Mathematical Model of Containers Placement in Rail Terminal Operations Problem

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***Abstract*—In the paper the increasing of efficiency for rail terminal operations is analyzed. The problem of optimization the placement of containers on railway platforms and in the storage area at railway transshipment yard is formulated. A combinatorial optimization model of the problem is constructed, its properties are discussed, an example is considered.**

***Keywords*—rail terminal operations, transshipment yard, container, mathematical model, combinatorial optimization.**

I. INTRODUCTION

Rail terminal operations are of great importance in modern logistics [1-2]. Most of them consist in processing of containers which have shape of parallelepipeds [3] at railway transshipment yards [4-7]. Starting from that one of actual problems is optimization of operations in the rail terminal during train service at railway transshipment yard. The operations which may be optimized are: scheduling of processes (arrival, departure, loading and unloading) of trains and placing containers from these trains on railway platforms and to storage area. In some situations additional requirements for balancing containers and objects inside them are formulated. The problems of this class are investigated (see e.g. [1–10] and references therein).

The servicing of freight trains includes the following processes [4-7]: the arrival of a train at the transshipment yard, the loading and unloading containers from the train and the departure of the train from the transshipment yard. The train may return to the station to complete the loading. Container handling consists in transferring a container directly from a train to another one, transferring a container from a train to the storage area, transferring a container from the storage area to a train.

A modern transshipment yard consists of several basic elements [6]:

1) a platform with a certain number of parallel tracks; trains are located on these tracks;
2) storage area for containers which is located parallel to the tracks;
3) gantry cranes that move containers directly between trains and between trains and storage area.

There are railway tracks, on which trains arrive and leave the transshipment yard, or await service in front of the transshipment station and behind it. In addition, transshipment stations can also be equipped with automobile access roads. Such a transshipment yard provides multimodal transport services. A scheme of a typical transshipment yard is shown in Fig.1 [6].

A transshipment yard is designed to enable trains exchanging containers on their way. A train with containers for other trains (source train) arrives at the station. Containers are reloaded into target trains directly at the station or to the storage area for the future loading. If there are containers for the target train in the storage area, then it should be moved to the free platforms of this train. After implementing all rail terminal operations a train leaves the transshipment station.

Paper [6] contains the review of the problems of work planning at transshipment yard. The authors analyze the actual problems which solving allows increasing the efficiency of the rail terminal operations at transshipment yard.

General problem of planning the work of the transshipment yard formulated in [6] consists in scheduling freight trains and planning rail terminal operations. Due to complexity of the general problem the following levels for its solving are proposed [5, 6]:

I. Schedule the service slots of trains (TYSP).
II. Assign each train to a railway track.
III. Decide on positions of the containers on trains.
IV. Assign container moves to portal cranes.
V. Decide on the sequence of container moves per crane.

Paper [6] proposes solution of the problem on the level I (TYSP). An approach for scheduling trains arriving to the transshipment yard is elaborated.

According to the approach trains are allocated to service slots. A mathematical model of the problem TYSP based on Boolean variables and heuristic algorithm for analysis of the mathematical model are constructed.

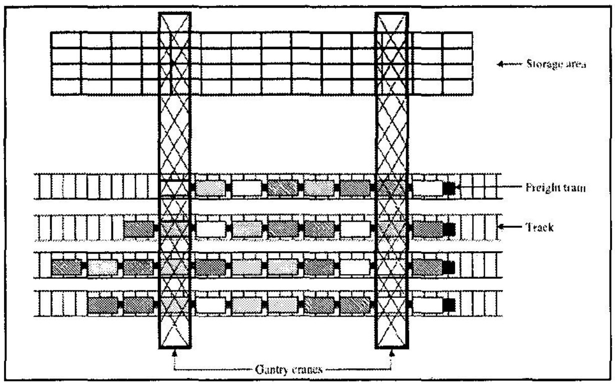

Fig. 1. A scheme of the transshipment yard

Paper [7] continues investigations of [6] and formulates the extended problem. Time slots for train service are formed and the allocation of each train on a certain railway track in such a service slot is implemented.

The approach corresponds to I and II levels of solving the general problem formulated in [6]. Mathematical model based on combinatorial configurations and solution strategy are constructed. For constructing mathematical model and solution strategy special classes of combinatorial configurations and their properties are used [11, 13–14].

The cost of moving containers from a source train to a target train depends on the distance between the trains. If the target train and the source are served in the same time slot, both trains should be allocated on the nearest railway tracks. If trains are served at different slots, it is also important to assign both trains so that the movement of the gantry crane is minimal. Starting from that the total cost of moving containers is estimated by the distance that the crane makes during the rail terminal operations [7]. The cost of moving containers from a source train to a target train depends on the distance between the trains. If the target train and the source train are served in the same time slot, both trains should be allocated on the nearest railway tracks. The total cost of the rail terminal operations revisits to the transshipment yard, split moves and movement of containers (depends on the location of trains on the tracks) is minimized.

The purpose of this paper is analysis of the general problem of planning the work of the transshipment yard based on approach [7] taking into account placement of containers on railway platforms and in the storage area.

II. SCHEDULING OF TRANSSHIPMENT YARD

In [6] transshipment yard scheduling problem (TYSP) is formulated. The problem consists in creating a train schedule for transshipment yard with assignment trains to time service slots. TYSP may be formulated as follows [6]: set of I trains with known number of freight platforms is specified. Trains that arrive at the station should be assigned to a service slots $t=1 \ldots T$, which consists of G simultaneously serviced trains. The number of trains in the slot is limited by the number of parallel tracks at the transshipment yard. Trains in one slot arrive and leave the station at the same time.

Containers from arriving trains are divided into those that have to be passed through the transshipment yard and those that require processing at the transshipment yard. If it is impossible to load all the containers to the train due to processing of the source train for it in next slot the problem of a revisit [6] of the train to the transshipment yard arises. Two alternative ways of solving the problem are considered.

I. Trains that have not received all the necessary containers have to leave the transshipment yard and then return to the transshipment yard in a further service time slot. In this case, the return visit delays the route as a whole and its duration should be reduced to a minimum. In case of necessity of the return visit such a train is processed after all other trains have visited the transshipment yard at least once. This provides that all the containers for such a train have already been delivered to the transshipment yard and more than one revisit to the station is not required.

II. All trains leave the transshipment yard even if they have not received all their containers and go to the destination. Thus, in the storage area of the station some containers remain, they should be assigned to other trains that head to the destination of the containers. This situation creates a long delay and is completely undesirable as the containers take up space in the storage area and it is necessary to provide for free platforms on another train. But in this case, a return visit to the transshipment yard is not required, which has a positive effect on the schedule of trains.

The next problem that arises when planning the work of a transfer station is the split moves [6].

The movements of the gantry crane [8] must be accurate, so the movement of the container [10] (and hence the processing of the train) takes a long time. If a source train and a target train are assigned into one service slot, the transfer of the container between the trains is provided by just one movement of the gantry crane. If such trains allocated into different slots, the gantry crane has to make a certain number of movements in order to transfer a container to the storage area [10], and then (in another time slot) repeat these movements and transfer the container from the storage area to the train. In this case, the service process is divided into several stages, and the processing time increases. The planning of the transshipment yard is aimed at avoiding split moves.

The freight trains scheduling problem described in [7], is formulated as follows. For TYSP in described [6] additional problem of assignment trains to tracks in a service slot is formulated and solved.

Each time slot t in [7] is described using tuple K^t, which contains the numbers of all trains assigned to slot t. In the problem of assigning each train to a certain track, their order is important, therefore $K^t=(K_1^t,K_2^t,\ldots,K_g^t,\ldots, K_G^t)$. Here $K_g^t \in I$ denotes the number of train that belong to slot t and is located on the track $g \in G$.

During forming a service time slot, G trains are selected from $|I|$ available, that is, tuple K^t is selected from a set of permutations $P_{|I|}^G$ (permutations of $|I|$ elements are taken from G at a time). An optimal time slot K^t is determined as solution of combinatorial optimization problem [11, 13, 14]. An optimal solution corresponds to combinatorial configuration from the set $K^t \in P_{|I|}^G$. As a result of solving the problem, time slots K^t, $t=1,2,\ldots,T$ are formed.

III. FORMULATION OF THE PROBLEM

This paper analyses peculiarities of the general problem of planning the work of the transshipment yard related to level III. It is proposed to choose optimal position of containers on railway platforms and in the storage area, during the processing of trains at the transshipment yard.

Placement of containers at the transshipment yard involves transferring containers from a train to a train, from a train to the storage area or from the storage area to a train with a choosing position for each container. The cost of container processing operations is determined by the distance passed by containers. Optimization of containers processing at level III consists in reducing the cost of moving containers based on choosing their positions on platforms and in the storage area.

Thus, the problem of placement of containers in the processing of trains at the transshipment yard consists in determining optimal location of containers on the target trains and in the storage area based on the solution of the problem of level I. Using the formed service time slots and the known positions of the trains on the tracks at the station [7], optimal positions of the containers on the target trains and in the storage area have to be determined.

In each service slot, certain containers from source trains and specified containers from the storage area should be moved and placed on platforms in appropriate target trains. Accepting platforms in target trains should be selected from free platforms and platforms on which containers are intended to be transferred to the storage area. Specified containers from the source trains should be moved to free positions in the storage area.

The problem consists in constructing a schedule for trains processing (assign trains to slots), determining the location of trains at the transshipment yard tracks (assign trains on tracks) and creating a schedule for containers placement in order to minimize the cost for trains processing.

In this paper, we propose the solution strategy which involves two stages: at the first stage, the problem scheduling freight trains in rail-rail transshipment yards with train arrangements is solved. A mathematical model and method for solving the problem is proposed in [7]. At the second stage, for each service slot formed at the first stage, optimal positions for containers on target trains and in the storage area are determined, minimizing the total cost of containers processing.

Containers placement in rail terminal operations problem (CPRTOP). For a given service slot of freight trains, determine the optimal container positions on target trains and in the storage area at the transshipment yard, minimizing the total cost of container processing.

IV. MATHEMATICAL MODEL OF CPRTOP

We introduce the following parameters and variables to formalize the problem:

- G is number of tracks and number of freight trains in a service slot;
- $J_n = \{1,2,\dots,n\}$ is index set;
- $N = [n_{ij}]$, $i,j \in J_G$ is a number of containers in source train i intended to target train j;
- l_j, $j \in J_G$, is a number of containers in the storage area intended to target train j;
- k_i, $i \in J_G$, is a number of containers in source train i intended to the storage area;
- m_j, $j \in J_G$, is a number of free platforms in target train j;
- d is a number of free positions for containers in the storage area.

Let A be set of containers connected with rail terminal operations on current service slot, $A=\{a_1,a_2,\dots,a_D\}$. Each container $a_i \in A$ is represented by 4 parameters, $a_i=(a_x^i, a_y^i, a_f^i, a_t^i)$. Here (a_x^i, a_y^i) are coordinates of the container a_i in the source train $a_f^i \in J_G$ or its position in the storage area $(a_f^i=0)$ and intended to the target train $a_t^i \in J_G$ or to the storage area $(a_t^i=0)$. We represent free platforms in target trains and free positions in the storage area for containers by the same technique as containers.

We order all the containers of set A in a such way: $A_1, A_2,\dots, A_G, A_{G+1}$. Here set A_i consists of the containers of train i assigned to service slot i, $i \in J_G$. Elements of set A_{G+1} are containers from the storage area intended to trains $1,2,\dots,G$. We order containers in each set A_i as follows:

$$A_i = \{a_1^i, a_2^i, \dots, a_{n_{11}}^i \mid a_{n_{11}+1}^i, \dots, a_{n_{11}+n_{12}}^i \mid \dots \mid a_{n_{11}+\dots+n_{1G-1}}^i, \dots, a_{n_{11}+\dots+n_{1G}}^i \mid a_{N_1+1}^i, \dots, a_{N_1+k_i}^i \mid a_{N_1+k_i+1}^i, \dots, a_{N_1+k_i+m_i}^i\},$$

where $a_1^i, a_2^i, \dots, a_{n_{11}}^i$ are containers of train i intended to train (and track) $j \in J_G$,1,..., $a_{n_{11}+\dots+n_{1G-1}}^i, \dots, a_{n_{11}+\dots+n_{1G}}^i$ are containers for train G, $a_{N_1+1}^i, \dots, a_{N_1+k_i}^i$ are containers of train i intended to the storage area, $a_{N_1+k_i+1}^i, \dots, a_{N_1+k_i+m_i}^i$ are free platforms in train i.

We define variables of the model. Let $U=\{u \in R^{\sigma}\}$, $u=(u_1, u_2,\dots, u_G)$ where $u_i=(X_i, Y_i, Z_i)$, $X_i = (X_1^i, X_2^i, \dots, X_{N_i}^i)$, $X_t^i = (x_1^{it}, x_2^{it}, \dots, x_{n_{it}}^{it}) \in A_{m_t}^{n_{it}}$, $N_i = \sum_{j=1}^{G} n_{ij}$, $A_{m_t}^{n_{it}}$ is set of arrangements of m_t elements which corresponds to free platforms in target train t by n_{ij} which is equal to number of containers in source train i intended to target train j, $x_j^{it} = (x_x^{itj}, x_y^{itj})$. $Z_i = (z_1^i, z_2^i, \dots, z_{k_i}^i) \in A_{d_i}^{k_i}$, where $z_j^i = (z_x^{ij}, z_y^{ij})$ are positions in the storage area for k_i containers from source train i, $d_i = d - \sum_{j=1}^{i-1} k_j$. Y_i are free platforms in train i for containers from all the source trains for which train i is target train $y_j^i = (y_x^{ij}, y_y^{ij})$.

$$Y_i = (y_1^i, y_2^i, \dots, y_{\bar{N}_i}^i, y_{\bar{N}_i+1}^i, \dots, y_{\bar{N}_i+l_i}^i) \in A_{m_i-n_{ii}}^{\bar{N}_i+l_i}, \quad \bar{N}_i = \sum_{j=1}^{G} n_{ji},$$

$$\sigma = \sum_{i=1}^{G} (\bar{N}_i + l_i + k_i + \sum_{t=1}^{N_i} n_{it}).$$

We assume that costs of rail terminal operations are determined by the distances passed by the gantry crane. To decrease the total cost we should minimize total distance passed by the gantry crane during all the rail terminal operations. Based on specifics of gantry crane moving (along two orthogonal axes) we use Manhattan distance for calculating length of way passed by the crane. Starting from that total cost for the operations with train i is equal to:

$$F_i(u)=\sum_{t=1}^{N_i}\sum_{s=1}^{n_{it}}\rho(a^{i}_{n_{i1}+\ldots+n_{it-1}+s},x^{it}_{s})+\sum_{s=1}^{k_i}\rho(a_{N_1+s},z^{i}_{s})+ \\ +\sum_{t=1}^{G+1}\sum_{s=1}^{n_{iti}}\rho(y^{i}_{s},a^{i}_{n_{1i}+n_{12}+\ldots+n_{i-1i}+s}) \qquad (1)$$

where $\rho(a_i,x_i) = | a_x^i - x_x^i | + | a_y^i - x_y^i |$.

Total cost of the operations with all the trains for current service slot is equal to

$$F(u)=\sum_{i=1}^{G}F_i(u)$$

Based on introduced designations and assumptions we represent mathematical model of CPRTOP as follows:

$$F(u)=\sum_{i=1}^{G}F_i(u)\to\min\,,\ u\in U\subset R^{\sigma} \qquad (2)$$

This combinatorial optimization problem is NP-hard and can be solved both by exact and heuristic methods. Some approaches for solving combinatorial optimization problems based on generation of combinatorial configurations with special properties are represented in [13].

The construction of a mathematical model represented in this paper is the first step in the investigation of the process of optimizing container placement at the transshipment yard. The next stage of the investigation should be analysis of the mathematical model by one of the methods of combinatorial optimization. Since the results obtained in this paper are extension of research represented in [7], a modification of the heuristic method proposed in [7] can be used to analyze the mathematical model. Software developed on the basis of both stages of the research can be used for trains scheduling at modern transshipment yard.

Example. Let service slot consists of two trains on two tracks and $n_{12}=0$, $n_{21}=2$; $l_1=2$, $l_2=0$; $k_1=0$, $k_2=2$; $m_1=4$, $m_2=0$; $d=3$; $A=\{a_1,a_2,a_3,a_4,a_5,a_6\}$. Coordinates of the containers are given in Table 1. Calculate cost of two different variants of containers moving at the service slot.

First variant. The cost of moving container a_1 to platform x_3 and container a_2 to platform x_4 is equal to $C_1=\rho(a_1, x_3)+\rho(a_2, x_4)=|1-4|+|2-1|+|3-5|+|2-1|=7$. The cost of moving container a_4 to platform y_1 and container a_3 to platform y_2 is equal to $C_2=\rho(a_4, y_1)+\rho(a_3, y_2)=|1-2|+|0-1|+|2-1|+|0-1|=4$. The cost of moving container a_5 to platform z_1 and container a_6 to platform z_2 is equal to $C_3=\rho(a_5, z_1)+\rho(a_6, z_2)=|2-3|+|2-0|+|4-1|+|2-0|=5$.

Total cost of the operations with all the trains for current service slot is equal to $F(u)=C_1+C_2+C_3=7+5+4=16$.

TABLE I. COORDINATES OF THE CONTAINERS

Containers	Free platforms
$a_1=(1,2,2,1)$ $a_2=(3,2,2,1)$	$x_1=(1,1)$, $x_2=(2,1)$ $x_3=(4,1)$, $x_4=(5,1)$
$a_3=(1,0,0,1)$ $a_4=(2,0,0,1)$	$y_1=(1,1)$ $y_2=(2,1)$
$a_5=(2,2,2,0)$ $a_6=(4,2,2,0)$	$z_1=(3,0)$ $z_2=(4,0)$ $z_3=(5,0)$

Second variant. Calculate the value of objective function if placement of containers a_3 and a_4 are changed. Container a_3 moves to platform y_1 and container a_4 moves to platform y_2 and $C_3=\rho(a_3, y_1)+\rho(a_4, y_2)=|1-1|+|0-1|+|2-2|+|0-1|=2$. Total cost of the operations with all the trains for current service slot is equal to $F(u)=C_1+C_2+C_3=7+5+2=14$.

Thus changing of placement of containers decreases the total cost of rail terminal operations.

V. CONCLUSION

The problem of analysis and modeling of rail terminal operations is considered. An approach for increasing efficiency of implementing rail terminal operations is proposed. According to the approach the increasing of efficiency of the operations is based on optimization of the placement of containers on railway platforms and in the storage area at railway transshipment yard. A combinatorial optimization model of the problem is constructed; properties of the problem are analyzed. Possibilities of analysis of the model are discussed.

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