Evaluation of Unsteady Non-Harmonic Fields in Microwave Devices. III. Decomposition in the Regular Modes

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Introduction

Described in the Part I evaluation of unsteady non-harmonic fields in a dispersive electrodynamic line with decomposition of ones in N the *partial modes* is a universal and wideband approach. However, the "truncation" errors (the Gibbs' phenomenon) might be excessive for a small number N. Increasing N inflates requirements in computer resources. Therefore, the continuous approximation [1] based on *regular modes* of the line is of use for regular lines (a periodic with the period D delay structure or uniform smoothbore waveguide).

The Continuous Approximation

A q-th regular mode $\mathfrak{A}_{gq}(x,y,z,\beta)$ of the line is defined as a complex envelope in the longitudinal direction of the line normal eigenmode in a q-th passband:

$$\mathfrak{A}_{rqm}(x,y,z) = \mathfrak{A}_{gq}(x,y,z,\beta_m)\exp(-\mathrm{i}\,\beta_m z)\,,$$

so:

$$\mathfrak{A}_{gq}(x,y,z,\beta_m) = \mathfrak{A}_{rqm}(x,y,z)\exp \mathrm{i}\,\beta_m z$$

where $\beta_m = 2\pi m/ND$ is the m-th normal eigenmode longitudinal wavenumber. $\mathfrak{A}_{gq}$ is a periodic function of z with the period of D (for a smoothbore line, $\mathfrak{A}_{gq}$ does not depend on z, and D is a conventional period defining a upper bound of the considered β).

The *generic potential* is evaluated by expansion of $\mathfrak{A}_{gq}$ in a Taylor series in β (the subscript 0 implies that an item is taken for $\beta = 0$):

$$\mathfrak{A}(t,x,y,z) = \sum_q \left(\mathfrak{A}_{gq0} u_{gq} + \frac{\mathrm{i}}{1!}\frac{\partial \mathfrak{A}_{gq0}}{\partial \beta}\frac{\partial u_{gq}}{\partial z} - \frac{1}{2!}\frac{\partial^2 \mathfrak{A}_{gq0}}{\partial \beta^2}\frac{\partial^2 u_{gq}}{\partial z^2} - \frac{\mathrm{i}}{3!}\frac{\partial^3 \mathfrak{A}_{gq0}}{\partial \beta^3}\frac{\partial^3 u_{gq}}{\partial z^3} + \frac{1}{4!}\frac{\partial^4 \mathfrak{A}_{gq0}}{\partial \beta^4}\frac{\partial^4 u_{gq}}{\partial z^4} + \ldots \right). \quad (1)$$

A temporal and longitudinal dependence $u_{gq}(t,z)$ of the q-th regular mode instantaneous value is a solution of the excitation equation [1]:

$$\frac{\partial^2 u_{gq}}{\partial t^2} + 2\frac{\partial}{\partial t}\left[\delta_{rq0} u_{gq} - \frac{1}{2!}\frac{d^2 \delta_{rq0}}{d\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2} + \frac{1}{4!}\frac{d^4 \delta_{rq0}}{d\beta^4}\frac{\partial^4 u_{gq}}{\partial z^4} - \ldots \right] + (\omega_{rq}^2)_0 u_{gq} - \frac{1}{2!}\frac{d^2(\omega_{rq}^2)_0}{d\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2} + \frac{1}{4!}\frac{d^4(\omega_{rq}^2)_0}{d\beta^4}\frac{\partial^4 u_{gq}}{\partial z^4} - \ldots = \frac{1}{2D}\int_{z-D/2}^{z+D/2} d\zeta \int_{S_\perp} dxdy \left[\frac{\mathfrak{A}^*_{gq0}(x,y,\zeta)}{\tilde{W}_{gq0}} \cdot j(t,x,y,\zeta) - \frac{\mathrm{i}}{1!}\frac{\partial}{\partial \beta}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0 \frac{\partial j}{\partial z} - \frac{1}{2!}\frac{\partial^2}{\partial \beta^2}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0 \frac{\partial^2 j}{\partial z^2} + \frac{\mathrm{i}}{3!}\frac{\partial^3}{\partial \beta^3}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0 \frac{\partial^3 j}{\partial z^3} + \frac{1}{4!}\frac{\partial^4}{\partial \beta^4}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0 \frac{\partial^4 j}{\partial z^4} - \ldots \right] \quad (2)$$

where $\omega_{rq}(\beta)$ and $\delta_{rq}(\beta)$ are the eigenfrequency and the damping factor of the normal eigenmode for the line q-th passband respectively;

$$\tilde{W}_{gq}(\beta) = \frac{\varepsilon_0}{2D}\int_D dz \int_{S_\perp} dxdy \left|\mathfrak{A}_{gq}(x,y,z,\beta)\right|^2$$

is a linear unit *pseudoenergy* of the q-th regular eigenmode, J·s^2/m. Other items are same as in the Part I. For the energy normalization, $\tilde{W}_{gq}(\beta) \equiv 1$ J·s^2/m and the right-hand side of (2) is simpler.

Expression (2) is a *generalized wave equation* for a regular dispersive and dissipative electrodynamic line of arbitrary geometry and dispersion characteristic. The classic wave equation, the Klein-Gorgon equation, and the Telegraphist's equation are subsets of one. The left-hand side of (2) is similar to the 1D wave equation at first sight. However, (1) and (2) are fully 3D expressions taking into account the delayed potentials in the transverse directions of the line as well as in the longitudinal one.

The Four-Vector Potentials

One of disadvantages of (1) and (2) is that the Lorentz gauge of the vector potential is guaranteed only if a PIC model exactly fulfills the current continuity law. If this is a burdensome routine, the relativistic four-vectors [2] may be useful as ensuring the Lorentz gauge inherently. The potential four-vector (marked hereinafter by four dots over a variable) $\ddot{\ddot{A}}(t,x,y,z) = (\phi/c, \vec{A})$ is a solution of the 4D Maxwell's equations (see [2]) $\Box^2 \ddot{\ddot{A}} = -\mu_0 \ddot{\ddot{j}}$; $\ddot{\ddot{\Box}} \cdot \ddot{\ddot{A}} = 0$, where $\ddot{\ddot{\Box}}$ is the 4D vector differential operator:

$$\ddot{\ddot{\Box}} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right);$$

$\ddot{\ddot{j}}(t,x,y,z) = (c\rho, \vec{j})$ is the current density four-vector ($\ddot{\ddot{\Box}} \cdot \ddot{\ddot{j}} = 0$).

When $\ddot{\ddot{A}}$ is used for the fields definition, most of the transverse eigenvalues (cutoff frequencies) of a regular smoothbore electrodynamic line are at least thrice degenerated. Conventionally, first two wave modes are TE_{ij} (H_{ij}) and TM_{ij} (E_{ij}) solenoidal ones having $(0,A_x,A_y,0)$ and $(0,A_x,A_y,A_z)$ components of $\ddot{\ddot{A}}$ respectively. The third mode may be denominated as *Zero Magnetic* (ZM_{ij}) or *Potential* (P_{ij}) having (A_t,A_x,A_y,A_z) components. This is a "virtual" wave having no electric or magnetic field components in the empty line, but usable as a base for decomposition of the electric field in the beam presence. The ZM (P) mode can be treated physically as inseparable combination of two waves having only A_t and only (A_x,A_y,A_z) divergent components respectively, which are coupled by the Lorentz gauge and suppress one another in an empty line, but their equilibrium locally upsets when free charges appear. Using of the single ZM wave instead of both abovementioned its constituents guarantees the Lorentz gauge notwithstanding computational errors as the $\ddot{\ddot{j}}$ components are evaluated.

The H, E, and P wave modes are the full set of the eigenfunctions for a regular simply connected smoothbore line. Any delay structure also has "potential" passbands in addition to the usual "solenoidal" ones. The TEM_i (T_i) modes have to be added to the set for a multiply connected line.

As a simplest example, the regular eigenmode $\ddot{\ddot{A}}_{gq}(x,y,\omega,\beta)$ components are given below for a rectangular ($\Delta X \times \Delta Y$) waveguide using the amplitude normalization ($A_0 = 1$ V·s/m). For the TE wave these are

$$A_{gqt}=0\,;$$

$$A_{gqx}=-A_0\frac{k_{yq}}{\sqrt{k_{xq}^2+k_{yq}^2}}\cos k_{xq}x\sin k_{yq}y\,;$$

$$A_{gqy}=A_0\frac{k_{xq}}{\sqrt{k_{xq}^2+k_{yq}^2}}\sin k_{xq}x\cos k_{yq}y\,;$$

$$A_{gqz}=0\,.$$

For the TM wave:

$$A_{gqt}=0\,;$$

$$A_{gqx}=-\mathrm{i}\,A_0\frac{k_{xq}\beta}{k_{xq}^2+k_{yq}^2}\cos k_{xq}x\sin k_{yq}y\,;$$

$$A_{gqy}=-\mathrm{i}\,A_0\frac{k_{yq}\beta}{k_{xq}^2+k_{yq}^2}\sin k_{xq}x\cos k_{yq}y\,;$$

$$A_{gqz}=A_0\sin k_{xq}x\sin k_{yq}y\,.$$

For the ZM wave:

$$A_{gqt}=A_0\sin k_{xq}x\sin k_{yq}y\,;$$

$$A_{gqx}=\mathrm{i}\,A_0\frac{\omega k_{xq}}{c(k_{xq}^2+k_{yq}^2+\beta^2)}\cdot\cos k_{xq}x\sin k_{yq}y\,;$$

$$A_{gqy}=\mathrm{i}\,A_0\frac{\omega k_{yq}}{c(k_{xq}^2+k_{yq}^2+\beta^2)}\cdot\sin k_{xq}x\cos k_{yq}y\,;$$

$$A_{gqz}=A_0\frac{\omega\beta}{c(k_{xq}^2+k_{yq}^2+\beta^2)}\cdot\sin k_{xq}x\sin k_{yq}y$$

where $k_{xq}=\pi i_q/\Delta X$; $k_{yq}=\pi j_q/\Delta Y$ are the transverse wavenumbers in x and y directions respectively; $i_q = 1, 2, \dots$ and $j_q = 1, 2, \dots$ are the indexes. For the TE wave, one of these indexes may be zero.

The linear pseudoenergy is calculated using the four-vector multiplication rule:

$$\tilde{W}_{gq}(\omega,\beta)=\frac{\varepsilon_0}{2}\int_{S_\perp}dxdy(A_{gqt}A_{gqt}^*-A_{gqx}A_{gqx}^*-A_{gqy}A_{gqy}^*-A_{gqz}A_{gqz}^*)\,.$$

E.g., for the TE wave in the waveguide this is

$$\tilde{W}_{gq}=-A_0^2\frac{\varepsilon_0\Delta X\,\Delta Y}{8}\,.$$

For the TM wave:

$$\tilde{W}_{gq}=-A_0^2\frac{\varepsilon_0\Delta X\,\Delta Y}{8}\left[1+\frac{\beta^2}{k_{xq}^2+k_{yq}^2}\right].$$

For the ZM wave:

$$\tilde{W}_{gq}=A_0^2\frac{\varepsilon_0\Delta X\,\Delta Y}{8}\cdot\left[1-\frac{\omega^2}{c^2(k_{xq}^2+k_{yq}^2+\beta^2)}\right].$$

Because $\overset{....}{A}_{gq}$ and $\tilde{W}_{gq}$ are both ω and β dependent, the generalized wave equation for a regular dispersive and dissipative electrodynamic line (2) must be rewritten as

$$\frac{\partial^2u_{gq}}{\partial t^2}+2\frac{\partial}{\partial t}\left[\delta_{rq0}u_{gq}-\frac{1}{2!}\frac{d^2\delta_{rq0}}{d\beta^2}\frac{\partial^2u_{gq}}{\partial z^2}+\frac{1}{4!}\frac{d^4\delta_{rq0}}{d\beta^4}\frac{\partial^4u_{gq}}{\partial z^4}-\dots\right]+(\omega_{rq}^2)_0u_{gq}$$
$$-\frac{1}{2!}\frac{d^2(\omega_{rq}^2)_0}{d\beta^2}\frac{\partial^2u_{gq}}{\partial z^2}+\frac{1}{4!}\frac{d^4(\omega_{rq}^2)_0}{d\beta^4}\frac{\partial^4u_{gq}}{\partial z^4}-\dots=\frac{1}{2D}\int_{z-D/2}^{z+D/2}d\zeta\int_{S_\perp}dxdy\left[\frac{\overset{....}{A}{}^*_{gq0}(x,y,\zeta)}{\tilde{W}_{gq0}}\right.$$
$$\cdot\overset{....}{j}(t,x,y,\zeta)+\frac{\mathrm{i}}{1!}\frac{\partial}{\partial\omega}\left(\frac{\overset{....}{A}{}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial\overset{....}{j}}{\partial t}-\frac{\mathrm{i}}{1!}\frac{\partial}{\partial\beta}\left(\frac{\overset{....}{A}{}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial\overset{....}{j}}{\partial z}-\frac{1}{2!}\frac{\partial^2}{\partial\omega^2}\left(\frac{\overset{....}{A}{}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^2\overset{....}{j}}{\partial t^2}$$
$$+\frac{1}{2!}\frac{\partial^2}{\partial\omega\partial\beta}\left(\frac{\overset{....}{A}{}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^2\overset{....}{j}}{\partial t\partial z}\left.-\frac{1}{2!}\frac{\partial^2}{\partial\beta^2}\left(\frac{\overset{....}{A}{}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^2\overset{....}{j}}{\partial z^2}+\dots\right]$$

where the subscript 0 implies that an item is taken for $\omega = 0$ and $\beta = 0$. The four-vector potential can be evaluated using the expansion of $\ddddot{A}_{gq}$ in a Taylor series in ω and β:

$$\ddddot{A}(t,x,y,z)=\sum_q\left(\ddddot{A}_{gq0}u_{gq}-\frac{\mathrm{i}}{1!}\frac{\partial\ddddot{A}_{gq0}}{\partial\omega}\frac{\partial u_{gq}}{\partial t}+\frac{\mathrm{i}}{1!}\frac{\partial\ddddot{A}_{gq0}}{\partial\beta}\frac{\partial u_{gq}}{\partial z}-\frac{1}{2!}\frac{\partial^2\ddddot{A}_{gq0}}{\partial\omega^2}\frac{\partial^2 u_{gq}}{\partial t^2}+\frac{1}{2!}\frac{\partial^2\ddddot{A}_{gq0}}{\partial\omega\partial\beta}\frac{\partial^2 u_{gq}}{\partial t\partial z}-\frac{1}{2!}\frac{\partial^2\ddddot{A}_{gq0}}{\partial\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2}-\dots\right).$$

The electric and the magnetic fields can be derived from the four-vector potential as components of the electromagnetic field tensor [2].

Conclusion

Described excitation equation using the potential and the current density four-vectors is effective only for non-relativistic devices. The right-hand side calculation errors growth for the high-voltage tubes, because $\tilde{W}_{gq}$ as well as $\ddddot{A}^*_{gq}\cdot\ddddot{j}$ tend to zero for the ZM wave if the longitudinal velocity of the beam is close to the *c*. Possibly, this singularity can be eliminated by some change of variables.

References

[1] A. V. Gritsunov, "Non-monochromatic fields in a dispersive electrodynamic line. II. The continuous approximation," in *Proc. Fifth IEEE Int. Vacuum Electronics Conf. (IVEC 2004)*, Monterey, CA, 2004, pp. 222-223.

[2] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*. Reading: Addison-Wesley, 1964, vol. 2, ch. 25, 26.

[3] *Ibid*, ch. 32.

Final Remarks

Suggested in the Parts I–III methods of solving the wave equation for unsteady non-harmonic fields use the variable separation (Fourier) method and maintain all well-known advantages of one. However, if any dielectric and dissipative elements present in the device, the temporal and the spatial variables might not separate, as well as other difficulties may arise with taking into account the frequency-dependent complex permittivity $\varepsilon(\omega)$. Instead, the polarization and the conduction current vectors [3] of the non-uniform dielectrics may be added to the right-hand side of the excitation equations. This allows expanding the fields in a mode basis derived for an "auxiliary" system having the same metallic walls, but filled with the vacuum.

A problem of the contemporary microwave and optical electronics is evaluation of unsteady fields (including short video pulses) in dispersive, dissipative, non-stationary, and nonlinear media. Suggested methods are capable of this challenge accepting, as it seems. For a nonlinear medium, the linear decomposition of "weak" fields in the partial modes or oscillets can be performed at first. Thereupon, nonlinear terms may be added to the excitation equations.

A nontrivial task is a "direct" solving the intervalues problem, i.e., finding the partial modes and the oscillets of an electrodynamic system without previous solving the eigenvalue problem.

All previously described recognizes the partial modes, the oscillets, and the regular modes as prospective bases for explorations of generation, amplification, and propagation of non-stationary non-harmonic fields including the UWB EMP in various microwave devices and media.