Airy Pulse Transformation by an Accelerated Medium Boundary

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***Abstract* — In the statement of a problem with a moving boundary there is one more idealization, namely, movement stationarity assuming that the movement has begun at infinite past time. Abandoning this idealization, by considering a movement that begins at a finite moment of time, leads to the appearance of new peculiarities in the wave transformation on a moving boundary. In this paper such peculiarities are considered with an abrupt uniform movement of a boundary beginning at zero moment of time, as well as with a smooth "turning on" of a boundary movement according to a relativistic uniformly accelerated law. In the latter approach the continuity of a boundary velocity change allows the development of the evolution of the wave transformation process to be traced.**

Keywords— uniform accelerated movement of a boundary; resolvent method; electromagnetic Airy pulse.

I. Introduction

Special features of wave transformations, become apparent in the case of smooth non-stationarity of the moment when the boundary velocity reaches the magnitude of the wave phase velocity continuously. Such non-stationarity is realized by non-uniform movement of the boundary. It is important to consider the scattering of electromagnetic waves by a boundary, which makes irregular movement.

The peculiarities of moving non-stationary boundary have already been described in works [1-3], but the study of the influence on the propagation of the wave of inhomogeneous motion in these works has not been carried out.

First, let's consider the problem of transforming the Airy pulse into a plane boundary that begins to move at a certain point in time, that is, the value of the speed drastically changes from zero to value. The position of the boundary is determined by the characteristic function $\chi(t,x)$, which is equal to the unit in half-space $x>0$ and zero in half-space $x<0$. It is supposed that at the zero moment the interface between the media was motionless, i.e. $\chi^{-}(t',x')=\theta(x)$.

Let's take a case, if there is a zero moment between each other. $\chi^{+}(t',x')=\theta(x-x_s(t))$, but the medium on both sides of the border remain stationary.

An integral equation describing an electromagnetic field in this case has the form:

$$E=F-\frac{v^2-v_1{}^2}{2v^2v_1}\frac{\partial^2}{\partial t^2}\times \\ \times\int_0^{+\infty}dt'\int_{-\infty}^{+\infty}dx'\theta\left(t-t'-\frac{|x-x'|}{v}\right)\chi^{+}(t',x')E(t',x'), \tag{1}$$

where v and v_1 - the phase velocities of the waves before and after the appearance of the boundary of separation of environment.

The resolvent of the integral equation (1) is described by the expression:

$$\hat{R}=-\chi(x-ut)\frac{v^2-v_1}{2v_1v^2}\frac{\partial}{\partial t}\left\{\frac{\partial}{\partial t}\theta\left(t-t'-\frac{|x-x'|}{v_1}\right)+\right. \\ \left.+\frac{v-v_1}{v+v_1}\frac{\partial}{\partial t}\theta\left(v_1t-x\frac{v_1-u}{v_1+u}(v_1t'+x')\right)\right\}\chi(x'-ut'), \tag{2}$$

As a result of the counter movement of the momentum and the boundary with the corresponding ratio between the velocities $|u|<v_1$ a secondary pulse is formed:

$$E_{Tr1}(t,x)=\frac{v_1}{v}\frac{v-u}{v_1-u}\frac{2v_1}{v+v_1}\mathrm{Ai}\left(-\frac{v_1}{v}\frac{v-u}{v_1-u}\frac{t}{T}+\frac{v-u}{v_1-u}\frac{x}{vT}\right). \tag{3}$$

In the case of a crossover movement with superluminal velocity $-u>v_1$ two secondary ones are formed:

$$E_{Tr2}(t,x)=\frac{v_1}{v}\frac{v-u}{v_1-u}\frac{v+v_1}{2v}\mathrm{Ai}\left(-\frac{v_1}{v}\frac{v-u}{v_1-u}\frac{t}{T}+\frac{v-u}{v_1-u}\frac{x}{vT}\right)+ \\ +\frac{v_1}{v}\frac{v-u}{v_1+u}\frac{2v}{v+v_1}\mathrm{Ai}\left(-\frac{v_1}{v}\frac{v-u}{v_1+u}\frac{t}{T}-\frac{v-u}{v_1+u}\frac{x}{vT}\right). \tag{4}$$

Thus, when the speed of the border is greater than the velocity of the pulse, it will reach the observation point later than the moment of the beginning of the movement of the limit. This means that at this moment splitting of the pulse into two others will occur as a result of a sharp change in the value of the dielectric permittivity of the medium. After passing the momentum of the boundary, two pulses are formed which propagate with a new velocity v_1 in opposite directions.

Reflected from the limit of momentum are found by substituting expressions (3) and (4) into the integral equation (1) when the observation point is in the region $x<ut$. In the case of $-u>v_1,v$ the reflected pulses can not

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overtake the limit due to the fact that the speed of the border exceeds the rate of propagation of the pulse. When $|u|< v_1, v$ reflected pulse has the form::

$$E_R(t,x) = -\frac{v-v_1}{v+v_1}\frac{v-u}{v+u}\mathrm{Ai}\left(\frac{v-u}{v+u}\left(-\frac{t}{T}+\frac{x}{vT}\right)\right), \quad (5)$$

Let us now consider the more complex case of the motion of the limit, namely, the case of smooth non-stationary, when the speed of the boundary gradually changes from zero to the value of the pulse speed.

With such a movement the velocity of a boundary is changing constantly from zero to relativistic magnitudes and thus passing constantly through all special points.

The law of boundary movement towards the impinging wave takes the form [4] $x_s(t) = -v\left(\sqrt{\xi^2+t}-\xi\right)$, where $\xi = c/w$; w is a value of the acceleration in a boundary reference system. The boundary movement velocity $u(t) = -vt\left(\xi^2+t^2\right)^{-\frac{1}{2}}$ with $t \to \infty$ tends to its limiting value $-v$ which is to the wave velocity in the background medium.

As the medium properties are not changed at the zero moment in the region $T_3{}'(\chi > v_1 t)$ the transmitted field is the same as before this zero moment, $E_2 = E_1$. In the region $(\chi < v_1 t)$ the nature of interaction are different. If $n = v/v_1 < 1$ (Fig. 1) then the resolvent characteristic reflected from the boundary world line is in the world region $\chi^{(+)} = \theta\left(x - x_s(t)\right) = 1$ during all time

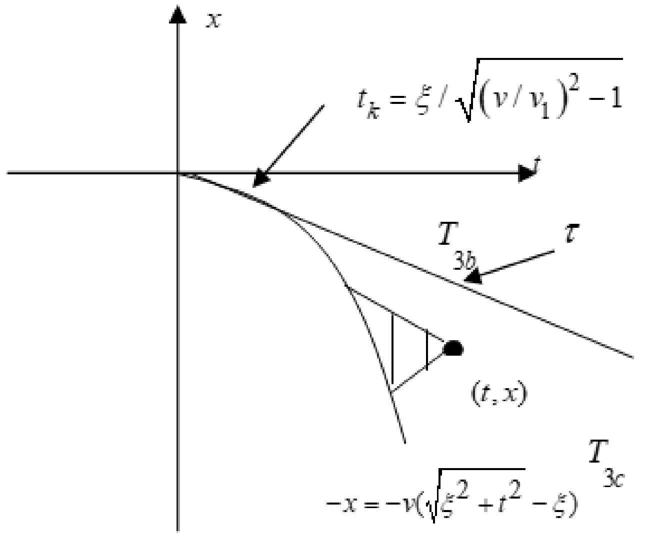

Fig. 1. Time-spatial zones in the case of smoothly accelerated movement of the boundary. Vertical hatching shows the light cone of the resolvent with the vertex at the observation point (t, x).

When $v/v_1 < 1$, the resolvent has the form [5-7]:

$$\begin{aligned}&\left\langle x \,|\, \hat{R} \,|\, x'\right\rangle = -\Theta(x - x_s(t)) \times \\ &\times \frac{v^2 - v_1}{2v_1 v^2}\frac{\partial}{\partial t}\left\{\frac{\partial}{\partial t}\Theta\left(t - t' - \frac{|x-x'|}{v_1}\right) + \right. \qquad (6)\\ &\left. + R_u(\tau)\frac{\partial}{\partial t}\Theta\left(\varphi(\tau) - t' - \frac{x}{v_1}\right)\right\}\Theta(x' - x_s(t')),\end{aligned}$$

where $\varphi(\tau) = 2t_1 - \zeta$, t_1 - the point of intersection of the lower characteristic of the resolvent with the world line of the boundary $t_{1,2}(\tau) = v_1 v(v^2 - v_1{}^2) \times \left(\pm\xi - \frac{v_1}{v}\tau + \sqrt{\xi^2 \pm 2\frac{v_1}{v}\xi\tau + \tau^2}\right)$, and the coefficient $R_u(\tau) = \frac{v-v_1}{v+v_1}\frac{v_1 - u_1(\tau)}{v_1 + u_1(\tau)}$ - the function of the observation point (the velocity of the boundary is taken at the moment of convergence with the resolution of the resolvent $u_1(\tau) = u(t_1)$).

When $v/v_1 > 1$ the reflected characteristic belongs to the region $\chi^+ = 1$ only up to the moment of touch $t_k = \frac{\xi}{\sqrt{(v/v_1)^2 - 1}}$, when the speed of the boundary coincides with the velocity of the pulse v_1. The beam $\tau = \xi\left(\frac{v^2+1}{v_1\sqrt{v^2 - v_1{}^2} - v/v_1}\right)$ separates from a region T_3 a subregion T_{3b}, in which the reflected characteristic no longer contributes to the resolvent, and the tangent $t_k = \frac{\xi}{\sqrt{(v/v_1)^2 - 1}}$ separates the region T_{3c}, in which both lines of the characteristics of the resolvents intersect with the boundary.

II. RESULTS AND DISCUSSION

Such a structure of the past field assumes that, if $v/v_1 > 1$ only in a subregion T_3 the field consists of one Airy pulse:

$$\begin{aligned}&E_3(t,x) = \frac{2v_1^2}{(v+v_1)^2}\left(1+\Omega^-\right)\times \\ &\times \mathrm{Ai}\left[-\frac{v_1}{v+v_1}\left(t - \frac{x}{v_1} - \xi + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2}\right)\right], \qquad (7)\end{aligned}$$

where $\Omega^- = \left(\frac{v}{v_1}\xi + t - \frac{x}{v_1}\right) / \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2}$

When $v/v_1 > 1$ in a subregion T_{3b} the past field consists of two pulses:

$$T_{3b}(t,x)=\frac{v_1}{2v}\left(1+\Omega^-\right)\times$$
$$\times\mathrm{Ai}\left[-\frac{v_1}{v+v_1}\left(t-\frac{x}{v_1}-\xi+\sqrt{\xi^2+2\xi\frac{v}{v_1}(t-\frac{x}{v_1})+(t-\frac{x}{v_1})^2}\right)\right]-$$
$$-\frac{v_1}{2v}\left(1-\Omega^-\right)\times \tag{8}$$
$$\times\mathrm{Ai}\left[-\frac{v_1}{v-v_1}\left(t-\frac{x}{v_1}+\xi+\sqrt{\xi^2+2\xi\frac{v}{v_1}(t-\frac{x}{v_1})+(t-\frac{x}{v_1})^2}\right)\right],$$

In this case, the splitting of the initial waves given by the free term F, also occurs at the boundary, and leads to the formation of reciprocal waves in the subregion T_{3c}:

$$E_{3c}(t,x)=T_{3b}(t,x)-$$
$$-\frac{v_1}{2v}\left(1-\Omega^+\right)\times$$
$$\times\mathrm{Ai}\left[\frac{v_1}{v-v_1}\left(t+\frac{x}{v_1}-\xi-\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}\right)\right]+ \tag{9}$$
$$+\frac{v_1}{2v}\left(1+\Omega^+\right)\times$$
$$\times\mathrm{Ai}\left[\frac{v_1}{v+v_1}\left(t+\frac{x}{v_1}+\xi+\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}\right)\right]$$

where $\Omega^+=\left(-\frac{v}{v_1}\xi+t+\frac{x}{v_1}\right)\Big/\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}$,

In Fig. 2 shows the distribution of the field $E_{3c}(t,x)$ before the boundary reaches the pulse velocity. before the boundary reaches the pulse velocity $p=-5$.

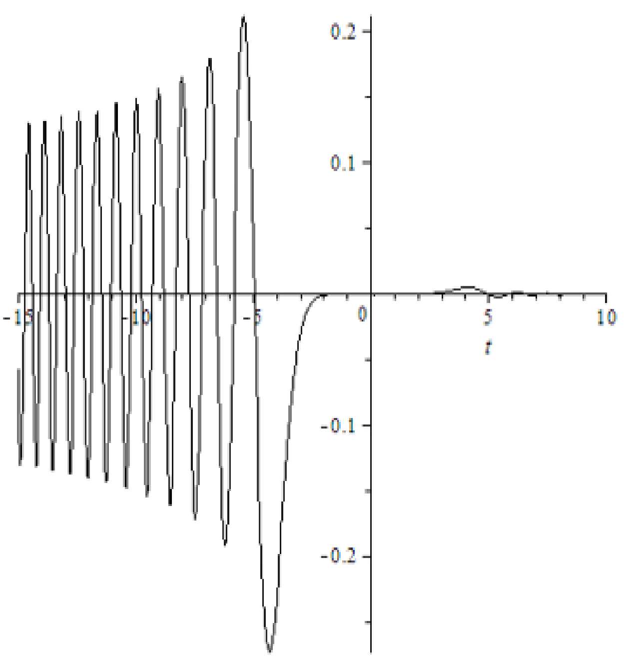

Fig. 2. The motion of the Airy pulse toward the moving boundary at the instant of equality their velocities when. $p=-5$.

Such a situation is equivalent to the case when the Airy pulse interacts with the boundary and their velocity is different from each other. Therefore, no visible changes in the shape of the pulse as a result of passing through the border does not occur.

Fig. 3 illustrates a situation where the speed of the boundary reached the pulse velocity at the moment of the interaction of the boundary with the forward front of the pulse.

Such an interaction is possible if a positive value of the starting parameter is close to zero, for example $p=1$. In this case, two endless discontinuities are formed, which can be clearly seen in Fig. 3. As a result of such interaction, the front line of the pulse, which forms the main petal, goes to infinity.

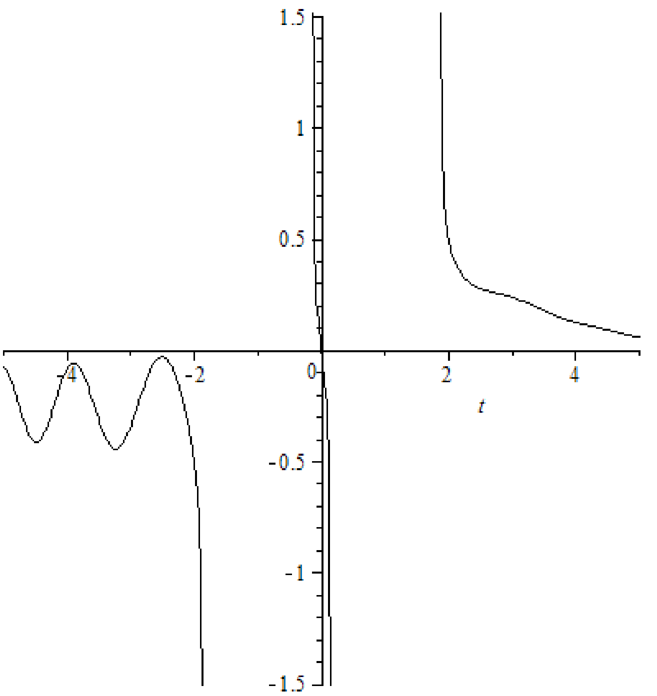

Fig. 3. The motion of the Airy pulse toward the moving boundary at the instant of equality their velocities at $p=1$

In Fig. 4 shows the situation when the boundary reached the pulse speed after it crossed its front.

As a result of this interaction, the decrease of the main lobe of the Aire pulse and its bifurcation occurs, while the periodicity of the oscillations of the pulse tail remains unchanged in comparison with the primary field.

At the moment of reaching the limit of the velocity of the pulse there is a tail break so that on one side of the boundary remains cut off the main pulses of the pulse, and on the other hand - the oscillating tail. This effect can be used to truncate the pulse tail in order to use its maximum energy located in its head lobe.

A comparison of the unperturbed Airy pulse and the past field $E_{3c}(t,x)$ is shown in Fig. 5

The origin of the past field discontinuities is laid down in expressions for Ω^+ and Ω^- in expressions (8) and (9), respectively.

From the analysis of these expressions it follows that the denominator Ω^{+} vanishes at two points

$$\hat{\tau}_1^{\,+} = t + \frac{x}{v_1} = 2\xi\left(\frac{v + \sqrt{v^2 - v_1^2}}{v_1}\right),$$
$$\hat{\tau}_1^{\,-} = t + \frac{x}{v_1} = 2\xi\left(\frac{v - \sqrt{v^2 - v_1^2}}{v_1}\right), \qquad (10)$$

and these expressions are valid for $v / v_1 > 1$. In the opposite case, the radical expression will be purely imaginary.

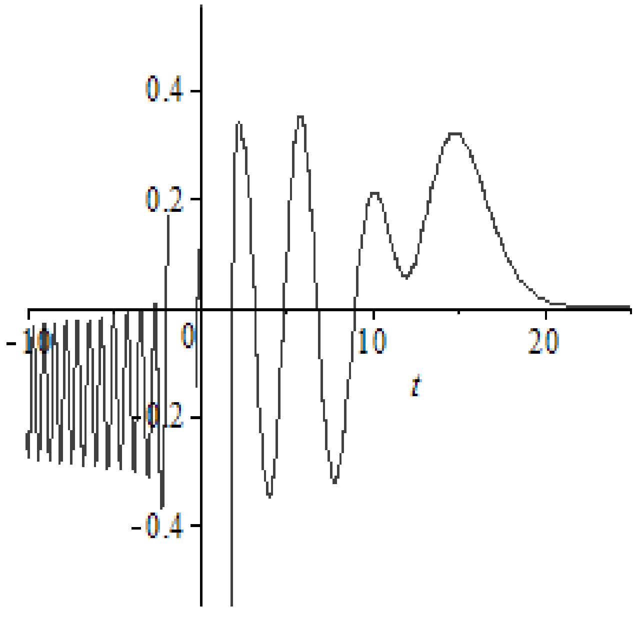

Fig. 4. – The counter-current motion of the Airy pulse and the limits at the instant when their velocities were equalized when $p = 10$.

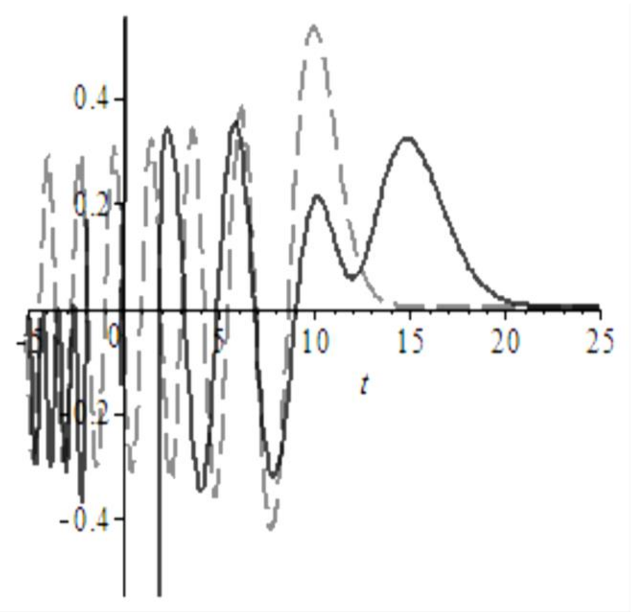

Fig. 5. Comparison of initial Airy pulse (dashed line) and pulse transmitted through the boundary $E_{3c}(t,x)$ (solid line) at the point $x = 1$ at $p = 10$.

Denominator Ω^{-} vanishes also at two points

$$\hat{\tau}_2^{\,+} = t - \frac{x}{v_1} = 2\xi\left(\frac{v + \sqrt{v^2 - v_1^2}}{v_1}\right),$$
$$\hat{\tau}_2^{\,-} = t - \frac{x}{v_1} = 2\xi\left(\frac{v - \sqrt{v^2 - v_1^2}}{v_1}\right), \qquad (11)$$

and determined also by $v / v_1 < 1$.

The presence of singular points explains the field breaks as a result of interaction with the boundary. Moreover, the break lines form two intersecting bands in the space-time impulse propagation region, where the impulse does not exist.

The proposed method can be applied to physico-mathematical studies of parameters modern lasers: micro- and nanalasers [8-9].

III. CONCLUSIONS

A complex problem is solved, when the medium separation limit gradually reaches the value of the pulse velocity. Expressions for the Airy transformed pulses are obtained and it is shown that at the moment of reaching the limit of the velocity of the pulse, two infinite gaps are formed. In this case, the interaction process is controlled by selecting the value of the starting parameter. At the moment of reaching the limit of the velocity of the pulse there is a tail break so that on one side of the boundary remains cut off the main pulses of the pulse, and on the other side - the oscillating tail.

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