lambda_i A_i\right)\vec{W}_{pj}(k)\right\}, \quad j = \overline{1, L}, \tag{5}$$

where $\mu_k \in R_+$; Pr{.} is a projector on the hypersphere of unit radius.

Necessary for application of (4), (5) matrices A_i and coefficients λ_i may be determined reasoning from a possible maximum estimation error of vectors $\vec{V}_i$.

Analysis of algorithm efficiency

For clearness we shall estimate the efficiency of algorithms (4), (5) as applied to the case of AAA performing separation of two independent narrowband signals $s_1(t)$, $s_2(t)$. In this case, the efficiency is considered to mean SINR $\eta(\vec{W}_1) = P_1/(P_2 + \sigma_n^2)$; $\eta(\vec{W}_2) = P_2/(P_1 + \sigma_n^2)$; where P_1, P_2 – power of signals $s_1(t)$, $s_2(t)$; σ_n^2 – a variance of a thermal noise, and taking into account an obvious AAA symmetry, we shall limit ourselves by considering of SINR $\eta(\vec{W}_1)$ only (we consider signal $s_1(t)$ to be a desired one and $s_2(t)$ – interference, respectively). To obtain concrete numerical values of $\eta(\vec{W}_1)$ we introduce a set of assumptions characterizing a signal-interference situation and the AAA proper: the antenna array is linear, equidistant, $N = 3$; the antenna elements are isotropic and not interacting; the arrival angle of signal $s_1(t)$, determined relative to the normal to the line of antenna elements arrangement, $\Theta_1 = 28^0$; the arrival angle of signal $s_2(t)$ $\Theta_2 = 45^0$ (fig.1), Θ_2 – is a variable value (fig.2); the input signal/interference ratio $10\lg(P_1/\sigma_n^2) = 20$ dB (fig.2), $10\lg(P_1/\sigma_n^2)$ – a variable value (fig.1); the input interference/noise ratio $10\lg(P_2/\sigma_n^2) = 20$ dB; vectors $\vec{V}_1$, $\vec{V}_2$ allow a scalar parameterization $\hat{\vec{V}}_1(\Theta_1)$, $\hat{\vec{V}}_2(\Theta_2)$, in this case $\Theta_1 - \hat{\Theta}_1 = \Theta_2 - \hat{\Theta}_2 = \Delta\Theta = 5^0$.

Fig. 1, 2 (curves 1) show output SINR $\eta(\vec{W}_1)$ versus input SNR (P_1/σ_n^2) and the angle of interference arrival (Θ_2) calculated with the use of assumptions. Algorithm (5) was used for calculations, and matrices A_1, A_2 were determined as

$$A_1 = \frac{1}{2\Delta\Theta}\int_{\Theta_1-\Delta\Theta}^{\Theta_1+\Delta\Theta} \vec{V}_1(\Theta)\vec{V}_1^H(\Theta)\,d\Theta, \qquad A_2 = \frac{1}{2\Delta\Theta}\int_{\Theta_2-\Delta\Theta}^{\Theta_2+\Delta\Theta} \vec{V}_2(\Theta)\vec{V}_2^H(\Theta)\,d\Theta.$$

For comparison, the dependencies obtained with a projective algorithm (curve 2) we also plotted in Fig. 1, 2. In both figures, curve 3 denotes the curve characterizing potential efficiency of separation.

The plots show that for rather wide class of signal-interference situations the efficiency of separation realized on the basis of algorithms (4, 5), is close to potentially achievable one and practically does not depend on the input signal/noise ratio.

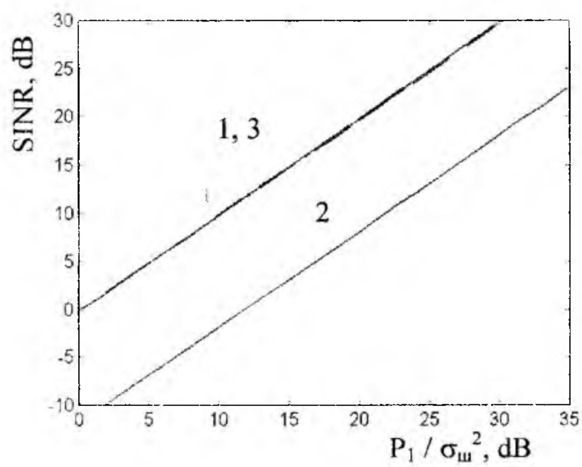

Fig. 1

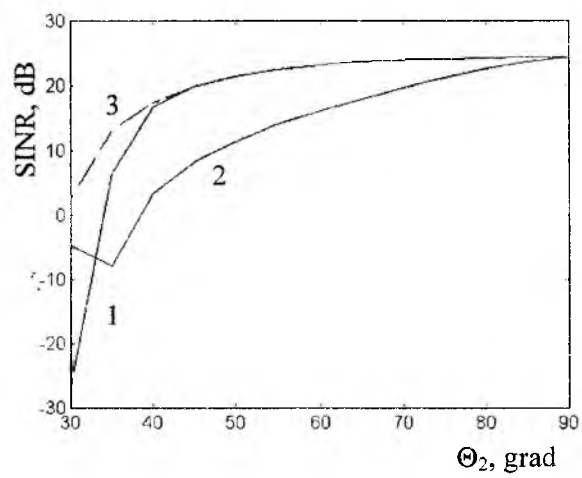

Fig. 2

References: 1. *Hacket C.M.* Adaptive arrays can be used to separate communication signal // IEEE Trans. Aerospace and Electronic Syst. 1981, V.AES – 17. № 2. P.234 –245. 2. *L.A.Marchuk, A.V.Efimov.* Algorithm of adaptive spatial separation of signals and interference // Radioelectronics and Communication System, 1996, т.39, №4, p.31-37. 3. *J.F Cardoso., A Souloumiac.* Blind beamforming for non-gaussian signals // Proc. Inst. Elec. Eng., pt.. F, vol. 140, №6, pp.362-370, December 1993. 4. *L.A.Marchuk, K.K.Venskauskas, V.A.Ivanov.* A polynomial-based approach to design of a small adaptive antenna array for blind separation of sources // Proc. 1997 IEEE AP-S International Symposium and USNC/URSI Na.ion Radio Science Meeting, July 21-26, 1998, Atlanta, USA. 5. *L.A.Marchuk.* Robust algorithms for adaptive spatial filtrating of signals with inaccurately know parameters // Telecommunications and Radio Engineering, 1997, №11, p.3-7.

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