

Mathematical Modeling of the Radiation of a Conical Multiple-Element Antenna

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Abstract—The results of studying a model strict formulation problem of electric radial dipole excitation of a conical periodic multi-element structure are presented. It is shown that the solution of the considered problem is equivalent to the solution of the singular integral equation with a logarithmic singularity. Field distribution patterns and curves of the surface current density are given.

Keywords—cone, longitudinal slot, singular, integral equations

I. INTRODUCTION

Theoretical studying and development of mathematical models that adequate to processes make possible not only to validate, but also often to predict results of experimental investigations of these processes. Researching problems of exciting irregular shape bodies is of interest for practical applications in antenna and electronic engineering. Irregular structures, that include structures with characteristic angular and geometrical parameters (cones, conical and plane angular sectors) occupy a special place among canonical surfaces. Antennas which surfaces represent cones, conical and plane angular sectors have wideband and superwideband properties, and also an extensive range of practical applications [1,2]. Model problems of wave diffraction by such surfaces were considered in works [3-5]. Changing the antenna surface properties one can expand the practical use of such antennas, and significantly complicate the solution of the corresponding mathematical problem. In this regard, it is required to work out new methods and approaches or modify existing ones for solving the corresponding mathematical problems. In the papers [6,7] methods for solving model electromagnetic boundary value problems for finite cones, cones with transverse inhomogeneities are proposed and developed. The solution of these boundary value problems is based on the use of the factorization method and eigenwave expansion one.

In this paper a mathematical model for the process of interaction between the field of an elementary source and a conical periodic structure on the period of which there are several longitudinal slots and conical strips of various angular dimensions is proposed. Interest to studying such model problems is caused, in particular, by the fact that the presence of conical strips leads to an expansion of the operating domain of a real antenna due to a variation in their angular dimensions. It should be noted that practically important cases of the conical considered structure are plane angular sectors (models of linearly

expanding antennas), bowtie antennas, as well as slot and wire cone antennas.

II. FORMULATION OF THE PROBLEM. SINGULAR INTEGRAL EQUATIONS (SIE)

Consider a model problem of axisymmetric excitation by an elementary source of a semi-infinite perfectly conducting periodic conical structure Σ (Fig. 1) consisting of N repeating systems, each of which contains M conical strips and M longitudinal slots of various angular dimensions.

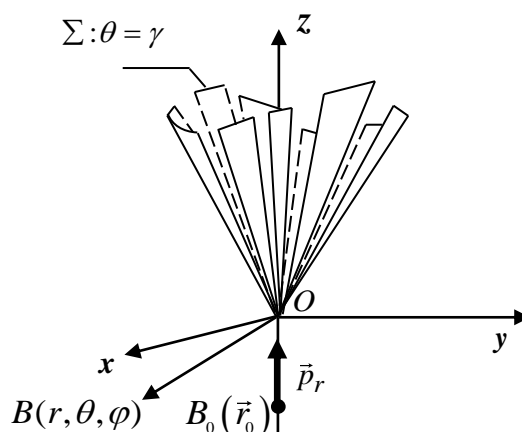


Fig. 1. Geometry of problem.

The slots are cut from the top of the cone along its generators. The value of the slot width (angular size) is equal to the value of the dihedral angle formed by the planes drawn through the axis of the cone and the edges of the strips. Let us 2γ denote the opening angle of the cone, $L = 2\pi / N$ is a period of structure, d_p is an angular slot width with number p ($p = 1 \div M$) on the period, $l_p^{(s)}$ is a conical strip with number p at the $(s+1)$ -th period ($s = 0 \div N-1$). In the introduced spherical coordinate system (r, θ, φ) with the origin at the cone tip, the conical surface Σ is defined as $\Sigma: \theta = \gamma$, and

$$l_p^{(s)} = \{\varphi: \rho \in (a_p + sL, b_p + sL)\}, a_p, b_p \in R,$$

$l^{(s)} = \bigcup_{p=1}^M l_p^{(s)}$ is the total surface of all strips on the period

with the number s , $l = \bigcup_{s=0}^{N-1} l^{(s)} = \bigcup_{s=0}^{N-1} \bigcup_{p=1}^M (a_p + sL, b_p + sL)$

is the total surface of the strips of the conical surface Σ .

The source of excitation is an electric radial dipole located on the cone axis, with moment \vec{p}_r (Fig. 1). The total electromagnetic field \vec{E}, \vec{H} in the presence of a cone and source is represented as

$$\vec{E} = \vec{E}_0 + \vec{E}_1, \quad \vec{H} = \vec{H}_0 + \vec{H}_1,$$

where \vec{E}_0, \vec{H}_0 is the dipole field that changes harmonically, \vec{E}_1, \vec{H}_1 is the field excited by the cone is expressed in terms of the Debye potential $v_1(\vec{r})$ associated with the Green's function [8]. The unknown potential v_1 satisfies the Helmholtz equation of everywhere out of conical strips and the source, the Dirichlet boundary condition at the strips, the condition at the infinity, the condition of local energy finiteness.

The solution of this boundary problem exists, is unique [9]. It is found in the form

$$v_1 = \frac{2}{\pi^2} \int_0^{+\infty} \tau \text{sh} \pi \tau C_\tau V_\tau(\theta, \varphi) \frac{K_{i\tau}(qr)}{\sqrt{r}} dr$$

where $K_{i\tau}(z)$ is the Mackdonald function, $q = -ik$, k is a wave number, C_τ are known coefficients,

$$V_\tau = \sum_{n=-\infty}^{+\infty} x_n \frac{P_{-1/2+i\tau}^{nN}(\pm \cos \theta)}{P_{-1/2+i\tau}^{nN}(\pm \cos \gamma)} e^{inN\varphi}, \quad (1)$$

where $P_{-1/2+i\tau}^{nN}(\cos \theta)$ is the associated Legendre function of the first kind, x_n are unknown coefficients, the upper signs in (1) correspond to the domain $0 < \theta < \gamma$ and the lower ones $\gamma < \theta < \pi$.

As a result of using the boundary condition at the cone strips and the condition of field continuity at the slots, we obtain functional equations for finding x_n of the form

$$\sum_{n=-\infty}^{+\infty} x_n e^{inN\varphi} = 1, \quad \varphi \in l^{(s)} \quad (2)$$

$$\sum_{n=-\infty}^{+\infty} N \cdot |n| (1 - \varepsilon_n) x_n e^{inN\varphi} = 0, \quad \varphi \in Cl^{(s)} \quad (3)$$

where

$$N |n| (1 - \varepsilon_n) = \frac{1}{\pi} (-1)^{nN} ch\pi\tau \frac{\Gamma(1/2 + i\tau + nN)}{\Gamma(1/2 + i\tau - nN)} \times$$

$$\times \frac{1}{P_{-1/2+i\tau}^{nN}(\cos \gamma) P_{-1/2+i\tau}^{nN}(-\cos \gamma)},$$

where $\Gamma(z)$ is a gamma function.

Solving the system (1) - (2) is difficult, since its function equations are defined at the different parts of the conical surface.

It should be noted that independence of the coefficients x_n on the wave number simplifies studying fields in the high-frequency region.

Let us introduce the function

$$F(\psi) = \sum_{n=-\infty}^{+\infty} N |n| (1 - \varepsilon_n) x_n e^{in\psi} e^{im\psi}, \quad \psi \in [-\pi, \pi], \quad (4)$$

$\psi = N\varphi$ and obtain, by virtue (2) - (3), SIE (for finding $F(\psi)$ (4) and coefficients x_n) of the form

$$\frac{1}{N} \int_{l^{(0)}} \ln \left| 2 \sin \frac{\psi - \xi}{2} \right| \cdot F(\xi) d\xi + \\ + \frac{1}{N} \int_{l^{(0)}} R_{r\tau}(\psi - \xi) \cdot F(\xi) d\xi = -\pi, \quad \psi \in l^{(0)}. \quad (5)$$

The resulting SIE (5) has a logarithmic singularity (the first term on the left side), while the second term has a regular kernel $R_{r\tau}(\psi - \xi)$ of the known form.

The physical meaning of the function $F(\psi)$ is that it defines the radial component of the density of the surface current induced on the strips. In contrast to (2) - (3) SIE (5) is defined at the strips ($s = 0$) only.

III. NUMERICAL RESULTS. FIELD AND CURRENT DISTRIBUTION

In the case of one longitudinal slot and one strip ($M = 1, N = 1$) the numerical solution of SIE (5) and the numerical values of the Fourier coefficients x_n are obtained using the discretization method. Figure 2 shows the dependences of these coefficients on the angular parameters of the conical structure. With the number n these coefficients decrease in absolute value, which makes it possible to limit them to a certain number when calculating V_τ (1) and taking into account the behavior of functions under the sum sign.

In the wave zone ($qr \gg 1, \theta > 2\gamma$) the components of the electromagnetic field take the form

$$E_\theta = \frac{e^{qr}}{r} F_\theta(\theta, \varphi, \gamma, d), \quad E_\varphi = \frac{e^{qr}}{r} F_\varphi(\theta, \varphi, \gamma, d), \quad (6)$$

where $F_\theta(\theta, \varphi, \gamma, d)$, $F_\varphi(\theta, \varphi, \gamma, d)$ are functions describing the field in the wave zone

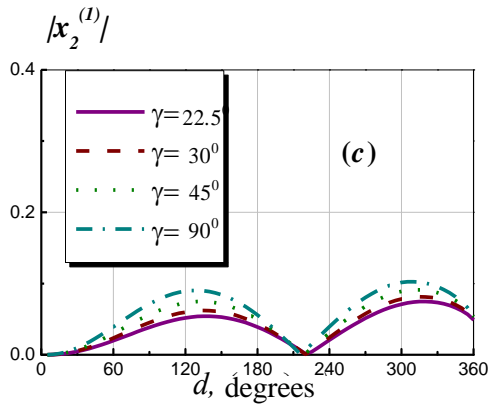
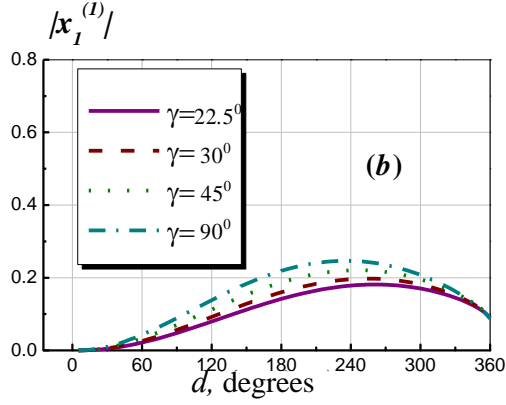
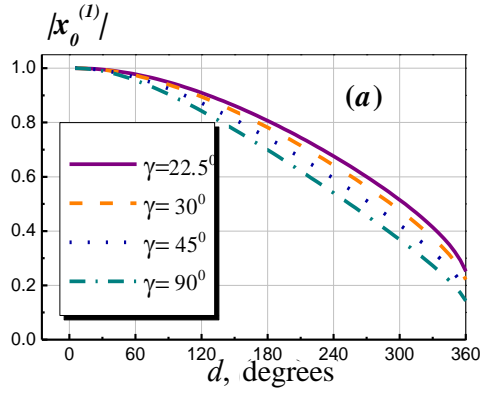


Fig. 2. Dependence of the coefficients $|x_n^{(1)}|$ on the width of slot d : a) $n=0$, b) $n=1$, c) $n=3$.

$$F_\theta(\theta, \varphi, \gamma, d) = \text{Const} \frac{e^{qr}}{r} \int_0^{+\infty} \tau \theta \pi \tau e^{\frac{\pi}{2}} K_{ir}(qr_0) \times$$

$$\times \left(\sum_{n=1}^{+\infty} x_n^{(1)}(\tau) \frac{d}{d\theta} \frac{P_{-1/2+ir}^{-n}(-\cos\theta)}{P_{-1/2+ir}^{-n}(-\cos\gamma)} 2 \cos n\varphi + \right.$$

$$\left. + x_0^{(1)}(\tau) \frac{-P_{-1/2+ir}^1(-\cos\theta)}{P_{-1/2+ir}(-\cos\gamma)} \right) d\tau \quad \square$$

$$F_\varphi(\theta, \varphi, \gamma, d) = \text{Const} \frac{e^{qr}}{r} \frac{1}{\sin\theta} \int_0^{+\infty} \tau \theta \pi \tau e^{\frac{\pi}{2}} K_{ir}(qr_0) \times$$

$$\times P_{-1/2+ir}(\cos\gamma) \sum_{n=1}^{+\infty} x_n^{(1)}(\tau) \frac{P_{-1/2+ir}^{-n}(-\cos\theta)}{P_{-1/2+ir}^{-n}(-\cos\gamma)} 2 \sin n\varphi d\tau.$$

For the time dependence in the form $e^{-i\omega t}$ the parameter q in (6) is taken in the form $q = -ik$ ($\text{Im}k \leq 0$); and in the case of time dependence $e^{i\omega t}$ $q = ik$ ($\text{Im}k \geq 0$).

Figure 3 shows the distribution of the total field (in the wave zone, $\theta > 2\gamma$) $|\vec{E}| = \sqrt{|\vec{E}_\theta|^2 + |\vec{E}_\varphi|^2}$ normalized to the maximum in the azimuthal plane for a cone ($\gamma = 22.5^\circ$) with one slot (the slot axis corresponds to the value of the azimuth angle $\varphi = 0^\circ$) for different values of slot width d . The slot effect on the field distribution in the wave zone turns out to be weak, which is explained by this case of excitation. When $d < 60^\circ$ the diagram is almost circular. With the expansion of the slot, a change in the diagram shape is observed and for it differs little from the diagram of a single extended scatterer in the form of a thin wire, the axis of which is shifted from the axis Oz by $\theta = 22.5^\circ$.

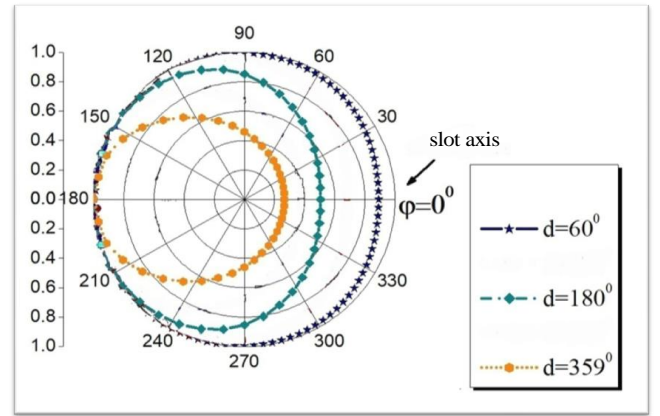


Fig. 3. Distribution of the total field in the wave zone ($\theta > 2\gamma$) for a cone whose half-opening angle is $\gamma = 22.5^\circ$, $\theta = 60^\circ$, $qr_0 = 1$ for different on the width of slot d .

To analyze the polarization of the scattered field, it is convenient to represent the polarization characteristic

$$Y(\theta, \varphi) = E_\theta / E_\varphi \text{ in the form } Y(\theta, \varphi) = |Y(\theta, \varphi)| e^{i \text{Arg} Y(\theta, \varphi)}.$$

The type of polarization is determined by the shape of the curve, which describes the end of the electric field strength vector in a plane perpendicular to the direction to the observation point. The value of the argument $\text{Arg} Y(\theta, \varphi) = \pm 90^\circ$, corresponds to the circular polarization of the scattered field, and for $\text{Arg} Y(\theta, \varphi) = 0, \pm 90^\circ$, is linear. In all other cases, the polarization is elliptical. The field scattered by a solid cone, when excited by an electric dipole, is linearly polarized. The appearance of a slot leads to changing the field polarization. Figure 4 shows the curve dependence of the argument and the modulus of the polarization characteristic $Y(\theta, \varphi)$ on the azimuthal coordinate φ and the values of slot width d . An analysis of the behavior

$ArgY(\theta, \varphi)$ shows that if $10^0 < d < 240^0$ then the scattered field polarization is elliptical.

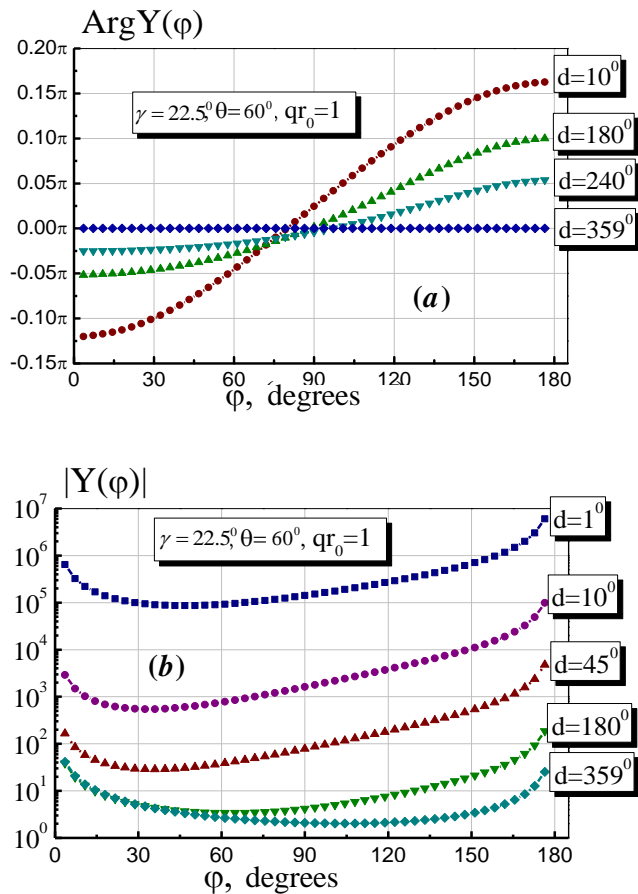


Fig. 4. Argument ($ArgY(\theta, \varphi)$) (a) and modulus ($|Y(\theta, \varphi)|$) (b) of the polarization characteristic $Y(\theta, \varphi)$ as a function of the azimuthal coordinate φ for the field scattered by the cone ($\gamma = 22.5^0$, $\theta = 60^0$, $qr_0 = 1$). The curves are given for different values of slot width d .

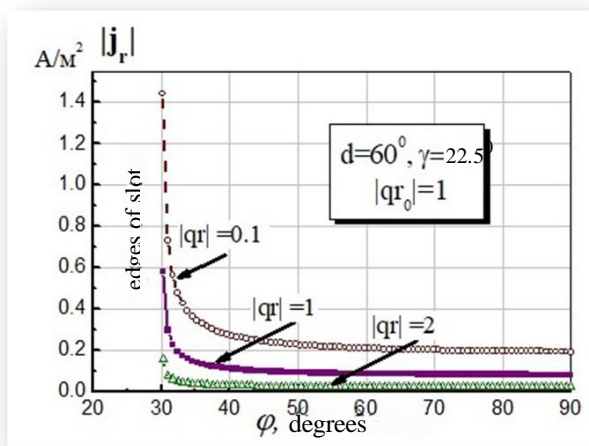


Fig. 5. Modulus of the density of distribution for surface current j_r . Curves are presented for different values of qr .

The distribution of the modulus of the only radial component of the current density is given in Fig. 5. Due to

the study of the density distribution depending on the distance to the cone tip, the current density decreases markedly and at a distance from λ/π wavelengths and more, the current density is small. Hence it follows that in the case of axisymmetric excitation of the cone, the results obtained in the case of a semi-infinite cone can be used in the case of a finite cone whose generatrix length exceeds the λ/π wavelength. As follows from Fig. 5 the density of current increases as it approaches the edge of the slot, that it agrees with the main statements of the classical theory of diffraction [9].

IV. CONCLUSIONS

As a result of the studies of the model problem considered, it is shown that the problem of axisymmetric electric radial dipole excitation of a conical open structure, on the period of which there are conical strips and slots of different angular widths, is reduced to solving a singular integral equation with a logarithmic kernel. Based on the SIE numerical solution obtained in the case of one slot per period, a field distribution diagram depending on the angular width of the slot is given. The behavior of the surface current density modulus depending on the distance to the top of the cone (in comparison with the wavelength) is studied. From the above material, we can conclude that in the case of electric radial dipole excitation of a cone with a longitudinal slot a weak slot effect on the field distributions is observed and the field diagram remains practically circular (a slot width is less than 60^0).

It follows from the curve dependence of the surface current modulus on the distance of the observation point to the cone tip that, in the case of axisymmetric radial electric dipole excitation of a cone with one slot, the results obtained can be used in the design and creation of real slot conical antennas, the operating length of which exceeds the λ/π wavelength.

V. REFERENCES

- [1] B. Xia, S. Ming, G. Zhaodong and L. Yuanan, "An UWB top-loaded monocone antenna for multiservice wireless applications," *Progress in Electromagnetics Research Letters*, vol. 73, pp. 91–97, 2018.
- [2] C. A. Balanis, "Antenna Theory: Analysis and Design," 4th Edition. Wiley, 1096, 2016.
- [3] P. Ya Ufimtsev, "Fundamentals of the Physical Theory of Diffraction," Wiley-IEEE Press, 496, 2014.
- [4] L. Kraus and L. Levine, "Diffraction by an elliptic cone," *Commun. Pure Appl. Math.*, vol. 14, pp. 49–68, 1961.
- [5] M. A. Lyalinov, "Electromagnetic scattering by a plane angular sector: I. Diffraction coefficients of the spherical wave from the vertex," *Wave Motion.*, vol. 55, pp. 10–34, June 2015.
- [6] D. B. Kuryliak, Z. T. Nazarchuk and V. O. Lysechko, "Diffraction of a plane acoustic wave from a finite soft (rigid) cone in axial irradiation," *Open Journal of Acoustics.*, vol. 5, no. 4, pp. 193–206, 2015.
- [7] D. B. Kuryliak and O. B. Trishchuk, "Modeling of intrinsic and transfer conductances of narrow grooves on the surface of the finite cone under axisymmetric electromagnetic excitation," *Journal of Mathematical Sciences.*, vol. 184, issue 2, pp. 208 – 214, 2012.
- [8] L. B. Felsen, "Radiation and Scattering of Waves," New Jersey: Prentice-Hall. Inc., vol.1, 548. vol. 2, 556, 1973.
- [9] H. Honl, A. W. Maue and K. Westpfahl, "Theorie der Beugung," Springer-Verlag-Berlin-Gottingen-Heidelberg, Band XXV/1, S. 218, 1961.