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ANOTHER DYNAMIC MODEL OF MARKOVIAN HUMAN-MACHINE-ENVIRONMENT SYSTEM THAT IS EFFECTED BY SOME HAZARD

Introduction

This work continues the series of works on modeling systems and processes for protection of operators and public from exposure to electromagnetic radiation in radio systems. The general problem and classification of models for radio protection are given in [1], and for the first time such a model is proposed in the paper [2].

It is based on the assumption that accidents and disasters (hereinafter – the "events") have the Markov property. The corresponding mathematical apparatus is for the first time systematically set out in the work of Khinchin [3], and further developed in queuing theory [4]. Currently, this model is extended to the case of different types of harmful effects and non-classical flow of events that is now more important than just protection from the effects of electromagnetic radiation.

The aim of this work is to assess the dynamic and steady-state (limiting) probability for all possible states of the system, that has a multi-stage recovery subsystem, including humans.

In accordance with the statement of the problem in this paper we consider Human-Machine-Environment systems of three types: with simplest flow of events, with Erlang flow and with a non-homogeneous flow. The reaction time to the events of the subsystems "machine" and "environment" by "operator person" is exponentially distributed with parameter μ. All these types of events lead with a certain probability to a change in the system state. The probabilities of the states at the given time, finite or infinite, are of interest. This work is based on general approach and formulas from the paper [5].

Time and quality of operations at the accident depends on the state of human operator's health, which, in turn, depends on the state of the system (and the harmful effects, or stress). We assume that the operator can be in two states, relatively speaking, "fully functional" and "partially functional". Probability of recovery efficiency in the process of liquidation of the accident is zero, and the probability of a working state when performing i-th operations is equal to b_i. After finishing the work, performance of operator is restored or he is replaced. Then the transition probabilities for each pair of states $i+1$ and $(i+1)_b$ with full and partial capability of the operator, respectively, are $(1-b_i)\mu_i$ and $b_i\mu_i$. System status, intensity and probability of recovery work with incomplete capability of the operator, respectively, are i_b, μ^b and P_{ib}.

System diagram of the possible states and transitions is shown in Fig. 1, similar to[5].

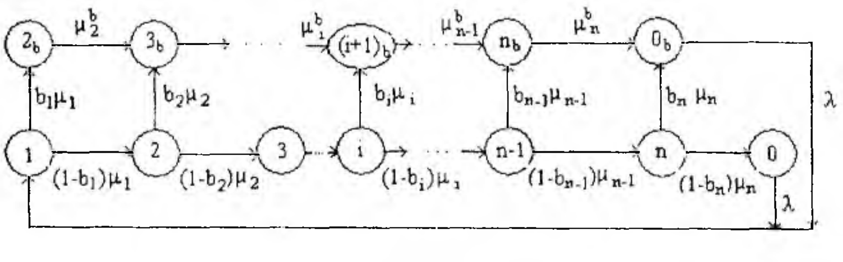

Figure 1

Note that the probability of disease of the human operator in the execution of the first function is not known and will be determined using the "principle of maximum information entropy". Stationary system of equations for the probabilities of the states has the form:

$$
\begin{cases}
\lambda P_{0b} + \lambda P_0 = \mu_1 P_1; \\
(1-b_1)\mu_1 P_1 = \mu_2 P_2 ; \\
b_1 \mu_1 P_1 = \mu_2^b P_{2b} ; \\
(1-b_2)\mu_2 P_2 = \mu_3 P_3 ; \\
\mu_2^b P_{2b} + b_2 \mu_2 P_2 = \mu_3^b P_{3b} ; \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
(1-b_i)\mu_i P_i = \mu_{i+1} P_{i+1} ; \\
\mu_i^b P_{ib} + b_i \mu_i P_i = \mu_{i+1}^b P_{i+1,b} ; \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
(1-b_n)\mu_n P_n = \lambda P_0 ; \\
\mu_n^b P_{nb} + b_n \mu_n P_n = \lambda P_{0b}.
\end{cases} \tag{1}
$$

And also

$$
\sum_{i=0}^{n} P_i + \sum_{i=2}^{n} P_{ib} + P_{0b} = 1. \tag{2}
$$

Of practical interest are the values of $P_i^* = P_i + P_{ib}, 1 - P_0^*$ and P_{0b}.

From the recurrence relations (1), using (2) we denote for $(k = 2 \ldots n)$:

$$
B_k = \prod_{j=1}^{k-1} (1-b_j), \quad A_k = 1 - B_k, \quad A_0 = A_1 = 0, \quad B_0 = B_1 = 1;
$$

$$
\Sigma_1 = \sum_0^n B_k / \mu_k, \quad \Sigma_2 = \sum_0^n A_k / \mu_k^b, \quad P_k = \frac{\Sigma_1}{(\Sigma_1 + \Sigma_2)} \text{ and } P_{k,b} = \frac{\Sigma_2}{(\Sigma_1 + \Sigma_2)}.
$$

So,

$$
P_k = \frac{B_k}{\mu_k (\Sigma_1 + \Sigma_2)}, \quad P_{k,b} = \frac{A_k}{\mu_k^b (\Sigma_1 + \Sigma_2)}, \quad \text{where } k = 1 \ldots n,
$$

and

$$
P_0 = \frac{B_{n+1}}{\lambda (\Sigma_1 + \Sigma_2)}, \quad P_{0,b} = \frac{A_{n+1}}{\lambda (\Sigma_1 + \Sigma_2)}, \quad P_0^* = \frac{1}{\lambda (\Sigma_1 + \Sigma_2)}.
$$

Now we model the behavior of the system when the intensity of the incoming stream of events depends on time $\lambda(t)$. For this case the system of Kolmogorov equations is set up. We find the probability b_i using the maximum information entropy principle, by solving the optimization problem.

Thus, suppose that, to eliminate the accident, an operator must perform five operations in sequence, that is $n = 5$.

The system of Kolmogorov equations[3] for this model looks as:

$$\left.\begin{array}{c}
\lambda(t)P_0(t)+\lambda(t)P_{0b}(t)-\mu_1P_1(t)=P_1'(t)\\
(1-b_1)\mu_1P_1(t)-\mu_2P_2(t)=P_2'(t)\\
(1-b_2)\mu_2P_2(t)-\mu_3P_3(t)=P_3'(t)\\
(1-b_3)\mu_3P_3(t)-\mu_4P_4(t)=P_4'(t)\\
(1-b_4)\mu_4P_4(t)-\mu_5P_5(t)=P_5'(t)\\
(1-b_5)\mu_5P_5(t)-\lambda(t)P_0(t)=P_0'(t)\\
b_1\mu_1P_1(t)-\mu_2^bP_{2b}(t)=P_{2b}'(t)\\
b_2\mu_2P_2(t)+\mu_2^bP_{2b}(t)-\mu_3^bP_{3b}(t)=P_{3b}'(t)\\
b_3\mu_3P_3(t)+\mu_3^bP_{3b}(t)-\mu_4^bP_{3b}(t)=P_{4b}'(t)\\
b_4\mu_4P_4(t)+\mu_4^bP_{4b}(t)-\mu_5^bP_{5b}(t)=P_{5b}'(t)\\
b_5\mu_5P_5(t)+\mu_5^bP_{5b}(t)-\lambda(t)P_{0b}(t)=P_{0b}'(t)
\end{array}\right\}\qquad(3)$$

In equilibrium case for this equations we obtain

$$\left.\begin{array}{c}
\lambda(t)P_0(t)+\lambda(t)P_{0b}(t)=\mu_1P_1(t)\\
(1-b_1)\mu_1P_1(t)=\mu_2P_2(t)\\
(1-b_2)\mu_2P_2(t)=\mu_3P_3(t)\\
(1-b_3)\mu_3P_3(t)=\mu_4P_4(t)\\
(1-b_4)\mu_4P_4(t)=\mu_5P_5(t)\\
b_1\mu_1P_1(t)=\mu_2^bP_{2b}(t)\\
b_2\mu_2P_2(t)+\mu_2^bP_{2b}(t)=\mu_3^bP_{3b}(t)\\
b_3\mu_3P_3(t)+\mu_3^bP_{3b}(t)=\mu_4^bP_{3b}(t)\\
b_4\mu_4P_4(t)+\mu_4^bP_{4b}(t)=\mu_5^bP_{5b}(t)\\
(1-b_5)\mu_5P_5(t)=\lambda(t)P_0(t)\\
b_5\mu_5P_5(t)+\mu_5^bP_{5b}(t)=\lambda(t)P_{0b}(t)
\end{array}\right\}\qquad(4)$$

In the system (3), the intensity of input is λ, as well as this situation all state probabilities are functions of the time.

Now we determine the type of function $\lambda(t)$. Obviously, $\lambda(t)$ depends on the type of disaster or any other external influences that cause accidents, or crash of the system. Consider an abstract external action (catastrophe) that causes the operations for accident liquidation. Suppose we only know that the intensity of the impact changes in the following way: during the first two hours there is an increase of the intensity about two-fold (from 0.01 to 0.02), and within twelve hours the intensity increases sharply to its maximum (0.8), then in five to ten days, it gradually decreases to near the initial value. We normalize the time interval so, that the quantity t that describes time varies from 0 to 100. Here we consider a time unit that corresponds to approximately two hours in the field. Under the given conditions using a package Matlab we define the function $\lambda(t)$ as follows:

$$\lambda(t)=\begin{cases}0.79e^{-0.12137354(x-7)^2}+0.01, & 0\le t\le 7,\\ 0.78e^{-0.005(x-7)^{1.8}}+0.02, & 7<t\le 100.\end{cases}$$

Coefficients and the exact form of the function selected in such a way as to meet the above requirements and to ensure sufficient smoothness for the numerical methods used in Matlab. Thus if $t=0$ $\lambda(t)\approx 0.01$, for $t=1$ (that is, two hours after the start of the interval) $\lambda(t)=0.02$, and at $t=7$ $\lambda(t)=0.8$ – this is the maximum. Graph of the function $\lambda(t)$ is shown in Fig. 2.

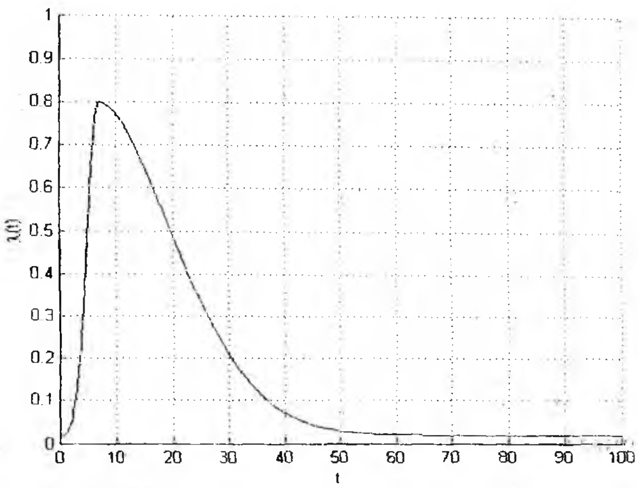

Figure 2

Now consider the intensity of a human operator in accident's elimination procedure. It can be assumed that the maximum efficiency of the operator responsible for about the middle of the shift. Then gradual decline starts to its initial level. By the end of shift efficiency can rise again, due to the haste and desire to get the job done quickly. Based on these considerations, we define the function of the intensity of operations to eliminate the accident as shown in Fig.3

This function $\mu(x)$ is given as follows:

$$\mu(x)=\begin{cases}0.4, & 0\le x\le 5-\pi,\\ 0.2\cos(x-5)+0.6, & 5-\pi<x\le 10.\end{cases}$$

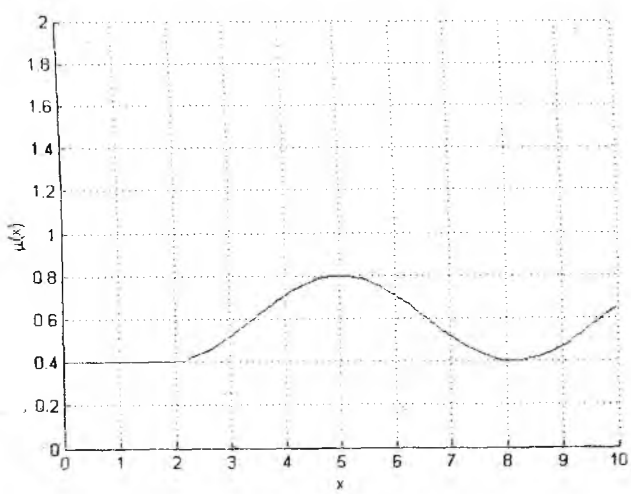

Figure 3

To get the values of coefficients for μ_i, $i=\overline{1,5}$, take values at characteristic points:

μ(0)= 0.4, μ(2.5)= 0.439, μ(5)= 0.8, μ(7.5)= 0.439, μ(10)= 0.657.

Since we consider the abstract system, the data on the relationship between the intensity of the operator in the healthy and diseased states do not exist. We therefore use the preliminarily assumption that □ib = 0.75 □i.

To find the probability b_i, we use the principle of maximum information entropy and form the optimization problem:

$$S_I = -\sum_{i=1}^{2} p_i \ln p_i, \quad p_i>0, \quad p_1+p_2=1, \quad 36.5p_1 + 38p_2 \le 37.$$

We solve this problem by using Matlab with tolerance up to 3 decimals and obtain the following values:

$$p_1=0.667, \quad p_2=0.333; \qquad \text{here } p_2 = b_i, \quad i=\overline{1,5}.$$

Now we obtain all the values necessary for the solution of (3). The result is a graph of time for probability of finding the system in each of the possible states. Time dependency of probability graphs are presented in Figures 4 – 14. The calculations show that when the intensity $\lambda(t)$ of the incoming stream of events is no longer change with time, the system reaches a steady state, that is, the probability of states P_i and P_{ib} become permanent.

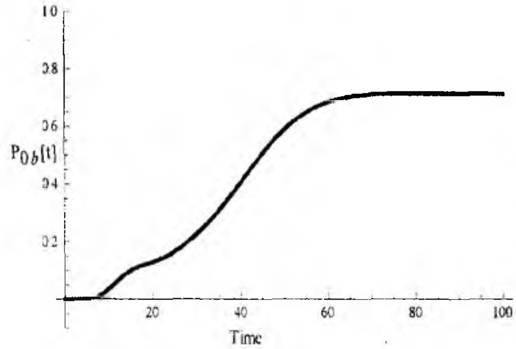

Figure 4

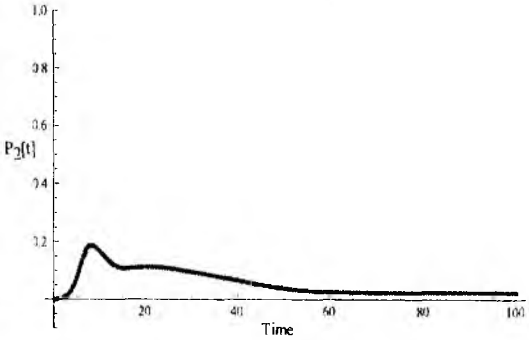

Figure 5

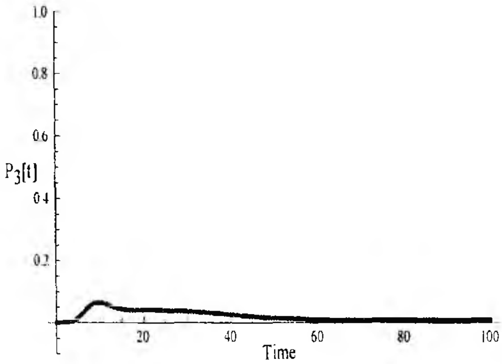

Figure 6

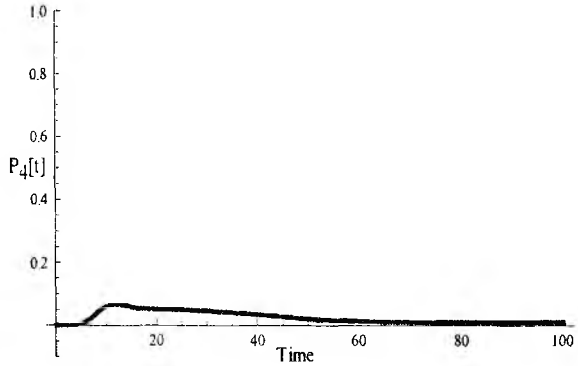

Figure 7

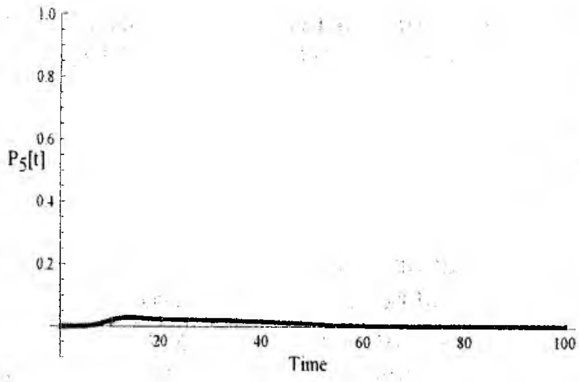

Figure 8

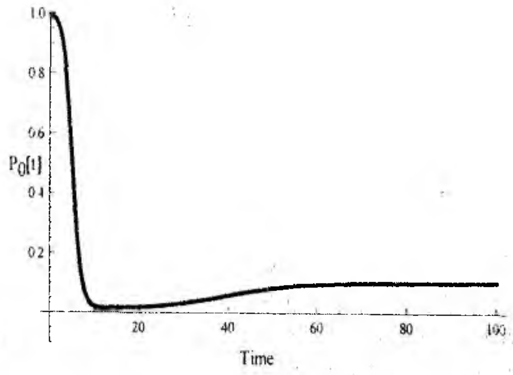

Figure 9

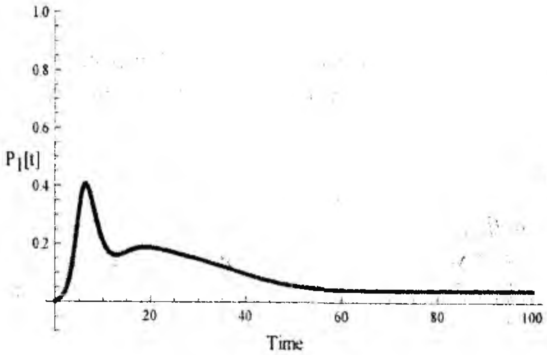

Figure 10

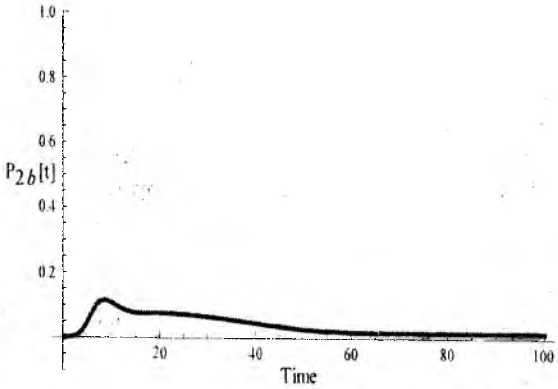

Figure 11

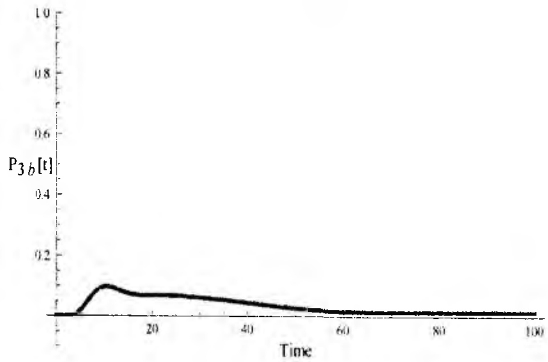

Figure 12

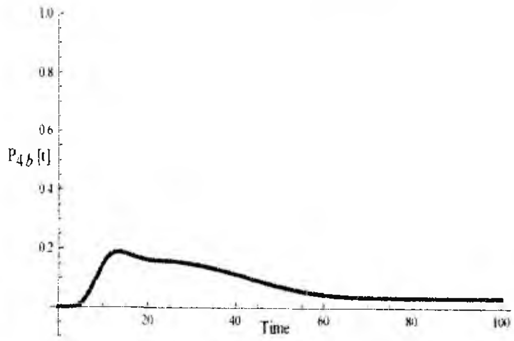

Figure 13

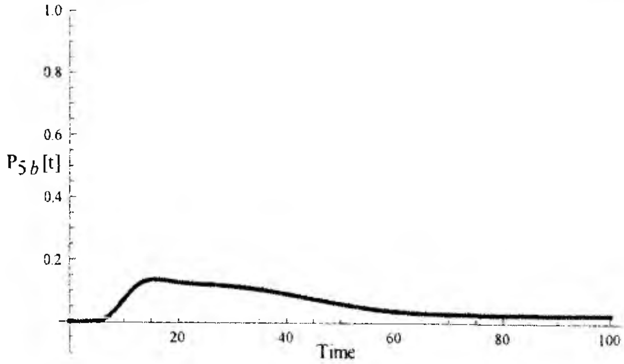

Figure 14

At the end of the considered time interval (system output in the steady state) the state probabilities are equal to: P_0=0.104, P_{0b}=0.712, P_1=0.040, P_2=0.025, P_{2b}=0.017, P_3=0.009, P_{3b}=0.015, P_4=0.011, P_{4b}=0.035, P_5=0.005, P_{5b}=0.027.

Conclusion

Within the given conditions of the system's configuration and the flow of incoming events, the probability that the operator performed all the operations and became hurt – P_{0b}=0.712 is the greatest; together with P_0 it gives 0.816 – probability that the accident is successfully eliminated. But the opposite event is rather probable too (~18%), so we cannot consider the situation safe. The only one parameter of the system, that we can impact to improve the situation, is $\mu(t)$. Or else, the general structure of the system must be changed.

References: 1. *Структуры* и типы моделей систем "человек–машина–среда" / Дзюндзюк Б.В., Наумейко И.В., Сердюк Н.Н., Стыценко Т.Е // Автоматизированные системы управления и приборы автоматики. – Харьков, 2007. – Вып. 138. – С. 47-50. 2. *Марковские* модели систем "человек – машина – среда" / Наумейко И.В., Сердюк Н.Н. // Электроника и информатика. – Харьков, 2005. Вып. 4. 3. *Хинчин А. Я.* Работы по математической теории массового обслуживания / Под ред. Б. В. Гнеденко. – М. : Физматгиз, 1963. – 236 с. 4. *Ивченко Г.И., Каштанов В.А., Коваленко И.Н.* Теория массового обслуживания. – М. : Высш. шк., 1982. – 256 с. 5. *К расчету* марковской модели эргатической системы / Наумейко И.В., Сова А.В. // Сб.науч.труд. 5-й Юбилейной Междунар. науч. конференции "Функциональная база наноэлектроники". – Харьков-Крым, 2012. – С. 236-239.

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