MATHEMATICAL METHODS FOR ENVIRONMENT REPRESENTATION IN COLLABORATIVE ROBOTICS: COMPARATIVE ANALYSIS AND APPLICATION RECOMMENDATIONS

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Annotation: The work examines the main mathematical methods used to describe the environment of collaborative mobile robots in digital twin and intelligent navigation systems. The analysis focuses on grid-based, geometric, graph-based, probabilistic, and dynamic models, including Occupancy Grid Map, Configuration Space SE(2) and SE(3), graph-based models, potential field models, probabilistic representations, point cloud models, and dynamic obstacle models. Their mathematical foundations, computational complexity, accuracy, scalability, and suitability for real-time applications are compared. The advantages and limitations of each approach are evaluated, and practical recommendations for their use in intelligent robotic systems are provided. The results are intended for researchers and engineers developing autonomous collaborative robotic systems operating in structured and dynamic environments.

Key words: collaborative robots, environment modeling, occupancy grid, configuration space, probabilistic model, point cloud, dynamic obstacles, digital twin.

МАТЕМАТИЧНІ МЕТОДИ ОПИСУ НАВКОЛИШНЬОГО СЕРЕДОВИЩА В КОЛАБОРАТИВНІЙ РОБОТОТЕХНІЦІ: АНАЛІЗ ТА РЕКОМЕНДАЦІЇ З ВИКОРИСТАННЯ

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Анотація: У роботі досліджено основні математичні методи опису навколишнього середовища колаборативних мобільних роботів у системах цифрових двійників та інтелектуальної навігації. Основну увагу приділено аналізу сіткових, геометричних, графових, ймовірнісних та динамічних моделей, включаючи Occupancy Grid Map, конфігураційні простори SE(2) і SE(3), графові моделі, потенціальні поля, ймовірнісні моделі, моделі на основі хмар точок та моделі динамічних перешкод. Проведено порівняння їх математичних властивостей, обчислювальної складності, точності та придатності до роботи в реальному часі. Надано практичні рекомендації щодо їх використання в інтелектуальних робототехнічних системах.

Ключові слова: колаборативні роботи, моделювання середовища, Occupancy Grid, конфігураційний простір, ймовірнісна модель, хмара точок, динамічні перешкоди, цифровий двійник.

An accurate mathematical representation of the environment is a fundamental requirement for the safe and efficient operation of collaborative robots in Industry 5.0 scenarios. The robot must continuously evaluate the spatial distribution of obstacles, free space, and dynamic objects to implement trajectory planning, collision avoidance, and collaborative interaction with humans and other robots. The Occupancy Grid Map model represents the environment as a discrete set of cells

$$m = \{c_i\}, P(c_i = occ|z_{1:t}) \tag{1}$$

where each cell stores a probability of occupancy based on sensor measurements. This method provides high robustness to sensor noise, but requires significant computational resources and memory for high-resolution environments.

The configuration space model describes the robot's motion in state space

$$x = (x, y, \theta) \in SE(2), x = (x, y, z, \phi, \theta, \psi) \in SE(3) \tag{2}$$

which allows explicit consideration of the orientation and geometry of the robot. The SE(2) space is computationally efficient and suitable for mobile robots moving in a plane, while SE(3) provides a complete spatial description but significantly increases the computational complexity.

Graph models represent the environment as a set of vertices and edges

$$G = (V, E) \tag{3}$$

where vertices correspond to robot states and edges correspond to permissible transitions between them. These models provide optimal trajectory planning using the A*, D*, and D* Lite algorithms, but require prior construction and updating when the environment changes.

The potential field model represents obstacles and targets using scalar potential functions

$$U(x) = U_{att}(x) + U_{rep}(x) \tag{4}$$

which provides efficient real-time navigation due to its low computational complexity. However, this method has the disadvantage of local minima and does not guarantee global optimality of the trajectory.

Probabilistic models represent the environment using probability distribution functions

$$P(x_t|z_{1:t}) \tag{5}$$

which allows for uncertainty and integration of noisy sensor data. These models are the mathematical basis of SLAM, Bayesian filtering, and digital twin synchronization, but require complex estimation algorithms.

Point cloud models represent the environment as a set of spatial points

$$P = \{p_i = (x_i, y_i, z_i)\} \tag{6}$$

obtained using LiDAR, stereo cameras or depth sensors. This approach provides high geometric accuracy and preserves the detailed spatial structure of the environment, but requires significant computational resources for processing and filtering.

Dynamic obstacle models describe moving objects using temporal state vectors

$$x(t) = [x, y, v_x, v_y]^T \tag{7}$$

which allows predicting their movement and implementing collision avoidance. These models provide adaptive navigation, but require continuous updating and the use of prediction algorithms such as the Kalman filter.

Comparative analysis shows that Occupancy Grid models provide high reliability and noise immunity, configuration spaces provide accurate geometric representation, graph models guarantee optimal planning, potential fields provide fast reactive navigation, probabilistic models provide operation under uncertainty, point cloud models provide high spatial accuracy, and dynamic obstacle models ensure safe operation in a changing environment.

Table 1 below shows a comparison of mathematical methods for describing the environment of collaborative robots.

Table 1 - Comparative analysis of environment representation methods for collaborative robots

Criterion	Occupancy Grid Map	Configuration Space SE(2) / SE(3)	Graph-based Model	Potential Field Model	Probabilistic Model	Point Cloud Mode	Dynamic Obstacle Model
Model type	Discrete, Probabilistic	Geometric, Kinematic	Discrete, topological	Analytical, continuous	Probabilistic	Geometric, 3D	Dynamic, time-dependent
Dimensionality	2D / 3D	SE(2): 3D, SE(3): 6D	Depends on the number of nodes	2D / 3D	Any	3D	2D / 3D + time
Computational complexity	Medium / High	Low (SE2), High (SE3)	Medium	Low	High	Very High	High
Memory requirements	High	Low	Medium	Low	Medium	Very High	Medium
Accuracy of description	Medium	High	Medium	Low / medium	High	Very High	High
Working with uncertainty	Yes	No	Limited	No	Yes (main advantage)	Limited	Yes
Working with dynamic objects	Limited	No	Limited	Yes (reactive)	Yes	Yes (with filtering)	Primary purpose
Suitability for digital twin	High	Very High	High	Low	Very high	Maximum	Very high
Main disadvantages	High Memory Needs	High complexity in SE(3)	Needs update	Local minima	Computationally complex	High CPU usage	Prediction complexity
Main areas of application	SLAM, Navigation	Kinematics, Planning	A*, D*, D* Lite	Reactive navigation	EKF, UKF, Bayesian SLAM	LiDAR, Digital Twin	HRC, autonomous navigation

CONCLUSIONS. For static structured environments, Occupancy Grid and graph models provide optimal accuracy and planning efficiency. SE(2) configuration space is recommended for mobile robots moving in a plane, while SE(3) is required for manipulators, drones, and robots operating in three-dimensional space. Potential field models are effective for real-time reactive navigation, but should be used in conjunction with global planning methods. Probabilistic models are critical when working under uncertainty and sensor noise. Point cloud models are recommended for digital twin systems and high-precision spatial modeling tasks. Dynamic obstacle models are required for collaborative robotic systems operating in environments with moving objects and people. The integration of multiple complementary mathematical models ensures maximum reliability, adaptability, and safety of collaborative robotic systems in complex dynamic environments

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