Implementation of the Extinction Theorem in a Problem of Airy Pulse Scattering by a Dielectric Layer

O.V. Kuryzeva, A.D. Tkach and A.G. Nerukh
Kharkiv National University of Radio Electronics
Kharkiv, Ukraine
E-mail: lyolya.90@bk.ru; nerukh@gmail.com

***Abstract*— A problem of Airy pulse transformation by a plane-parallel layer is considered. Solving of this problem is made by the Volterra integral equation method in two stages. Finding of an electrical field inside a layer is implemented on the first stage. The fields of reflected as well as transmitted pulses are calculated on the second stage. The virtual structure of the pulse field inside the layer consists of some terms one of which extinctions the initial pulse according to the Ewald-Oseen extinction theorem.**

***Keywords*— Airy pulse; extinction theorem; dielectric layer**

I. Introduction

The phenomena of the electromagnetic pulses interaction with an inhomogeneous medium represents a considerable interest to many problems in electrodynamics. Special interest is turned on the Airy pulses [1], which are actively studied in recent years. This is due to a number of their extraordinary properties in a paraxial approximation, such as non-diffractive propagation, self-acceleration and self-healing [2]. Asymmetrical form of the Airy pulse is characterized by an exponentially fading front and oscillating "tail", decaying at infinity. It allows investigating the inhomogeneous medium interaction process starting at a fixed moment of time. The research of the electromagnetic waves interaction with the inhomogeneous and nonstationary medium requires an adequate and realizable mathematical modeling, which is optimal for the problem. In the given paper such approach is based on the Volterra integral equations method [4], which is capable of physical clarity and descriptive completeness. The latter consists of, besides the other pros, the automatic involving of both the initial and the boundary conditions and the versatility regarding the free term of the equation (the initial pulse). Applying of this approach in the given paper is allowed to obtain a solution of the one-dimensional problem about interraction of an electromagnetic Airy pulse with a dielectric layer, reveal a virtual structure of the transformated pulse. The structure of the found inner field demonstrates the implementation of the Ewald-Oseen extinction theorem [6].

To show that a problem on transformation of an electromagnetic pulse $E_0(t,x)$ by a dielectric layer is considered. The initial pulse propagates normally to the layer in the environment characterised by the permittivity ε. A dielectric medium in the layer located in the interval $0<x<a$ is characterised by the permittivity ε_1. The main goal of the paper is determination of a transformed field structure.

II. The Problem Solution

A. The Volterra integral equation method

In the problem statement as above the electromagnetic field inside as well outside the layer is described by the integral equation

$$E(t,x)=E_0(t,x)-W\frac{\partial^2}{\partial t^2}\int_0^\infty dt'\int_0^a dx'\theta\left(t-t'-\frac{|x-x'|}{v}\right)E(t',x') \quad (1)$$

where $W=(v^2-v_1^2)/(2v_1v^2)$, $v=c/\sqrt{\varepsilon}$, $v_1=c/\sqrt{\varepsilon_1}$, c is the light velocity in vacuum and $\theta(t,x)$ is the Heaviside unit function. The equation (1) contains initial and boundary conditions for the problem.

In the case when the time-spatial observation point (t,x) is located inside the layer the expression (1) represents, in fact, the integral equation. In the contrary case when the observation point is located outside the layer the expression (1) is a formula for calculation of reflected and transmitted pulses.

Further the Airy pulse

$$E_0(t,x)=\mathrm{Ai}(-t+\frac{x}{vT})=\frac{T}{2\pi}\int_{-\infty}^{+\infty}e^{i\frac{(\omega T)^3}{3}-i\omega t+i\omega x/vT}d\omega \quad (2)$$

where T is a temporal scale will be considered as the initial one. The form of the Airy function allows considering a situation when the initial pulse begins to interact with the layer after the zero moment of time.

The solution to equation (1) i.e. the field inside the layer can be found by means of the resolvent [4]

$$\hat{R}=\theta(x)\theta(a-x)(\hat{R}_1+\hat{R}_{mn})\theta(x')\theta(a-x') \quad (3)$$

978-1-5090-1579-5/16/$31.00 ©2016 IEEE

The first part of the resolvent has the same form as in the case of the unbounded space

$$\hat{R}_1 = W\frac{\partial^2}{\partial t^2}\theta\left(t-t'-\frac{|x-x'|}{v_1}\right)\theta(t-t') \qquad (4)$$

The second part of this resolvent involves pulse re-reflections from the layer boundaries. The whole time-spatial region corresponding to the problem is divided into zones which are labelled by two indexes as in Fig. 1. The first index indicates how many times the pulse was reflected from the lower boundary of the layer (the second index indicates the reflections from the upper boundary)

$$\hat{R}_{mn} = W\frac{\partial^2}{\partial t^2}\left\{\sum_{k=1}^{m} R^k\theta\left[t-t'-h_k^{(+)}(x,x')\right]+ + \sum_{k=1}^{n} R^k\theta\left[t-t'+h_k^{(-)}(x,x')\right]\right\}\theta(t-t') \qquad (5)$$

where $h_k^{(\pm)}(x,x') = \frac{1}{2v_1}\{2x+(-1)^k\left(a-2x'\right)\pm(2k\mp1)a\}$

and the coefficient $R=(v-v_1)/(v+v_1)$ takes into account the contribution of each reflection.

B. The transformed inner field

Taking into account all these circumstances and using the resolvent (3) the electric field inside the layer is given by virtual terms

$$E(t,x) = E_0(t,x)+ +\theta(x)\theta(a-x)\int_0^\infty dt'\int_0^a dx'\hat{R}E_0(t',x') = E_0+E_1+E_{mn} \qquad (6)$$

Here,

$$E_1(t,x) = W\theta(x)\theta(a-x)\frac{\partial^2}{\partial t^2}\int_0^\infty dt'\int_0^a dx'\theta\left(t-t'-\frac{|x-x'|}{v_1}\right)E_0(t',x') \qquad (7)$$

is the “pure” field without influence of the layer boundaries and

$$E_{mn} = W\theta(x)\theta(a-x)\frac{\partial^2}{\partial t^2}\int_0^\infty dt'\int_0^a dx'\left(\sum_{k=1}^{m} R^k\theta\left[t-t'-h_k^{(+)}\right]+ \sum_{k=1}^{n} R^k\theta\left[t-t'+h_k^{(-)}\right]\right)E_0(t',x') \qquad (8)$$

is the field created by the pulse re-reflections from the boundaries.

For distinctness let the observation point (t,x) be located in the zone 44, Fig. 1. The initial pulse is located in the region bounded by the time-spatial trajectory shown in Fig. 1 by the dashed blue line. The transformed field in this zone calculated via the resolvent consists of the main contribution determined by the integration over the region $t-t'-|x-x'|/v_1>0$ shown in Fig. 1 by the red hatching.

Using the Fourier representation of the Airy pulse (2) the main component calculated by the formula (8) is given by the expression

$$E_{00}^{(44)}(t,x) = E_0(t,x)-\mathrm{Ai}\left(-\frac{t}{T}+\frac{x}{vT}\right)+E_{00}(t,x) \qquad (9)$$

where

$$E_{00}(t,x) = \frac{v_1+v}{2v}\mathrm{Ai}\left(-\frac{t}{T}+\frac{x}{v_1T}\right)+ \frac{v_1-v}{2v}\mathrm{Ai}\left(-\frac{t}{T}-\frac{x}{v_1T}+a\frac{v_1+v}{v_1v}\right) \qquad (10)$$

The first calculated term in (9) has the same form as the incident wave but with the opposite sign. So, this term turns off the initial pulse in accordance with the Ewald-Oseen extinction theorem [6], and another two terms (10) describe the transformed field in this region. The pulse propagation in this case is determined by the new velocity v_1 corresponding to the permittivity inside the layer.

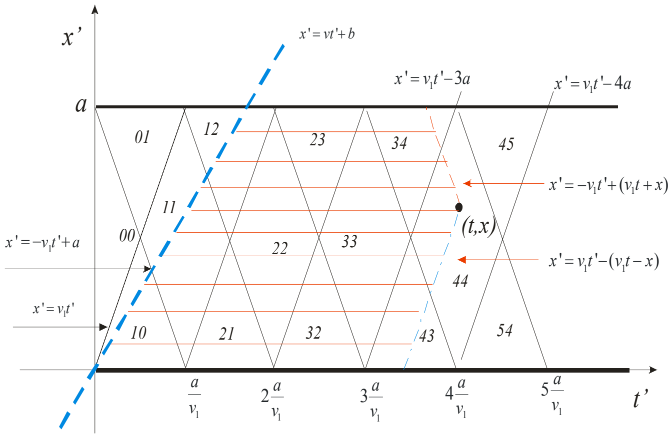


Fig. 1. Time-spatial zones corresponding to re-reflections of the pulse from each boundary of the layer after beginning of the pulse layer interaction.

The first term in the first sum of the formula (8), $k=1$, determines the field in the region shown in Fig. 2 by red hatching and it gives the contribution to the field caused by one reflection with the weight R of the pulse from the upper boundary. Calculating the integral over this region gives the field inside the layer with one re-reflection from the upper boundary:

$$E_{10}^{(44)}(t,x) = R\frac{v_1+v}{2v}\mathrm{Ai}\left(-\frac{t}{T}-\frac{x}{v_1T}+\frac{2a}{v_1v}\right) \qquad (11)$$

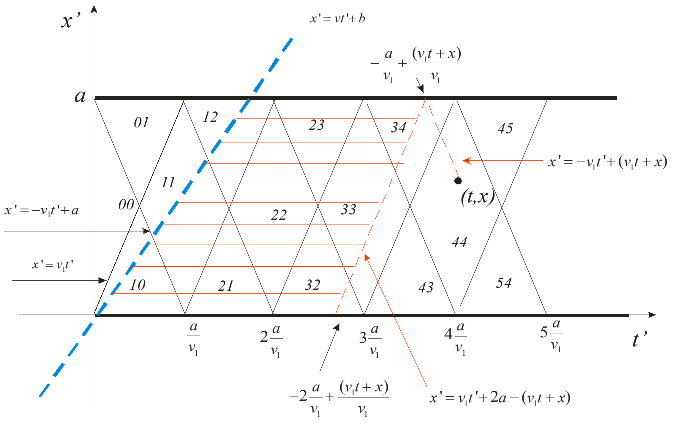


Fig. 2. The integration region for the inner field with one re-reflection from the upper boundary of the layer.

The first term of the second sum in (8), $k=1$, is determined the field in the region shown in Fig. 3 by blue hatching and gives also the contribution with the weight R, but as the result of the pulse reflection from the lower layer boundary. So, contributions of one reflection from the upper and one from the lower layer boundaries consists of four terms

$$E_{00}^{(44)}(t,x)+E_{11}^{(44)}(t,x)=\frac{2v_1}{v_1+v}\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1T})+$$
$$\frac{v_1-v}{2v}\mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1T}+a\frac{v_1+v}{v_1v})-$$
$$R\frac{v_1+v}{2v}\mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1T}+\frac{2a}{v_1v})+ \tag{12}$$
$$R\frac{v_1-v}{2v}\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1T}+a\frac{v_1+v}{v_1v})$$

So, the field on this stage is represented by the sum of two pairs of the waves propagating with the same velocity but in different directions.

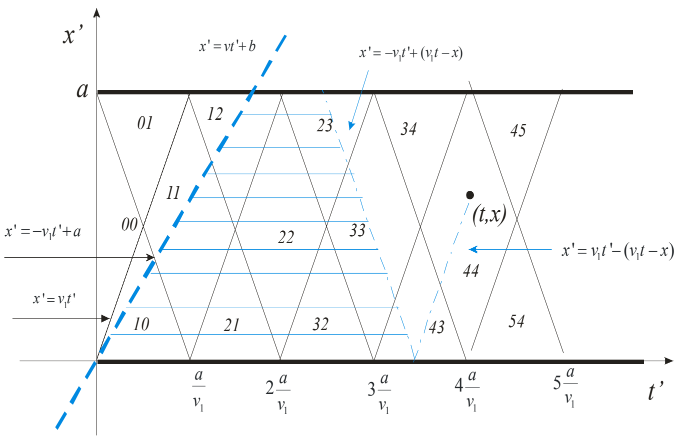


Fig. 3. The integration region for the inner field with the one re-reflection from the lower boundary of the layer.

Other terms of two sums in the formula (8) are proportional to higher powers of the coefficient R and give very small contributions to the field as $R<1$.

III. Results and Discussion

The initial pulse in the form of the Airy pulse is just adequate to the statement when the initial pulse begins to interact with the layer after zero moment of time. The temporal behavior and spatial distribution of the initial Airy pulse is shown in Fig.4.

Numerical comparison of an initial Airy pulse and the inner pulse at various points of the layer is shown in Fig.5 - Fig.8. Two cases of the medium parameters are considered: a) a less transparent layer when $v_1/v=0.75$ and b) more transparent layer when $v_1/v=1.5$. The normalized layer width $L=k(b-a)$ is used. The structure of a pulse inside the layer depending on the number of re-reflections from boundaries of the layer is also shown.

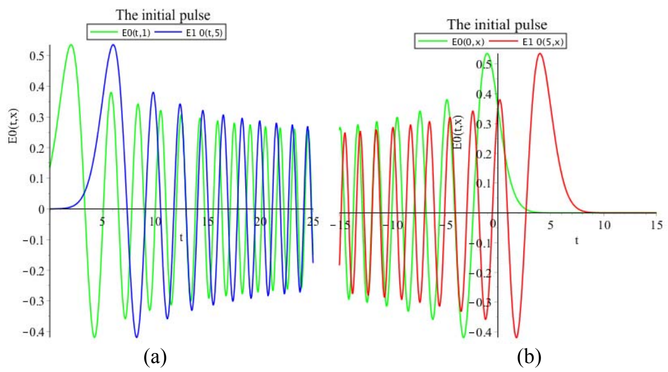
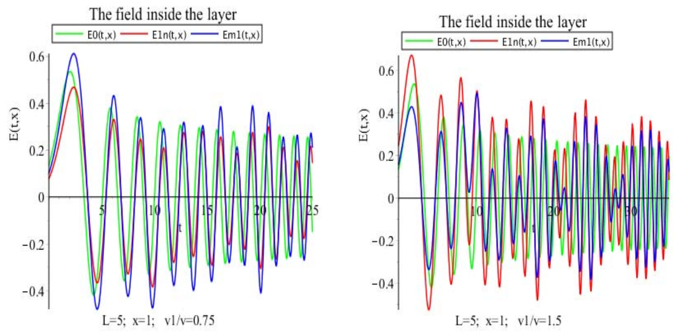


Fig. 4. The temporal behaviour of the initial Airy pulse at various points: (a) *x/vT*=1 (green) and *x/vT*=5 (blue). (b) The positions of the initial Airy pulse at various moments: *t*/T=0 (green) and *t*/T=5 (red). The layer thickness is supposed to be equal to L=10.

Calculation of the transformed pulse form inside the layer shows that it differs from the form of the initial one in the case of the thin layer only. Nearer location of the layer boundaries causes the appearance of a signal modulation. With increasing the layer thickness the inner field practically coincides with the initial pulse by form and size.

It is shown in Fig. 5a that near the lower boundary in the case of the less optically dense medium the inner field has practically the same form as the incident wave. Re-reflection of the pulse from the upper boundary decreases the modulation amplitude, Fig. 5a (red), and increases near the lower boundary, Fig. 5a (blue).

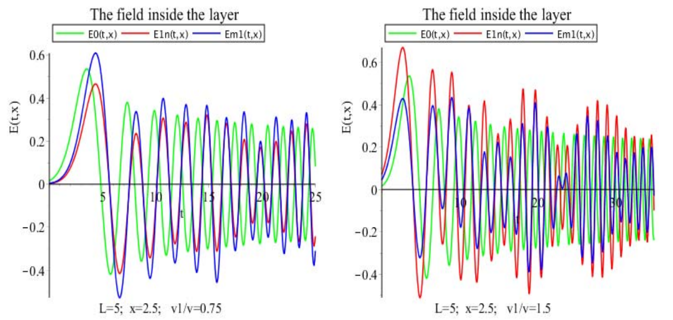
A typical form of the Airy pulse is violated and appears "chaotic" fluctuations at the more optical dense medium Fig.5b. The main "lobe" of the pulse keeps its form, but significantly changes the oscillation amplitude of the "tail". In this case re-reflection of the pulse from the upper boundary increases the oscillation amplitude, Fig. 5b (red), and decreases near the lower boundary, Fig. 5b (blue).



(a) (b)

Fig. 5. Airy pulse evolution near the lower boundary: the temporal dependence of the inner field on the number of re-reflections from boundaries of the layer at the less optical dense medium (a) and at the more optical dense medium (b).

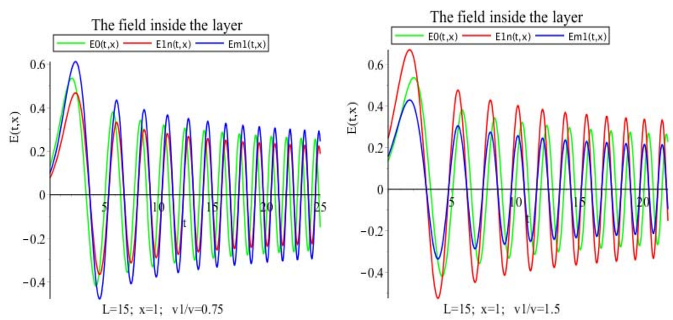
The pulse does not change its form propagating across the layer, Fig. 6. In the case of the less optical dense medium, Fig. 6a, the pulse near the lower boundary is characterized by a smaller number of oscillations than in the case of the medium with the larger refraction coefficient, Fig. 6b.



(a) (b)

Fig. 6. Airy pulse evolution in the middle of the layer: the temporal dependence of the inner field on the number of re-reflections from boundaries of the layer at the less optical dense medium (a) and at the more optical dense medium (b).

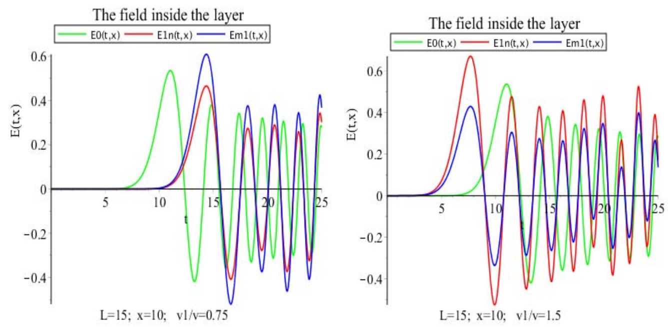
Near the lower boundary of the thick layer the signal almost coincides with the initial pulse but dependence on the re-reflections from the layer boundaries changes, Fig 7a,b.



(a) (b)

Fig. 7. Airy pulse evolution in the thick layer: the temporal dependence of the inner field near the lower boundary on the number of re-reflections from boundaries of the layer at the less optical dense medium (a) and at the more optical dense medium (b).

It is visible clearly that the retardation of the initial pulse is compared to the inner signal in the thick layer. The undisturbed pulse retards in relation to the signal inside the layer of the medium with the refraction coefficient $v_1 / v < 1$, Fig.8a, and leads in the medium with the refraction coefficient of $v_1 / v > 1$, Fig.8b.



(a) (b)

Fig.8. Airy pulse evolution in the middle of a thicker layer: the temporal dependence of the inner field on the number of re-reflections from boundaries of the layer at the less optical dense medium (a) and at the more optical dense medium (b).

CONCLUSIONS

Solution and analysis of a problem about transformation of an electromagnetic pulse by a dielectric layer show unusual behavior of the transformed Airy pulse which depends on the dielectric layer size as well as on an optical density of the layer medium. In main, in result of the interaction of the pulse with a layer three waves are formed, one of which extinguishes the incident pulse, confirming the implementation of the extinction theorem. The form of the pulse modulation is more pronounced compared to the undisturbed Airy pulse for a thin layer of the more optical dense medium.

References

[1] Y. Kaganovsky, E. Heyman, “Airy pulsed beams”, J. Opt. Soc. Am. A, 28, 2011, pp. 1243-1255.

[2] G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, “Observation of Accelerating Airy Beam Ballistics”, OSA/CLEO/QELS, 2008, №1. pp. 2085–2089.

[3] M. V. Berry, N. L Balazs, “Nonspreading wave packets”, Am. J. Phys., vol. 47, 1979, pp. 264.

[4] A. G. Nerukh, N. A. Khyzhniak, Modern problems of nonstationary macroscopic electrodynamics, HNPO «Test-radio», 1991.

[5] A. G. Nerukh, “Electromagnetics waves in a dielectric layer with the time-dependent parameters”, Journal of technical physics, 1987.

[6] M. Born, E. Wolf, Principles of optics, Cambridge Univercity Press, 1959, p 106.

[7] O. V. Kuryzheva, A. D. Tkach, A. G. Nerukh, “Spectral features of a dielectric layer in paraxial approximation”, MSMW, 978-1-5090-2267-0/16, 2016 IEEE.