ДОДАТОК А

Апробація наукових результатів досліджень

THE DIGITAL TWIN TO REPRESENT THE HEAT EXCHANGER AS THE AUTOMATION OBJECT THROUGH THE PARAMETRIC IDENTIFICATION

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***Annotation*:** This research is about the development of the digital twin envisaged to represent the heat exchanger as the automation object. The digital twin is developed on the basis of the simplified assumptions, that the heat transfer between the heater and coolant mediums inside the heat exchanger is through the equivalent plane wall. The relations between the processes in the heat exchanger and the mathematical model representing this heat exchanger as the automation object are discussed.

Key words: Heat exchanger, digital twin, automation object, mathematical modelling, heat conduction, parametric identification, initial boundary value problem.

The usage of the digital twins for automation engineering is in agreement with the green and digital transitions declared as the principal priorities of the European Union development until 2030 year, as it is highlighted in the political guidelines of the European Commission [1, 2]. Indeed, such approach allows to exclude the energy-consuming tests and material-consuming physical models from the researches. So, the relevance of this research is due to the agreement with the modern trends supported by the European green and digital transitions.

The digital twins are used usually to illustrate the properties of the researched objects, as it is discussed in the research [3] for the heat exchanger stations. The digital twins are based on the mathematical modelling of the processes inherent for the researched object, as it is discussed in the research [4] to consider the optimal controls for the transient processes of the heat conduction. The general approaches for the mathematical modelling of the heat exchangers as the automation objects proposed in the research [5] are based actually on the usage of the digital twins, so that the results of computer simulations of heat exchangers are used as data for the parametric identification of the linearized mathematical model representing these heat exchangers as the automation objects. The purpose of this research is in the continuation and development of the research [5] through the consideration of the more detailed representations of the heat exchangers as the automation objects.

The schematization of the heat exchanger is under the assumption, that the heat conduction is through the equivalent plane wall (Fig. 1), as it is in the research [5]. Such schematization leads to the following representations of the processes in the heat exchanger (Fig. 1):

$$\frac{\partial T}{\partial t}=\frac{\lambda}{c\rho}\frac{\partial^2 T}{\partial t^2},\ T\left(0,z\right)=T^{(0)}\left(z\right),\ a\le z\le b; \qquad (1)$$

$$\alpha_a T\left(t,a\right)-\lambda\frac{\partial T}{\partial z}\left(t,a\right)=\alpha_a T_a^{(inp)}\left(t\right),\ \alpha_b T\left(t,b\right)-\lambda\frac{\partial T}{\partial z}\left(t,b\right)=\alpha_b T_b^{(inp)}\left(t\right); \qquad (2)$$

$$T_a^{(out)}\left(t\right)=T_a^{(inp)}\left(t\right)-\frac{A_a\alpha_a}{c_a\rho_a v_a}\left(T_a^{(inp)}\left(t\right)-T\left(t,a\right)\right),\ T_b^{(out)}\left(t\right)=T_b^{(inp)}\left(t\right)+\frac{A_b\alpha_b}{c_b\rho_b v}\left(T\left(t,b\right)-T_b^{(inp)}\left(t\right)\right) \qquad (3)$$

where t,z are the time and the coordinate along the height of the wall; a,b are the coordinates of the boundary surfaces wall; $T\left(t,z\right),T^{(0)}\left(z\right)$ are the temperature and its initial value; λ,c,ρ are the

heat conductivity, the heat capacity and the density of the material of the wall: $\alpha_a, A_a, c_a, \rho_a$ and $\alpha_b, A_b, c_b, \rho_b$ are the heat transfer coefficient, the heat exchanging area, the heat capacity, the density of the heater and coolant mediums; $T_a^{(inp)}, T_a^{(out)}$ and $T_b^{(inp)}, T_b^{(out)}$ are the input and output temperatures of the heater and coolant mediums.

The mathematical model (1)–(3) allows to define the output temperatures for the given input temperatures of the heater and coolant by means the related numerical methods allowing the correspondent computer simulations, so this mathematical model represents the digital twin the heat exchanger.

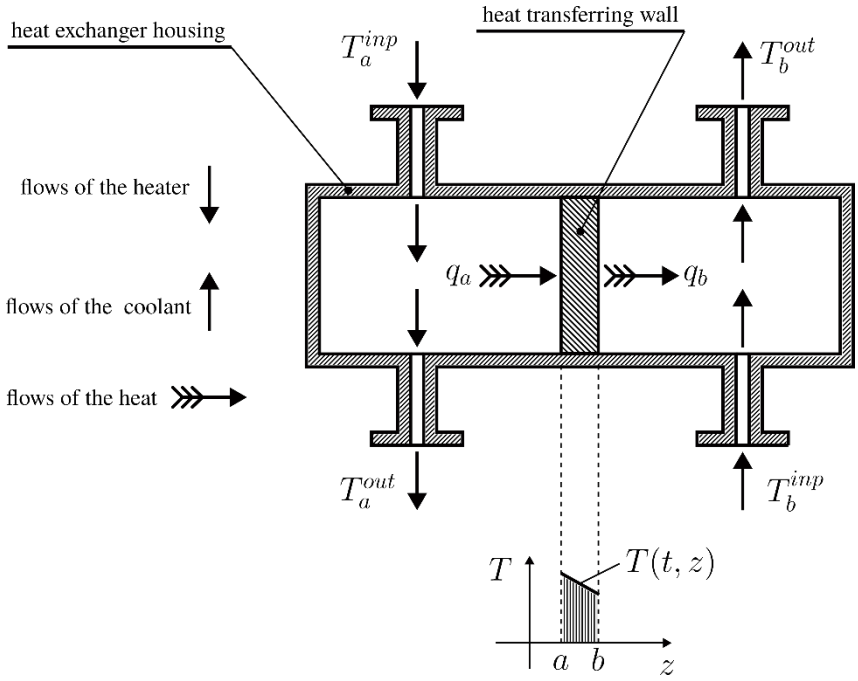

Figure 1 – The schematization of the heat exchanger and inherent heat conduction processes

The input temperatures of the heater and coolant must be considered as the controlling parameters, but the output temperatures of the heater and coolant must be considered as the controlled parameters of the heat exchanger.

CONCLUSIONS. The digital twin is developed on the basis of the simplified assumptions, that the heat transfer between the heater and coolant mediums inside the heat exchanger is through the equivalent plane wall.

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ДОДАТОК Б

Коди для використання в середовищі SCILAB

Модель 1

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=5; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,T0,"k--","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=15; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
```

```
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,T0,"k-","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=255; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,T0,"k:","linewidth",2);

titlex="$t,\mathrm{s}$"; titley="$T(t,a),\mathrm{K}$";
titlec="";
LEGEND=["$n=5$","$n=15$","$n=255$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res1.scg");
```

Модель 2

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=5; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tm,"k--","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=15; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
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n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tm,"k-","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=255; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tm,"k:","linewidth",2);

titlex="$t,\mathrm{s}$"; titley="$T\left(t,\frac{b-a}{2}\right),\mathrm{K}$";
titlec="";
LEGEND=["$n=5$","$n=15$","$n=255$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res2.scg");
```

Модель 3

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=5; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tn1,"k--","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=15; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
```

```
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tn1,"k-","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=255; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,Tn1,"k:","linewidth",2);

titlex="$t,\mathrm{s}$"; titley="$T\left(t,b\right),\mathrm{K}$";
titlec="";
LEGEND=["$n=5$","$n=15$","$n=255$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res3.scg");
```

Модель 4

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=5; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
x=X.values; t=T.time;
show_window(1);
plot(t,x,"k-","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=15; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
```

```
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
x=X.values; t=T.time;
show_window(1);
plot(t,x,"k--","linewidth",2);

a=0; b=0.005; c=586; ro=7830; lambda=48; T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000; Ab=1.5; rob=1000; cb=415; vb=0.5;
h=b-a; n=255; dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n); A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5; A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;
Context.tf=0.5; Context.dt=0.001;
scicos_simulate(scs_m,Context);
x=X.values; t=T.time;
show_window(1);
plot(t,x,"k:","linewidth",2);

titlex="$t,\mathrm{s}$"; titley="$T^{(2)}_{out}\left(t\right),\mathrm{K}$";
titlec="";
LEGEND=["$n=5$","$n=15$","$n=255$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res4.scg");
```

Модель 5

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005;
c=586; ro=7830; lambda=48;
T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000;
Ab=1.5; rob=1000; cb=415; vb=0.5;
n=255
h=b-a;
dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n);
A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5;
A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;

Context.tf=5;
scicos_simulate(scs_m,Context);

T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,T0,"k--","linewidth",2);
plot(t,Tm,"k-","linewidth",2);
plot(t,Tn1,"k:","linewidth",2);
titlex="$t,\mathrm{s}$"; titley="$T(t,z),\mathrm{K}$";
titlec="";
LEGEND=["$T\left(t,a\right)$","$T\left(t,\frac{a+b}{2}\right)$","$T\left(t,b\right)$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res5.scg");
```

Модель 6

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model2.zcos");
typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005;
c=586; ro=7830; lambda=48;
T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000;
Ab=1.5; rob=1000; cb=415; vb=0.5;
n=255
h=b-a;
dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n);
A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5;
A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;

Context.tf=15;
scicos_simulate(scs_m,Context);

T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;
show_window(1);
plot(t,T0,"k--","linewidth",2);
plot(t,Tm,"k-","linewidth",2);
plot(t,Tn1,"k:","linewidth",2);
titlex="$t,\mathrm{s}$"; titley="$T(t,z),\mathrm{K}$";
titlec="";
LEGEND=["$T\left(t,a\right)$","$T\left(t,\frac{a+b}{2}\right)$","$T\left(t,b\right)$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res6A.scg");
```

```
TA=X.values(:,1); x=X.values(:,2); TB=X.values(:,3); t=X.time;
show_window(2);
plot(t,x,"k-","linewidth",2);
plot(t,TB,"k:","linewidth",2);
titlex="$t,\mathrm{s}$"; titley="$T(t,z),\mathrm{K}$";
titlec="";
LEGEND=["$T{(2)}_{out}\left(t\right)$","$T{(2)}_{in}\left(t\right)$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res6B.scg");
```

Модель 7

```
clear; clc;

loadXcosLibs(); loadScicos();
importXcosDiagram("model3.zcos"); typeof(scs_m);
exec('FormatCharts.sce',-1);

a=0; b=0.005;
c=586; ro=7830; lambda=48;
T_0=300;
Ta=600; Tb=400;
alpha_a=30000; alpha_b=30000;
Ab=1.5; rob=1000; cb=415; vb=0.5;
n=255
h=b-a;
dz=h/(n+1);
m1=2*lambda/dz/(alpha_a+3/2*lambda/dz)-2;
m2=1-lambda/dz/2/(alpha_a+3/2*lambda/dz);
m3=alpha_a/(alpha_a+3/2*lambda/dz);
m4=1-lambda/dz/2/(alpha_b+3/2*lambda/dz);
m5=2*lambda/dz/(alpha_b+3/2*lambda/dz)-2;
m6=alpha_b/(alpha_b+3/2*lambda/dz);
A=zeros(n,n);
A(1,1)=m1; A(1,2)=m2;
for i=2:1:(n-1)
   A(i,i-1)=1; A(i,i)=-2; A(i,i+1)=1;
end
A(n,n-1)=m4; A(n,n)=m5;
A=A*lambda/c/ro/(dz^2);
B=zeros(n,2); B(1,1)=m3; B(n,2)=m6; B=B*lambda/c/ro/(dz^2);
n1=2*lambda/dz/(alpha_a+3/2*lambda/dz);
n2=-lambda/dz/2/(alpha_a+3/2*lambda/dz);
n3=alpha_a/(alpha_a+3/2*lambda/dz);
n4=-lambda/dz/2/(alpha_b+3/2*lambda/dz);
n5=2*lambda/dz/(alpha_b+3/2*lambda/dz);
n6=alpha_b/(alpha_b+3/2*lambda/dz);
C=zeros(2,n);C(1,1)=n1; C(1,2)=n2; C(2,n-1)=n4; C(2,n)=n5;
D=[n3,0;0,n6];
l1=alpha_b*Ab/rob/cb/vb;

Context.tf=15;
scicos_simulate(scs_m,Context);



T0=T.values(:,1); Tm=T.values(:,round((n+2)/2)); Tn1=T.values(:,n+2); t=T.time;

TA=X.values(:,1); x=X.values(:,2); TB=X.values(:,3); t=X.time;

ksi=x; u=TA; nn=length(ksi)-1; dt=t(2)-t(1);

a11=sum(ksi(1:nn).*ksi(1:nn)); a12=-sum(ksi(1:nn).*u(1:nn));
a22=sum(u(1:nn).*u(1:nn)); AA=[a11,a12;a12,a22];
b1=-sum(ksi(2:nn+1).*ksi(1:nn)); b2=sum(ksi(2:nn+1).*u(1:nn));
bb=[b1;b2];
```

```
xx=AA\bb;

a0=(xx(1)+1)/dt; b1=xx(2)/dt;
disp(a0,b1);
ksi_0=ksi(1);

importXcosDiagram("model4.zcos"); typeof(scs_m);

Context.tf=15; scicos_simulate(scs_m,Context);
ksi=XX;

show_window(1);
plot(t,x,"k-","linewidth",2);
plot(ksi.time,ksi.values,"k--","linewidth",2);
titlex="$t,\mathrm{s}$"; titley="$T(t,z),\mathrm{K}$";
titlec="";
LEGEND=["$T^{(2)}_{out}\left(t\right)$","$\xi\left(t\right)$"];
FormatCharts(titlex,titley,titlec,LEGEND,4);
xsave("res7.scg");
```

Модель 8

```
clear; clc;

loadXcosLibs(); loadScicos();
exec('FormatCharts.sce',-1);

function res=sumulation(u)
   importXcosDiagram("model5.zcos"); typeof(scs_m);
   Context.u=u;
   scicos_simulate(scs_m,Context);
   res=X;
endfunction

u=1; x1=sumulation(u);
u=2; x2=sumulation(u);
u=3; x3=sumulation(u);

show_window(1);
plot(x1.time,x1.values,"k-","linewidth",2);
plot(x2.time,x2.values,"k--","linewidth",2);
plot(x3.time,x3.values,"k:","linewidth",2);

titlex="$t,\mathrm{s}$"; titley="$T^{(2)}_{out}\left(t\right),\mathrm{K}$";
titlec="";
LEGEND=["$\tilde{u}=1$","$\tilde{u}=2$","$\tilde{u}=3$"];
FormatCharts(titlex,titley,titlec,LEGEND,1);
xsave("res8.scg");
```

№ док.	Позначення	Найменування	Кільк. арк..	Прим.
	ГЮІК. 401144. 037 ПЗ	Текстові документи		
1		Пояснювальна записка	62 с.	ел. ф.
2		Додаток А Апробація наукових результатів	2 с.	ел. ф.
3		Додаток Б Коди програм для використання у SCILAB	13 с.	ел. ф.
4		Додаток В Демонстраційний матеріал	15 с.	ел. ф.

					ГЮІК. 401144. 037 ВД		
Зм.	Арк.	№ докум.	Підпис	Дата			
Розроб.		Максим Р			Розроблення математичної моделі для системи автоматичного регулювання теплообмінного апарату Відомість атестаційної роботи	Лит.	Арк. 1 / Аркушів 1
Перевір.		Ромашов Ю.В					
Т.контр.						ХНУРЕ Кафедра КІТАР	
Н.контр.		Демська Н. П					
Затв.		Невлюдов І. Ш					