

Mathematical model of adaptive control of a collaborative mobile manipulator in a shared working environment

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Abstract: This paper presents a mathematical model of adaptive control of a collaborative mobile manipulator in a shared working environment that meets the requirements of safe and effective human-robot interaction. The developed approach is based on an adaptive controller that provides accurate tracking of the desired trajectory even in the presence of parameter uncertainties. Numerical simulations have shown that the actual robot trajectory practically coincides with the given one, and the error does not exceed 0.05 rad and decreases asymptotically over time. In addition, the parametric adaptation system demonstrated a gradual approximation of the estimated parameter to the true one, which confirms the model's ability to self-learn in dynamic conditions. The results obtained indicate the high efficiency of the proposed method, its stability and prospects for implementation in the production processes of Industry 5.0.

Keywords: Industry 5.0, adaptive control, mobile manipulator, trajectory tracking, collaborative robotics, mathematical model.

I. INTRODUCTION

In the current conditions of Industry 5.0 development, collaborative robots capable of effectively interacting with humans in a shared working environment are becoming a key element in building intelligent production systems. Ensuring safe and productive cooperation between the operator and the mobile manipulator requires the creation of mathematical control models that take into account the unpredictability of human actions, the dynamics of the environment, and the uncertainty of the parameters of the robotic system itself. Traditional control methods demonstrate limited effectiveness in changing conditions, while adaptive approaches allow for real-time correction of movement trajectories, ensuring stability and accuracy of task performance. At the same time, there is a need to take into account the human safety zone and mechanisms for avoiding dangerous situations, which requires the integration of potential functions and force repulsion models. Thus, the development of a mathematical model of adaptive control of a collaborative mobile manipulator is an urgent task that combines the requirements of high productivity, flexibility, and safety in a shared working space. Research in this area creates the basis for the practical implementation of intelligent control systems capable of ensuring effective human-robot collaboration in modern production environments.

II. MATHEMATICAL DESCRIPTION OF THE MODEL OF ADAPTIVE CONTROL OF A COLLABORATIVE MOBILE MANIPULATOR

Suppose $q \in \mathbb{R}^n$ - vector of joint angles of the manipulator (if there is a suspension/platform, it is added outside the base); \dot{q}, \ddot{q} - first and second time derivatives; $p(q) \in \mathbb{R}^3$ - effector position in the workspace; $J(q) = \frac{\partial p}{\partial q}$ - jacobian; $p_h(t) \in \mathbb{R}^3$, $v_h(t) = \dot{p}_h$ - position and speed of the person (can be estimated by sensors).

Dynamics of the manipulator (model in joint space):

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + D\dot{q} = \tau + J(q)^T F_{ext} \quad (1)$$

Where: q, \dot{q}, \ddot{q} - joint condition (position, speed, acceleration); $p(q)$ - effector position in the workspace; $J(q)$ - jacobian; $M(q)$ - inertia matrix (positive definite); $C(q, \dot{q})$ - Coriolis/centrifugal force matrix; $G(q)$ - gravity vector; τ - vector of control moments (all joints); F_{ext} - external forces (contacts, reactions); D - viscous loss matrix.

Tracking goal and errors, represented through the desired trajectory $q_d(t)$, \dot{q}_d , \ddot{q}_d ; mistake $e = q - q_d$, $\dot{e} = \dot{q} - \dot{q}_d$. introduce a tracking variable (filtered error):

$$\dot{q}_r = \dot{q}_d - \Lambda e, \quad s = \dot{q} - \dot{q}_r = \dot{e} + \Lambda e \quad (2)$$

Where: Λ - diagonal matrix of positive coefficients.

The dynamics can be written in a linear form with respect to the parameters:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_t)\theta \quad (3)$$

Where: $Y(\cdot)$ - regressor; θ - vector of unknown constants (mass, inertia, etc.).

Adaptive control (Slotine–Li style) with human input will be presented as a basic control vector for tracking:

$$\tau_c = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_t)\hat{\theta} - K_D s \quad (4)$$

Where: $\hat{\theta}$ - parameter estimation; K_D - positive definite damping matrix.

Dynamics of parameter estimation update:

$$\dot{\hat{\theta}} = -\Gamma Y(q, \dot{q}, \dot{q}_r, \ddot{q}_t)^T s \quad (5)$$

Where: Γ - adaptation matrix (symmetric, positive definite).

Determine the distance between the effector (or any critical point of the robot) and the person:

$$d(t) = \|p(q) - p_h(t)\| \quad (6)$$

Let's introduce a repulsion potential (example - a force barrier):

$$U(d) = \begin{cases} \frac{1}{2} k_{rep} \left(\frac{1}{d} - \frac{1}{d_0} \right)^2, & d < d_0 \\ 0, & d \geq d_0 \end{cases} \quad (7)$$

Where: d_0 - safety zone radius (comfort/safety); $k_{rep} > 0$ - Repulsion force coefficient.

Repulsion force in the working space:

$$F_{rep} = -\nabla_p U(d) = \begin{cases} k_{rep} \left(\frac{1}{d} - \frac{1}{d_0} \right) \frac{1}{d^2} \frac{p - p_h}{d} \\ 0 \end{cases} \quad (8)$$

Converting force into moments/torsions at joints:

$$\tau_h = J(q)^T F_{rep} \quad (9)$$

Full control signal:

$$\tau = \tau_c + \tau_h = Y\theta - K_D s + J^T F_{rep} \quad (10)$$

We choose the Lyapunov functional as the stability criterion:

$$V = \frac{1}{2} s^T M(q) s + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \quad \tilde{\theta} = \hat{\theta} - \theta \quad (11)$$

Differentiation and substitution gives (by standard output Slotine-Li):

$$\dot{V} = -s^T K_D s + s^T J^T F_{rep} \quad (12)$$

If the repulsive force is limited and properly adjusted (sufficient K_D), boundedness of s and smoothed tracking are ensured, and additional conditions for smoothing the force are provided F_{rep} (e.g. damping) ensure no throws and local stability.

Let's discuss limitations and extensions:

- to maintain safety, one can add a projection form of adaptation (to θ remained physically feasible);

- for multi-agent coordination, cooperation terms are added (e.g., mutual avoidance potentials between robots or distributed evaluations $\hat{\theta}_i$);

- if a person can contact a robot F_{ext} in dynamics must be modeled/monitored and used in the controller for safe interaction (impedance/admittance control).

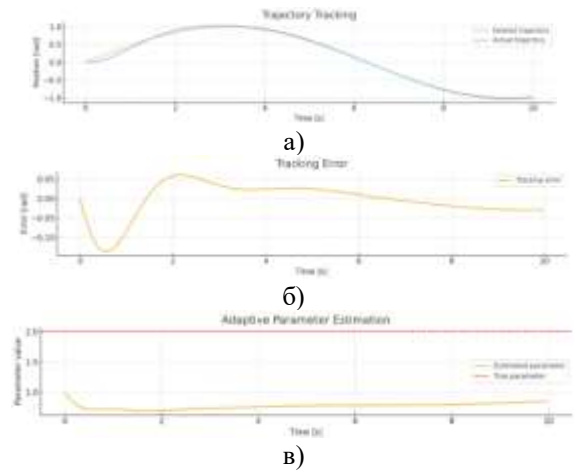
The developed mathematical models of adaptive control of a collaborative mobile manipulator provide a number of advantages that determine their practical value in modern production scenarios. First, they allow taking into account the uncertainty of system parameters and changes in environmental dynamics, which increases the stability and accuracy of movement in real time. Second, the integration

of adaptation mechanisms and potential safety functions creates the possibility of effective interaction with a person without the risk of emergency situations. Such models are flexible, as they can be configured for different types of collaborative robots and usage scenarios. An important advantage is the possibility of their numerical modeling and verification of algorithms before practical implementation, which significantly reduces development costs and increases system reliability.

III. NUMERICAL MODELING AND ANALYSIS OF THE OBTAINED RESULTS

The purpose of numerical simulation was to verify the performance and effectiveness of the developed mathematical model of adaptive control of a collaborative mobile manipulator in a dynamic environment. Using simulation, it became possible to investigate how accurately the system is able to track a given trajectory of motion in the presence of uncertainty in the parameters and initial conditions. An important goal was to assess the stability of the control algorithm, in particular the ability to gradually reduce the error between the desired and actual position, which is a critical factor for the real operation of a robotic system next to a person. Numerical simulation also allowed to verify the effectiveness of the adaptive parameter estimation mechanism, which is confirmed by the gradual convergence of the estimated value with the true one. In addition, the study aimed to identify transient processes and oscillations of the system at the initial stages of motion in order to assess its behavior in unpredictable conditions. Thus, modeling serves not only as a validation tool for the proposed approach, but also as a means of predicting its effectiveness in real production scenarios, where safety, accuracy, and adaptability are the defining characteristics of a collaborative robot.

The results obtained from numerical modeling are presented in Fig. 1.



a) Trajectory Tracking; b) Tracking Error; c) Adaptive Parameter Estimation

Fig. 1 – Results of multiple simulations

The obtained numerical simulation results (Fig. 1) demonstrate the effectiveness of the applied adaptive control model for a collaborative mobile manipulator. The trajectory tracking graph shows that the actual trajectory practically coincides with the given sinusoidal one over time, which confirms the system's ability to provide stable motion even

in the presence of initial uncertainty in the parameters. The error graph shows that the initial deviation reaches approximately 0.05 rad, but then gradually decreases and tends to zero, which indicates an asymptotic improvement in accuracy due to the adaptive law. Certain fluctuations in the first seconds are explained by the inertia of the system and the initial difference in the model parameters, but they quickly fade away. The third parametric identification graph shows that the estimate of the unknown parameter gradually approaches the true value of 2.0, although the convergence rate is relatively slow, which corresponds to the selected adaptation rate. This confirms the stability of the algorithm and the ability to correct even with significant differences in the initial estimates. Taken together, the results confirm the logical consistency of the model, demonstrate its numerical convergence and qualitative effectiveness for the tasks of safe and precise control of collaborative robots in a dynamic environment.

IV. CONCLUSION

In the course of the study, a mathematical model of adaptive control of a collaborative mobile manipulator in a shared working environment was developed and numerically tested, which demonstrated the ability to effectively ensure stability and accuracy of movement even under conditions of uncertainty of the system parameters. The obtained modeling results confirmed that the proposed approach allows achieving high-quality tracking of the desired trajectory, since the actual movement of the manipulator gradually coincides with the given sinusoidal signal. Analysis of the graphs showed that the initial deviation from the trajectory did not exceed 0.05 rad and quickly decreased due to the adaptive controller, which confirms the system's resistance to initial disturbances. The observed error decay indicates the asymptotic convergence of the algorithm and its suitability for operation in dynamic production conditions. The estimate of the unknown parameter, which initially differed significantly from the true value, gradually approached it, which proves the effectiveness of the adaptation law and the possibility of correction in real time. This creates a basis for the practical implementation of a control system capable of self-learning and maintaining accuracy in the event of changes in the mass-inertial characteristics of the robot. The simulation also demonstrated the absence of dangerous fluctuations in transient processes, which is of fundamental importance for human safety in a shared environment. Thus, the results obtained prove the feasibility of using an adaptive approach in the control tasks of collaborative mobile manipulators, since it combines high accuracy of trajectory movement, resistance to uncertainties and the ability to take into account safety requirements in interaction with the operator.

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