Mathematical Model and Method for Covert Estimation of Aerial Object Coordinates Using Two Optical-electronic Stations

Oleg Zemlyaniy
Department for Nonlinear Dynamics of Electronic Systems
O. Ya. Usikov Institute for Radiophysics and Electronics
National Academy of Sciences of Ukraine
Kharkiv, Ukraine
zolvas@ukr.net

Andrei Tevyashev
Department of Applied Mathematics
Kharkiv National University
of Radio Electronics
Kharkiv, Ukraine
andrew.teviashev@nure.ua

Igor Shostko
Department of Infocommunication Engineering
Kharkiv National University
of Radio Electronics
Kharkiv, Ukraine
iss010166@gmail.com

Anton Koliadin
Department of Applied Mathematics
Kharkiv National University
of Radio Electronics
Kharkiv, Ukraine
antonkoliadin@gmail.com

***Abstract*—The work is devoted to a method for determining the coordinates of an aerial object in a polygon coordinate system based on the results of data processing obtained using optoelectronic trajectory measurement stations, united into a single infocommunication network without the use of laser rangefinders to measure the slant distance.**

***Keywords*—aerial object, infocommunication network, optoelectronic trajectory measurement station.**

I. INTRODUCTION

Currently, the scientific direction of developing tools for testing modern samples of missile and artillery equipment during their field trials is developing intensively. A new solution for the methodology of building an information-measuring system (IMS) is proposed in [1–5] to solve this problem. For control and automation of IMS it is suggested to create it on the principle of a sensor infocommunication network of optoelectronic stations of trajectory measurements (OESTM).

The network of such type of spatially spaced OESTM provides detection of targets in the visible and infrared range of the spectrum, tracking and issuance of coordinates in real time in conditions of direct visibility for several stations. IMS OESTM can be used for various flight experiments, certification of aviation and missile and artillery systems, to provide information about the trajectory and video information to monitor the characteristics of various munitions with subsequent analysis of their test results.

One of the tasks that are solved when conducting field trials is to determine the coordinates of the air object in the polygon coordinate system. In this work the corresponding technique is developed, and the results on its verification with use of the software for the personal computer allowing carrying out modeling of coordinate's measurement process of aerial objects are given as well.

II. THE PROBLEM STATEMENT AND BASIC CALCULATION FORMULAS

OESTM are located on the test site along the test track. All OESTM are combined into a single information and measurement network and grouped into clusters, each of which contains at least two stations (Fig. 1). If necessary, increasing the number of stations in each cluster, as well as the organization of the network topology, in which each OESTM belongs to more than one cluster, allows achieving higher accuracy when measuring the parameters of the trajectory of the aerial object.

The following are the calculations for the case when the aerial object at any time is in the field of view of at least two stations from their set, located along the main direction of the polygon. The result of the measurements is a vector of object trajectory parameters in the local Cartesian coordinate system (LCCS):

$$X_{nt} = \left(X_{Mt}, Y_{Mt}, Z_{Mt}, V_{Xt}, V_{Yt}, V_{Zt} \right),$$

where $t = t_0, t_1, t_2, ..., t_i, ..., T$, $t_i = t_0 + i \cdot \Delta t$, $t_i = t_0 + i \cdot \Delta t$ – sampling interval; t_0, T are the times of start and end of the object trajectory measurement; $T - t_0 = N \cdot \Delta t$, N is the number of points on the trajectory at which measurements were made; X_{Mt}, Y_{Mt}, Z_{Mt} are the coordinates of the point of the object's trajectory at the time t; V_{Xt}, V_{Yt}, V_{Zt} is the projection of the object's velocity on the LCCS axes at the time t.

In Fig. 1, which shows a cluster consisting of two OESTM, the designations are as follows: $OX_MY_MZ_M$ is the local b (polygon) coordinate system; X_MOZ_M is the plane, which is tangent to the surface of the reference ellipsoid at the measurement site; $\left(x_i^S, y_i^S, z_i^S\right)$ are the coordinates of the station i in local coordinate system $OX_MY_MZ_M$; Y_{Mt}, y_i^S, y_{i+1}^S are heights above the plane X_MOZ_M of the object, stations i and $i+1$, accordingly; α_i is the angle between the direction from station i to station $i+1$ and the projection of the line of sight of the object from the station i to the plane X_MOZ_M; α_{i+1} is the angle between the direction from station i to station $i+1$ and the projection of the line of sight of the object from the station $i+1$ to the plane X_MOZ_M; β_i is the angle between the direction to the object from the station i and the plane passing through the point of the station i location parallel to the plane X_MOZ_M; d_i is the slant distance to the object from station i; $l_{i,i+1}$ is the distance between stations; $L_{i,i+1}$ is the distance between stations i and $i+1$ projections onto a plane X_MOZ_M.

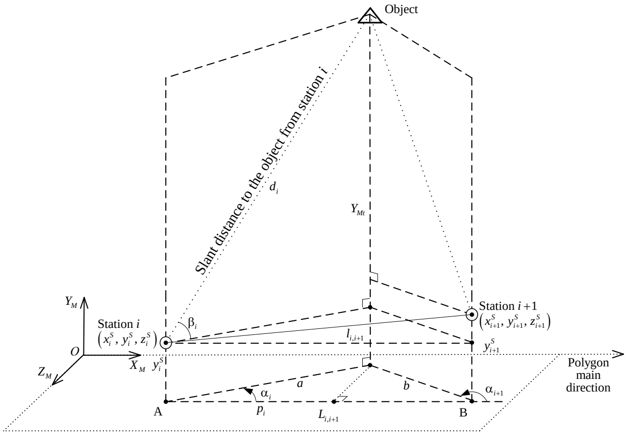


Fig.1. A cluster consisting of two OESTM

The calculation of the height of the object Y_{Mt} and its slant distance d_i from the station i at the time t with the given coordinates of the stations $\left(x_i^S, y_i^S, z_i^S\right)$, $\left(x_{i+1}^S, y_{i+1}^S, z_{i+1}^S\right)$, and the known values of α_i, α_{i+1}, β_i, obtained as a result of measurements, is carried out as follows.

For a triangle, lying in the plane X_MOZ_M, by the law of sines, we have:

$$\frac{a}{\sin\alpha_{i+1}} = \frac{b}{\sin\alpha_i} = \frac{L_{i,i+1}}{\sin\left(\alpha_{i+1} - \alpha_i\right)}. \tag{1}$$

Hence:

$$a = L_{i,i+1}\frac{\sin\alpha_{i+1}}{\sin\left(\alpha_{i+1} - \alpha_i\right)}, \tag{2}$$

$$b = L_{i,i+1}\frac{\sin\alpha_i}{\sin\left(\alpha_{i+1} - \alpha_i\right)}. \tag{3}$$

It follows from Fig. 1, that the height of the object and the distance between the object and the station i are determined in the following way:

$$Y_{Mt} = L_{i,i+1}\frac{\sin\alpha_{i+1}\mathrm{tg}\beta_{\mathrm{i}}}{\sin\left(\alpha_{i+1} - \alpha_i\right)} + y_i^S. \tag{4}$$

$$d_i = L_{i,i+1}\frac{\sin\alpha_{i+1}}{\sin\left(\alpha_{i+1} - \alpha_i\right)\cos\beta_i}. \tag{5}$$

The distance between the projections of the stations i and $i+1$ onto the plane X_MOZ_M according to the known coordinates of the stations is determined by the formula:

$$L_{i,i+1} = \sqrt{\left(x_{i+1}^S - x_i^S\right)^2 + \left(z_{i+1}^S - z_i^S\right)^2}. \tag{6}$$

Substituting (6) into (4) and (5), we finally get:

$$\begin{aligned} Y_{Mt} = {} & \sqrt{\left(x_{i+1}^S - x_i^S\right)^2 + \left(z_{i+1}^S - z_i^S\right)^2} \times \\ & \times \frac{\sin\alpha_{i+1}\mathrm{tg}\beta_{\mathrm{i}}}{\sin\left(\alpha_{i+1} - \alpha_i\right)} + y_i^S, \end{aligned} \tag{7}$$

$$\begin{aligned} d_i = {} & \sqrt{\left(x_{i+1}^S - x_i^S\right)^2 + \left(z_{i+1}^S - z_i^S\right)^2} \times \\ & \times \frac{\sin\alpha_{i+1}}{\sin\left(\alpha_{i+1} - \alpha_i\right)\cos\beta_i}. \end{aligned} \tag{8}$$

Let's calculate the coordinates of the object in LCCS.

Using the method of polar intersection [6] for determining the unknown coordinates of a point by two anchor nodes (stations i and $i+1$), knowing the angles α_i and γ_i (directional angle of the line AB between stations i and $i+1$), we have (see Fig. 2):

$$X_{Mt} = x_i^S + a\cos\left(\alpha_i + \gamma_i\right), \tag{9}$$

$$Z_{Mt} = z_i^S - a\sin\left(\alpha_i + \gamma_i\right). \tag{10}$$

After completing all the substitutions, we finally get:

$$\begin{aligned} X_{Mt} = {} & x_i^S + \sqrt{\left(x_{i+1}^S - x_i^S\right)^2 + \left(z_{i+1}^S - z_i^S\right)^2} \times \\ & \times \frac{\sin\alpha_{i+1}\cos\left(\alpha_i + \gamma_i\right)}{\sin\left(\alpha_{i+1} - \alpha_i\right)}, \end{aligned} \tag{11}$$

$$Z_{Mt} = z_i^S - \sqrt{\left(x_{i+1}^S - x_i^S\right)^2 + \left(z_{i+1}^S - z_i^S\right)^2} \times$$
$$\times \frac{\sin\alpha_{i+1}\sin\left(\alpha_i + \gamma_i\right)}{\sin\left(\alpha_{i+1} - \alpha_i\right)}. \qquad (12)$$

In this case $t = (i-1)k+1, (i-1)k+2, ..., ik$, where $i = 1, 2, ..., N/k$, k is the number of points on the trajectory obtained during measurements from each cluster.

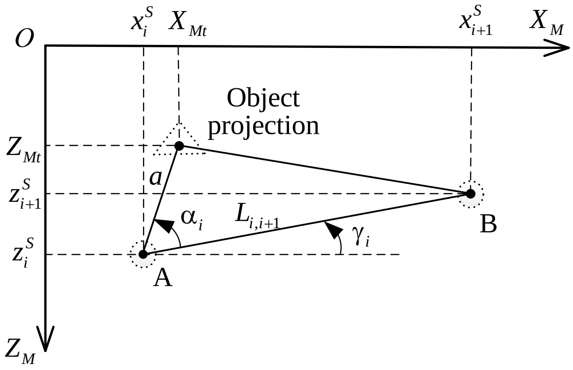


Fig. 2. Schematic of the polar intersection method

Thus, according to the results of measuring the angles of direction to the aerial object, obtained from two OESTM, the coordinates of which are known, the coordinates of the aerial object at each time t are uniquely calculated by formulas (7), (11), (12). If necessary, the projections of the object's velocity onto the LCCS axis are calculated from the results of two successive measurements of coordinates with known time interval (sampling interval Δt) between them:

$$V_{Xt} = \frac{X_{Mt} - X_{M(t-1)}}{\Delta t}, \qquad (13)$$

$$V_{Yt} = \frac{Y_{Mt} - Y_{M(t-1)}}{\Delta t}, \qquad (14)$$

$$V_{Zt} = \frac{Z_{Mt} - Z_{M(t-1)}}{\Delta t}. \qquad (15)$$

III. ALGORITHM AND ITS SOFTWARE IMPLEMENTATION

The algorithm for determining the coordinates of an aerial object in a polygon coordinate system based on the results of data processing obtained using the OESTM network is depicted in Fig. 3.

To verify the algorithm, software for a personal computer was created that allows modeling the process of determining the coordinates of an aerial object and verifying the data received and processed during field trials on a polygon. An interactive graphical user interface (GUI) is integrated into the software module, with the help of which the data and the parameters for modeling are entered, as well as the visualization of calculation results (Fig. 4) is performed.

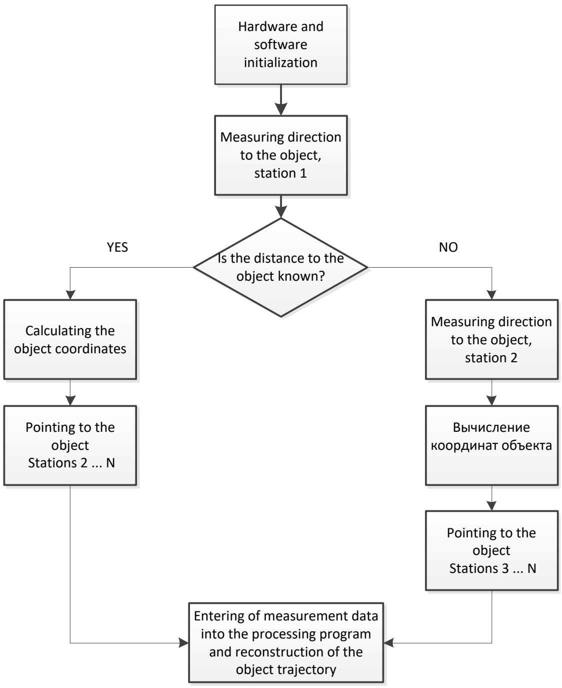
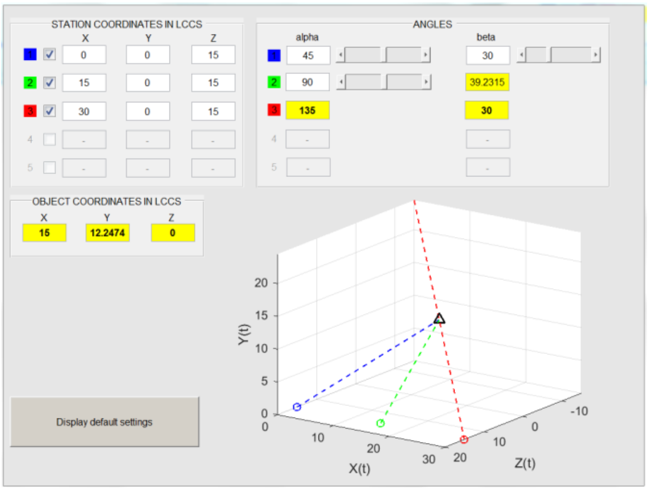


Fig. 3. The algorithm for determining the coordinates of an aerial object in a polygon coordinate system based on the results of data processing obtained using the OESTM network



STATION COORDINATES IN LCCS

		X	Y	Z
1	☑	0	0	15
2	☑	15	0	15
3	☑	30	0	15
4	☐	-	-	-
5	☐	-	-	-

ANGLES

	alpha	beta
1	45	30
2	90	39.2315
3	135	30
4	-	-
5	-	-

OBJECT COORDINATES IN LCCS

X	Y	Z
15	12.2474	0

Fig. 4. Graphical user interface that simulates the process of measuring the coordinates of an aerial object

Modeling procedure consists of several interrelated stages:

- when the program is launched for execution, a GUI window opens, in which all parameters are set by default;
- the user enters the coordinates of the stations and the angles of the directions to the object in the corresponding fields on the GUI panels;
- the results of calculating of the object coordinates are displayed in digital form;
- the location of the object and the direction of the optical axes of the video cameras of the stations are

visualized on a three-dimensional chart with the possibility of detailed analysis by rotating it along each of the three axes.

IV. CONCLUSIONS

The paper presents the mathematical model and the method for covert estimation of an aerial object coordinates by two OESTM. It is shown that the estimation of the coordinates of the aerial object can be carried out by measuring the object azimuth and elevation by each OESTM without the use of laser rangefinders to measure the slant distance between the aerial object and OESTM. The absence of irradiation by a laser beam makes it impossible for onboard systems to determine the fact of the aerial object detection, which allows covert monitoring of the airspace.

REFERENCES

[1] Shostko I., Tevyashev A., Kulia, Y., Koliadin A. Optical-electronic system of automatic detection and high-precise tracking of aerial objects in real-time // The Third International Workshop on Computer Modeling and Intelligent Systems, CMIS, 2020. P. 784–803.

[2] Tevyashev Andrey, Shostko Igor, Neofitniy Mihail, Koliadin Anton. Laser Opto-Electronic Airspace Monitoring System in The Visible and Infrared Ranges // 2019 IEEE 5th International Conference Actual Problems of Unmanned Aerial Vehicles Developments, APUAVD 2019 – Proceedings, 2019. С. 170–173, DOI: 10.1109/APUAVD47061.2019.8943887.

[3] Mathematical model and method of optimal placement of optical-electronic systems for trajectory measurements of air objects at test / A. D. Tevjashev, I. S. Shostko, M. V. Neofinyi, S. V. Kolomiyets, I. Yu. Kyrychenko, Yu. D. Pryimachov // Odessa Astronomical Publications. Vol. 32 (2019). С. 171–175. https://doi.org/10.18524/1810-4215.2019.32.182231

[4] Shostko I., Tevyashev A., Neofitnyi M., Kulia Y. Information-measuring system of polygon based on wireless sensor infocommunication network // Chapter in the book Lecture Notes on Data Engineering and Communications Technologies, 48. Publisher: Springer Nature, 2021. С. 649–674.

[5] Shostko I., Tevyashev A., Neofitnyi M., Ageyev D., Gulak S. Information and Measurement System Based on Wireless Sensory Infocommunication Network for Polygon Testing of Guided and Unguided Rockets and Missiles // 2018 International Scientific-Practical Conference on Problems of Infocommunications Science and Technology, PIC S and T 2018 – Proceedings, 2019. С. 705–710.

[6] Шостко И. С., Тевяшев А. Д., Неофитный М. В., Куля Ю. Э., Колядин А. В. Методы позиционирования узлов беспроводной сенсорной сети // Проблеми телекомунікацій, № 1 (24), 2019. С. 68–89