

The work is aimed at establishing the boundaries of the use of models for describing signals in optoelectronic systems in calculating efficiency.

A description of the signal formation process is proposed, taking into account the corpuscular and wave properties when registering signals in a wide range of intensities.

A description of the statistical features of the output signals depending on the energy properties of the signal and noise components is proposed. It is shown that when describing the output signals of optoelectronic systems that register signals with different properties, Poisson and Gaussian distributions are used. The invariance of Poisson flows determines the description of an additive mixture of signal and background flows using Poisson flow.

The efficiency of optoelectronic systems is calculated by the signal-to-noise ratio criterion based on the corpuscular and wave description of signals. Efficiency calculations have shown the expedience of using this criterion, provided that the statistical properties of signal and background flows are stabilized. It is shown that under the condition of changes in the energy characteristics of signals, from the point of view of the wave and corpuscular models, the statistical characteristics of the signals have different descriptions.

The analysis of theoretical methods of signal analysis in optoelectronic systems is carried out, which is aimed at an adequate characteristic of the system operation, depending on the conditions of its operation. Taking into account the method of describing the process of receiving and processing signals will take into account additional statistical characteristics of signals, for example, an increase of the variance of the output signal. The use of adaptive methods for describing signals will make it possible to increase the efficiency of systems when receiving strong signals in a difficult interference environment, as well as when receiving weak signals

Keywords: *optoelectronic system, corpuscular theory, wave theory of light, statistical model, detection*

INFLUENCE OF A SIGNAL DESCRIPTION MODEL ON THE CALCULATIONS OF THE EFFICIENCY INDICATORS OF OPTOELECTRONIC SYSTEMS

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1. Introduction

The dynamic development of technology and equipment in recent decades has contributed to the technical implementation of the potentially high capabilities of optoelectronic systems. Modern directions of improving the efficiency of the system combine elements of theories of general construction of systems [1], detection [2] and stochastic-deterministic signal processing [3, 4].

Theoretical methods of signal description are the basis for the development of the latest processing algorithms, identification, and calculation of the system efficiency. The models describe the spatial-temporal properties of energy-related signals, both signal and noise components, received by the systems. With the help of modern methods of measuring optical radiation, the parameters of signals are estimated. Using the well-known, well-proven methods and technologies of statistical analysis, statistical parameters of signals

are calculated, which form the basis for calculating the efficiency of optoelectronic systems by any criterion.

In general, the models for describing signals in optoelectronic systems are based on two different approaches based on the corpuscular and wave theories.

The model based on the wave theory of light takes into account the process of interaction of optical radiation with matter. The model describes the processes of diffraction and interference. Signals are presented as a continuous process. The statistical characteristics of signals and interferences are taken into account as separate processes that obey the central limit theorem.

The model, based on the corpuscular theory of light, also takes into account the process of interaction of radiation with matter. It is taken into account that signals and noises have corpuscular properties. It is assumed that the fluctuations of the useful signal are part of the interference. Signals are delivered as a discrete stream.

The approach to determining the efficiency of systems, for example, by the criterion of the signal-to-noise ratio, both from the point of view of the corpuscular and from the point of view of the wave theory, is chosen the same.

However, with changes in observation conditions, in cases of registration of weak signals at the boundaries of the dynamic range, the results of determining the efficiency of systems based on the corpuscular and wave theory are interpreted differently. This leads to the impossibility of applying a unified approach to assessing the effectiveness of systems and the appearance of errors in signal processing. The physical interpretation of the results obtained has not yet been sufficiently studied. The disagreements can be associated with the corpuscular properties of optical radiation, which determine the statistical parameters of the output signals when the energy and space-time characteristics of the signals change.

The research of the authors is aimed at determining the application limits between the application of theoretical models for describing signals in optoelectronic systems. This will increase the efficiency of systems in difficult conditions of receiving and processing signals.

2. Literature review and problem statement

Models for describing signals in optoelectronic systems include: characterization of energy indicators and space-time properties of both signals and interference; the logical structure of the interaction of signals with elements and paths of systems. That is, the model aimed at creating an adequate description of signals and interference takes into account the operating conditions of the systems. For example, in the case of energy detection of signals in optoelectronic systems, it is necessary to take into account the limited dynamic range of photodetectors. The limiting capabilities of the systems at the upper limit of the dynamic range is the charge saturation mode, and at the lower limit – the presence of internal noise of photodetectors.

At the present stage, there are various theoretical models for describing the process of signal formation in systems. Each of the approaches makes it possible to characterize multiparameter signals in optoelectronic systems. The statistical characteristics of the signals are influenced by many factors, including the temporal characteristics of the photodetection. To describe the statistical characteristics of

optical signals, one can use both elements of the corpuscular theory of light and elements of the wave theory.

The development of modern elemental base of optoelectronics and photonics (Semiconductor Photodetectors, Photodiodes for Visible and Infrared light, Phototransistors; Ultraviolet Photodetectors; Nano Optoelectronic Sensors; CCD Image Sensors, CMOS, Quanta Images Sensors) is directed and based on the development of highly sensitive elements in single photons. In works [5–7] it is shown that the use of highly sensitive elements provides an opportunity to study in more detail the properties of optical radiation. The studies are based on the corpuscular model, taking into account the influence of the internal noise of the photodetector, but the statistics of the output stream are not considered. Attention is not paid to the analysis of the processes of statistical detection of signal photons against the background of noise. Conclusions about the Poisson statistics of signals are made on the basis of the analysis of the process of registration of rare photons in a very short period of time, but the statistics of the signal flow over the entire observation interval is not considered. In [8, 9], a statistical analysis of signals was carried out when the radiation interacts with the substance of the photodetector during image formation. However, the probabilistic characteristics of signal detection have not been calculated.

In works [10–12], methods of describing optical radiation from the standpoint of wave theory are presented. Using this approach, diffraction phenomena, interference, spectral and polarization properties of optical signals are described. The absorption process in the photocathode substance is described using deterministic coefficients. However, the works did not take into account the processes of probabilistic interaction of optical radiation with the elements of the system. Also, the additional components of the additive mixture of interference – fluctuations of the signal component – are not taken into account. This method does not allow taking into account the changes in the statistics of the output signals when the intensity of the incoming signal changes. As a consequence, such a model cannot be considered complete. Works [13, 14] describe optical radiation based on the corpuscular theory. With the help of this model, phenomena of a probabilistic nature are taken into account, for example, generation of radiation, absorption in a substance of a photosensitive element. However, the analysis of the influence of the proposed model on the performance indicators of the system is not carried out.

In monographs [13, 15], when analyzing the space-time properties of signals, it was shown that the use of the wave model when describing signals of low intensity leads to results that contradict the experimental data. However, there is no explanation regarding the processes that can affect the variance changes of the additive mixture of the original signals. Papers [16, 17] discuss the statistical characteristics of the output signals under various operating conditions of the system based on the corpuscular model. The study characterizes the process of registering signals only with a sufficient level of intensity. The adopted model does not allow taking into account changes in the variance of the output signals with changes in the intensity of optical radiation. Statistical parameters of signals are characterized based on Poisson and Gaussian statistics. As shown in [18, 19], signals, which are recorded on the edge of the dynamic range of optoelectronic systems, can obey not only Gaussian statis-

tics, but also different probabilistic distribution laws when the energy parameters of signals and noise are changed.

However, the factors that can lead to disagreements in determining the efficiency of optoelectronic systems from the standpoint of the wave theory and the corpuscular theory of light when registering signals in a wide range of intensities are not considered. To date, none of the studies considered solves the problem of comparing the results of applying the corpuscular and wave approaches to assessing the efficiency of optoelectronic systems according to a specific criterion. The systematization of theoretical knowledge about the processes of signal formation in systems is important for establishing the boundaries of the application of existing methods in calculating the efficiency of systems.

3. The aim and objectives of research

The aim of research is to establish the boundaries of the application of methods for describing signals in calculating the efficiency of optoelectronic systems.

To achieve the aim, the following objectives are set:

- to offer a description of the signal formation process, taking into account the corpuscular and wave properties;
- to offer a description of the statistical features of the output signals depending on the energy properties of the signal and noise components;
- to determine the efficiency of optoelectronic systems by the criterion of the signal-to-noise ratio using the signal description method.

4. Descriptions of the process of forming the output signal of the optoelectronic system, taking into account the corpuscular and wave properties

4.1. Description of the process of forming the output signal of the optoelectronic system based on the corpuscular theory

According to the corpuscular theory, an optical signal is considered as a stream of discrete particles – photons, and the output signal is a discrete stream of charge carriers. According to the accepted mathematical models in [15, 17, 20, 21], the flow of discrete particles is described using the Poisson distribution law. The main properties that determine the Poisson process are [22, 23]:

1. The probability does not change over time, so the corresponding sequence does not trend.
2. The probability of simultaneous occurrence of two or more events is very small.
3. The probability of an event occurring in the interval $(t, t+h)$ does not depend on what happened before the moment t ; in particular, it does not depend on the time elapsed since the previous event occurred.

Thus, in order for the number of realizations of an event to depend on this law, it is sufficient that these realizations are independent over time intervals. Let's introduce the following notation:

- $\mu(t)$ – intensity of photoelectron flux (average number of photoelectrons formed per unit time);
- $\bar{n}$ – average number of photoelectrons formed in the interval τ .

These values are related by the ratio:

$$\bar{n} = \int_0^{\tau} \mu(t) dt.$$

The intensity of the photoelectron flux, the number of photoelectrons formed during the time τ will be random. According to the provisions of quantum mechanics, the probability of the formation of photoelectrons in a short time interval is proportional to the duration of this interval and the intensity of the input signal $I(t)$. Therefore, the intensity of the photoelectron flux is proportional to the intensity of the light flux, that is, $\mu(t) \sim I(t)$. Through the quantum nature of the signal and the quantum nature of the interaction of light with the photodetector, the moments of formation of photoelectrons at the output of the photodetector are determined taking into account the time dependence of the intensity of the light flux [14].

The model of the formation of the output signal can be characterized based on the approaches described in [10, 13, 15, 17] and using the provisions of geometric optics and corpuscular theory. The flow of backgrounds from a weak incoherent source has three properties: stationarity, ordinarity, and absence of aftereffect, therefore, it is Poisson. Statistical parameters (mathematical expectation and variance) of a random variable distributed according to Poisson's law, according to expression (1), are equal to each other and are determined by the distribution parameter:

$$m_1 = \mu_2 = \lambda. \quad (1)$$

The probability density of time intervals between adjacent points in a Poisson flow is exponential:

$$p(\tau) = \lambda e^{-\lambda\tau}. \quad (2)$$

The mean and variance of the intervals between points is:

$$M(\tau_k) = \frac{1}{\lambda}; \quad D(\tau_k) = \frac{1}{\lambda^2}. \quad (3)$$

To determine the statistical characteristics of stochastic flows generated by both the signal and the background, let's consider the process of converting a stochastic signal by a propagation medium and elements of an optoelectronic system.

The stages of the image formation process are shown in Fig. 1. The path of propagation of radiation is divided into regions, in each of which the interaction of light with matter can be described using one mechanism. When the light flux $\xi(t)$ interacts with the medium, amplitude and spatial effects appear, which are determined by the phenomena of absorption, radiation, reflection and transmission of quanta. The next area of radiation propagation is the object of observation. Depending on the energy characteristics of objects, the phenomena of absorption, emission, reflection and transmission of quanta are also detected. Further, the luminous flux interacts with the optical link, which is characterized by the following parameters: the diameter of the entrance aperture of the optical system; focal length of the optical system; coefficient of aberrations; luminous flux transmission coefficient; absorption coefficient of neutral filters (NF) $k \geq 1$.

Then the luminous flux interacts with the photodetector, characterized by: the size of the distribution element (resolution), the coefficient of quantum efficiency q ; frame duration T_f . The image formed by the flow $\xi(t)$ represents pulses, the energy level of which is determined by an additive

mixture of the signal and background components. A step-by-step consideration of the formation of the output signal of optoelectronic systems will make it possible to determine the parameters characterizing the signal $\xi_{s+p}(t)$ and the interference $\xi_p(t)$.

The luminous flux $\xi_0(t)$ is characterized by the average value $\bar{N}_0$ – the number of photons per unit time:

$$\bar{N}_0 = \frac{P_{f0}}{h\nu}, \quad (4)$$

where P_{f0} – the radiation density per unit area S ; $h\nu$ – light quantum energy.

Let's consider the passage of a stream of photons. Let's represent the implementation of the photon flux as a set of points on the time axis (Fig. 2, *a*). The implementation of the flow at the outlet of the region of interaction with the propagation medium is shown in Fig. 2, *b*. Taking into account the phenomena of absorption, emission, reflection and transmission of quanta, the output random photon flux is characterized by a changed number of photons (events). The intervals between points are distributed exponentially.

The flow $\xi_0(t)$, passing through the propagation medium with the transmission coefficient τ_{med} and the observation object with the transmission coefficient τ_{ob} is attenuated in a probabilistic manner and the implementation of the flow $N_{c1}(t)$ is not equal to the implementation of the input flow $N_0(t)$, attenuated by $1/\tau_\Sigma$ times:

$$N_{c1}(t) \neq N_0(t)\tau_\Sigma, \quad (5)$$

where τ_Σ – the total transmittance of the medium and the object.

However, for the mean values of these flows, the following equation will be valid

$$\bar{N}_{c1} = \bar{N}_0\tau_\Sigma. \quad (6)$$

After passing through the optical link, the luminous flux $\xi_{c2}(t)$ can be determined:

$$\xi_{c2}(t) = \xi_{c1}(t) \frac{1}{k \cdot \tau_{omp} \cdot k_{ab}} = \frac{\xi_0(t)\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}}, \quad (7)$$

where k_{ab} and τ_{omp} – the aberration and transmission coefficients of the optical-mechanical path; k – deterministic attenuation coefficient of the optical link.

To implement the flow $N_{c2}(t)$ and the average value $\bar{N}_{c2}$, the following relations are valid:

$$N_{c2}(t) \neq \frac{N_0(t)\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}}, \quad (8)$$

$$\bar{N}_{c2} = \frac{\bar{N}_0\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}}. \quad (9)$$

A flux of light quanta is incident on the light-sensitive surface of an elementary photodetector with an area S_d , which is converted by a photodetector with a quantum efficiency S_λ into an electron flux $\xi_{c3}(t)$, which is equal to:

$$\xi_{c3}(t) = \xi_{c2}(t)S_\lambda S_d. \quad (10)$$

Using expression (7), let's obtain:

$$\xi_{c3}(t) = \frac{\xi_0(t)\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}} S_\lambda S_d. \quad (11)$$

Similarly to (6) and (9), to implement the flow of charges into the photodetectors, let's write:

$$N_{c3}(t) \neq \frac{N_0(t)\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}} S_\lambda S_d, \quad (12)$$

for the average value of the charge flux, let's write

$$\bar{N}_{c3} = \frac{\bar{N}_0\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}} S_\lambda S_d. \quad (13)$$

Since the photodetector is characterized by the accumulation time T_a , equal to the frame formation time T_k , the registered number of photo carriers $N_{c4}(nT_k)$ can be determined:

$$N_{c4}(nT_k) = \sum_{(n-1)T_k}^{nT_k} N_{c3}(t), \quad (14)$$

where the symbol $\sum_{(n-1)T_k}^{nT_k}$ means the summation of the stream of pulses on the interval T_k ; n – frame number.

Thus, the expression for the average value of the output stream $\bar{\xi}_{c_{out}}$ can be written:

$$\bar{\xi}_{c_{out}} = \bar{N}_{c_{out}} = \frac{P_{rand}}{h\nu} \frac{\tau_\Sigma}{k \cdot \tau_{omp} \cdot k_{ab}} S_\lambda S_d T_a. \quad (15)$$

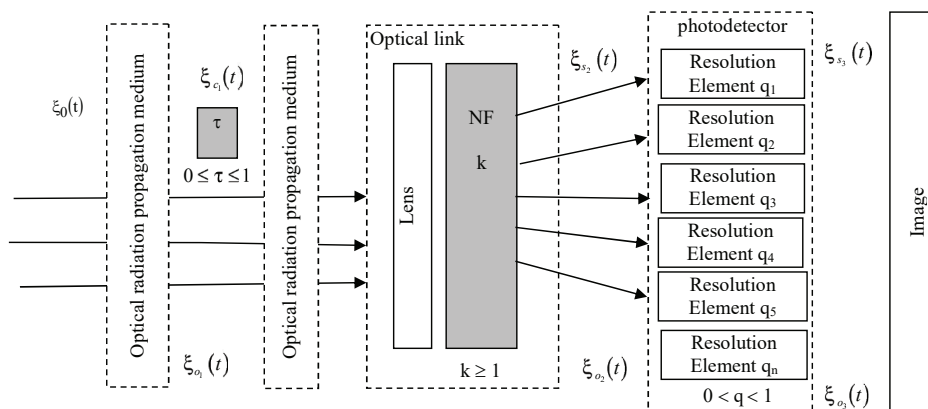


Fig. 1. Diagram of signal formation in optoelectronic systems

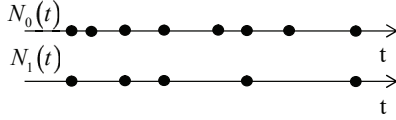


Fig. 2. Realizations of random streams:
 a – photon flux as a set of points on the time axis;
 b – photon flux after interaction with
 the propagation medium

When forming a stream, it does not interact with the object of observation, the expression for determining its average value, similar to the previous presentation, can be written:

$$\bar{\xi}_{p_{out}} = \bar{N}_{p_{out}} = \frac{P_{rand}}{h\nu} \frac{\tau_{ob}}{k \cdot \tau_{omp} \cdot k_{ab}} S_k S_d T_a. \quad (16)$$

When forming a response of the photodetector when observing transparent objects, the intensity formed by signal photons in the resolution elements of the photodetector will be less than the intensity formed by the background photons. Accordingly, the average value $\bar{N}_{res}$ of the result flow can be determined:

$$\bar{\xi}_{res} = \bar{N}_{res} = \bar{N}_{p_{out}} - \bar{N}_{c_{out}} = \bar{N}_0 \frac{S_k S_d}{k \cdot \tau_{omp} \cdot k_{ab}} T_k (1 - \tau_{ob}). \quad (17)$$

Let's find an expression for the distribution law of the random variable $\xi_{res}(nT_k)$. The random variable $\xi_{res}(nT_k)$ is the difference between the random variables $\xi_{out}(nT_k)$ and each of which is distributed according to Poisson's law. The distribution law for the difference of independent random variables having a Poisson distribution with parameters λ_1 and λ_2 is determined by the expression [4]:

$$p(\eta = n) = \begin{cases} e^{-(\lambda_1 + \lambda_2)} I_0(2\sqrt{\lambda_1 \lambda_2}), & \text{at } n = 0, \\ e^{-(\lambda_1 + \lambda_2)} \left[\left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{n}{2}} + \left(\frac{\lambda_2}{\lambda_1} \right)^{\frac{n}{2}} \right] \times \\ \times I_n(2\sqrt{\lambda_1 \lambda_2}), & \text{at } n = 1, 2, \dots, \end{cases} \quad (18)$$

where $I_n(2\sqrt{\lambda_1 \lambda_2})$ – the n -th order Bessel function.

Let's note that in this case

$$\lambda_1 = \bar{\xi}_{s_{out}} = \bar{N}_{s_{out}}$$

and

$$\lambda_2 = \bar{\xi}_{p_{out}} = \bar{N}_{p_{out}},$$

and

$$\eta = \xi_{res} = N_{res}.$$

In contrast to the Poisson distribution, the envelope of which is asymmetric, especially for small values of λ , the envelope of the probability distribution of the difference of Poisson fluxes is directed towards symmetric for large λ_1 and λ_2 [10].

4. 2. Description of the process of forming the output signal of the optoelectronic system based on the wave theory

Receiving signals in optoelectronic systems are analyzed from the standpoint of the wave theory based on the system of Maxwell's differential equations. The signals describe the strength of the electric and magnetic fields, electric and magnetic induction and the density of the electric charge. The system of Maxwell's equations also includes material equations that characterize the behavior of various media in an electromagnetic field. Taking into account the material equations and boundary conditions, the system of Maxwell's equations is complete and allows one to study all the properties of the electromagnetic field and many processes of the interaction of the field with substances. The wave theory allows to thoroughly explain the processes of interference and diffraction of light.

Optical radiation from the object enters the input of the optical system. When passing through the system, light is diffracted by the apertures of the optical elements. Let's consider the phenomenon of diffraction for linearly polarized light provided that the dimensions of the obstacle are much larger than the wavelength of the radiation used.

At the present stage, not only plane and spherical waves, but also modes of a laser cavity are used more and more often as a light source. The hole shape can be arbitrary, as well as the phase distribution. The set of microsources of the initial field in the plane of sources $X'Y'$ is described by the distribution of the amplitude $A_0(x', y')$ and the phase $\Phi_0(x', y')$. The interference from all microsources forms at each point on the observation plane XY at a distance z the value of the complex amplitude [12], which can be written using the Kirchhoff integral:

$$A(x, y, z) \sim \iint A_0(x', y') \frac{\exp(-i(kr + \Phi_0(x', y'))}{r} dx' dy', \quad (19)$$

where r – the radius vector,

$$r = \sqrt{(x - x')^2 + (y - y')^2 + z^2},$$

k – the wavenumber. The resulting complex amplitude can be represented by the expression

$$A(x, y, z) = U(x, y, z) + iV(x, y, z),$$

of which the modulus of the amplitude

$$|A(x, y, z)| = \sqrt{U(x, y, z)^2 + V(x, y, z)^2}$$

can be determined. When registering a signal, the intensity is always measured, which is proportional to the square of the modulus of the complex amplitude $I(x, y, z) \approx |A(x, y, z)|^2$.

The Kirchhoff integral is calculated using high precision numerical integration methods. If there is a need to determine the properties of a topological object of a light field in a region with dimensions of several wavelengths, then its analytical solution gives an unambiguous answer [24].

For the well-known problem of diffraction of a plane wave at a round hole of radius a , the Fresnel region is determined by the distance measured from the hole to $z < a^2/\lambda$. In this region, with a change in the observation distance, the diffraction pattern also changes significantly. The next

Fraunhofer region $z \geq \frac{a^2}{\lambda}$ is qualitatively different in that the diffraction pattern in it is similar to the flow of the entire region to infinity.

Depending on the distance at which the diffraction pattern is studied, formula (19) can be modified. The first simplification can be made on the assumption of small angles, which is mathematically formulated in the form of irregularities $|x-x'|/z \ll 1$, $|y-y'|/z \ll 1$ and obtain the expression:

$$A_0(x', y') \times A(x, y, z) \sim \iint \exp \left[-i \left(\frac{k(x-x')^2 + k(y-y')^2}{2z} + \Phi_0(x', y') \right) \right] dx' dy'. \quad (20)$$

Expression (20) is called the Fresnel integral and is used in cases where the observation distance is much larger than the image size. The following simplification of the Fresnel integral is possible if to observe an object at infinity or in the focal plane of the lens, then $z \rightarrow \infty$. Such an expression is called the Fraunhofer integral and has the form:

$$A_0(x', y') \times A(x, y, z) \sim \iint \exp \left[-i \left((kxx' + kyy') + \Phi_0(x', y') \right) / z \right] dx' dy'. \quad (21)$$

The use of a specific record for calculating the diffraction field (19)–(21) is determined by the goal and objectives of the study.

As an example, a fragment of the intensity of the calculated diffraction field (Fig. 3, *a*) and obtained experimentally in the optical laboratory (Fig. 3, *b*) for the problem of plane wave propagation through a double phase wedge from [25] is shown. The experimental image contains noises from the camera and unwanted horizontal fringes, as a manifestation of the flaws of the optical system. Their spatial parameters can be determined from the data of the intensity distribution.

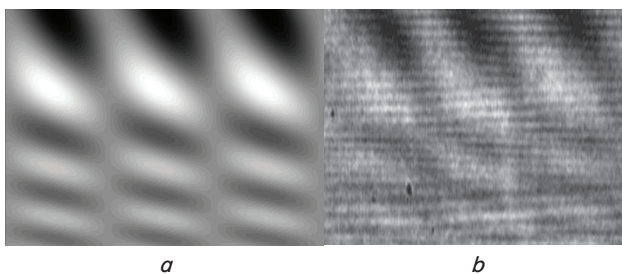


Fig. 3. Distribution of intensity: *a* – calculated; *b* – experimental

In the simplest case, the stochastic intensity distribution can be observed in an unperturbed laser beam immediately behind the laser exit window in the form of a speckle field structure. It occurs due to the interference of waves that are reflected from a rough surface in the laser cavity. The mathematical description of this phenomenon can be written in the form of adding a random phase function in the field $\Phi_{rnd}(x', y')$ to the wave front of the beam:

$$\Phi(x', y') \sim kxx' + kyy' + \Phi_0(x', y') + \Phi_{rnd}(x', y'). \quad (22)$$

Formula (22) can also be used to describe the passage of a light beam through an inhomogeneous medium, for example, a matte plate, and also to calculate fluctuations in the intensity of a beam that has passed through a turbulent medium. In these cases, the influence of the medium on the phase is determined through the function of fluctuations of the refractive index. The speckle field is used in photoacoustic tomography [26] and for measuring the elastic and thermal properties of composite materials [27].

Let's note that each type of laser can exhibit its own unique intensity fluctuations due to its design. For example, a change in the spatial intensity of the beam with time may indicate an unstable laser operation, sometimes observed in the first minutes of its operation. For solid-state lasers, such intensity fluctuations are possible due to fluctuations in the pump wave. Fluctuations of the pump wave are transferred to the lasing wave and cause multi-frequency lasing. The description of the space-time distribution of the intensity is possible using the integral:

$$A(x, y, t, z) = \int_{-\infty}^{\infty} G(x, y, v, z) \exp(i2\pi vt) dv, \quad (23)$$

where v – the frequency of the wave, the function $G(x, y, v, z)$ is determined using the inverse Fourier transform:

$$G(x, y, v, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(f_x, f_y, v, z) \times \exp(2\pi i(xf_x + yf_y)) df_x df_y. \quad (24)$$

Information on the definition of a function $g(f_x, f_y, v, z)$ can be found in [28].

5. Statistical characteristics of signals at the output of the photodetector

Signal characteristics at constant luminous flux intensity. If the intensity of the light signal does not fluctuate in time $I(t) = I_0$, then the probability of the formation of photoelectrons in the interval τ depends only on its duration and does not depend on the location of the interval on the time axis. Such a flux of photoelectrons can be considered stationary. The following moments of photoelectron formation are independent of the previous ones (no aftereffect). In this case, the distribution of the number of electrons n over an interval of any duration is described by Poisson's law [14, 15, 23]:

$$P(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \quad (25)$$

where $P(n)$ – the probability of emission of n photoelectrons in the interval τ .

The mathematical expectation of the number of photoelectrons in the interval

$$M(n) = \sum_{n=0}^{\infty} n P(n) = \bar{n}.$$

Variance of the number of photoelectron ejections

$$\sigma^2 = \langle (n - \bar{n})^2 \rangle = \sum_{n=0}^{\infty} (n - \bar{n})^2 P(n) = \bar{n}.$$

Variance and mathematical expectation are equal to the average number of photoelectrons over the observation interval. In addition to stationarity and the absence of after-effect, the Poisson flow is characterized by ordinarity (the passage of photoelectrons alone, and not in pairs or triplets).

An important property of Poisson's law is that the sum of N processes with Poisson divisions is also a Poisson process with an average value equal to the sum of the average values of N processes [29, 30].

For $n \gg 10$, the Poisson distribution is well approximated by the normal distribution law [22, 23]:

$$P(n) = \frac{1}{\sqrt{2\pi n}} e^{-\frac{(n-\bar{n})^2}{2n}}. \quad (26)$$

Signal characteristics with random fluctuations of the luminous flux intensity. Let's consider the case when $\mu(t)$ is a random function, observed with fluctuations of the light flux that hits the input of the photodetector. In this case, the probability of the appearance of photoelectrons in a small observation (exposure) interval τ will depend on the value of $\mu(t)$, that is, it will change in time in a random manner.

For a more convenient description, let's introduce a normalized random function $\xi(t)$ such that $\mu(t) = \xi(t)\bar{\mu}$, where:

$$\bar{\mu} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mu(t) dt,$$

$$\bar{\xi} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \xi(t) dt = 1.$$

The random function $\xi(t)$ can be characterized by the correlation interval τ_c . The statistical characteristics of the signal at the output of the photodetector depend on the ratio of the intervals τ and τ_c .

If the observation time (implementation duration) $\tau \ll \tau_c$, then on this interval the value of $\xi(t)$ can be considered constant and independent of time. The average number of photoelectrons in the observation interval τ will be $n_1 = \xi \bar{n}$, where $\bar{n} = \bar{\mu}\tau$.

The distribution of photoelectrons in the interval at a fixed value of ξ corresponds to the Poisson distribution:

$$P(n/\xi) = \frac{(\xi \bar{n})^n}{n!} e^{-\xi \bar{n}}. \quad (27)$$

In order to find the distribution $P(n)$, it is necessary to average the joint distribution of n and $\xi P(n, \xi)$ over ξ . Wherein:

$$P(n, \xi) = P(n/\xi)P(\xi).$$

The response of the signal reflected from the object and the background at the receiver input have normal fluctuations. This is due to the fact that the signal from the object is created as a result of the interference of statistically independent fields reflected from a large number of "shiny points" of the scattering surface. Background radiation is also generated by a large number of randomly placed scatterers (particles). Under normal fluctuations of the intensity, the instantaneous intensity of the signal, and, consequently, the intensity of the photoelectron flux, which are characterized by the value of ξ , fluctuates according to the exponential law:

$$P(\xi) = e^{-\xi}, \quad (28)$$

$$P(n) = \int_0^\infty P(n/\xi)P(\xi)d\xi = \int_0^\infty \frac{(\xi \bar{n})^n}{n!} e^{-\xi \bar{n}} e^{-\xi} d\xi = \frac{1}{n+1} \left(\frac{\bar{n}}{n+1} \right)^n. \quad (29)$$

This distribution is called geometric, or Bose-Einstein distribution. The mathematical expectation of this distribution $M(n) = \bar{n}$, and the variance:

$$\sigma^2 = \bar{n}(1 + \bar{n}). \quad (30)$$

Another practically important case occurs when the observation time $\tau \gg \tau_c$. The value of the electron flux intensity $\mu(t)$ at time intervals $\Delta t \gg \tau_c$ is statistically independent. If to divide the observation time τ into $m = \frac{\tau}{\tau_c}$ intervals then on each interval there is a distribution $P(n)$, which obeys the Bose-Einstein law with an average value $\bar{n}_m = \bar{\mu}\tau_c = \bar{n}/m$. Statistically independent quantities are added to the interval τ , each of which has a Bose-Einstein distribution. In this case, the probability of the appearance of n carriers is described by a negative binomial distribution:

$$P(n) = \frac{(m+n-1)!}{(m-1)!n!} \left(\frac{\bar{n}_m}{\bar{n}_m+1} \right)^n \cdot \left(\frac{1}{\bar{n}_m+1} \right)^m. \quad (31)$$

The mathematical expectation of this distribution is equal to the average number of photoelectrons emitted over the interval τ :

$$M(n) = \bar{n} = \bar{n}_m \cdot m,$$

and the variance

$$\sigma^2 = \bar{n} \left(1 + \frac{\bar{n}}{m} \right). \quad (32)$$

Thus, the statistical representations of the output signals have different characteristics. According to the description methods, the output signals based on the wave theory are approximated by the Gauss distribution. When using the corpuscular properties of the optical field, Poisson, Bose-Einstein and exponential statistics are used.

6. Determination of the efficiency of optoelectronic systems

Traditional methods of signal detection in optoelectronic systems based on the signal-to-noise ratio criterion are based on signal threshold processing. That is, by comparing the magnitude of the response of photosensitive elements to the effect of an additive mixture of the signal and background components with a set limit value, the value of which is due to the selected criterion of the quality of decision making. The decision to detect a signal is made when the amplitude of the electrical signal generated by one difference element of the photodetector matrix

exceeds the set value of the threshold [11, 14, 19]. Let's determine the efficiency of optoelectronic systems based on the signal-to-noise ratio criterion based on the corpuscular and wave description of signals.

When using the corpuscular model for describing the output signals of optoelectronic systems, it can be concluded that when $\bar{n}_p \gg \bar{n}_s$ optical signals are detected, the signal-to-noise ratio at the receiver output can also be entered as a detection parameter in the form:

$$\phi = \bar{n}_s / \sqrt{\bar{n}_p + \bar{n}_s}. \quad (33)$$

When using the wave model for describing the output signals of optoelectronic systems, expression (33) coincides with the expression for the signal-to-noise ratio at $x(t) \sim \mu_c(t)$ with completely known parameters; it is observed in white noise with a spectral density $N_0/2 \sim \mu_n$:

$$q = \sqrt{2E / N_0}, \quad (34)$$

where $E = \int_0^{\tau} x^2(t) dt$ – energy of the signal.

Then for the signal at $x(t) \sim \mu_s(t) = \mu_s$, $N_0/2 \sim \mu_p$:

$$q = \sqrt{\mu_s^2 \tau / \mu_p} = \mu_s \tau / \sqrt{\mu_p \tau} = \bar{n}_s / \sqrt{\bar{n}_p}.$$

The possibility of using the value of the signal-to-noise ratio ϕ as a parameter for detecting optical signals only under the condition $\bar{n}_p \gg \bar{n}_s$ is illustrated by the graph in Fig. 4.

A typical dependence of the ratio $\bar{n}_s / \sqrt{\bar{n}_p + \bar{n}_s}$ on $\bar{n}_p$ at fixed probabilities of correct detection D and the probability of false detection F . As follows from the analysis of Fig. 4, already at the value $\bar{n}_p > 100$ of the ratio stabilizes, and, therefore, uniquely characterizes the value of D and F . When this condition is met to determine D and F , one can calculate the parameter ϕ and use the classical relations for the characteristics of detecting a signal with known parameters in white noise. In practice, the condition $\bar{n}_p > 100$ is always met if photodetectors with an internal photoelectric effect are used (photodiodes, photoresistors, etc.).

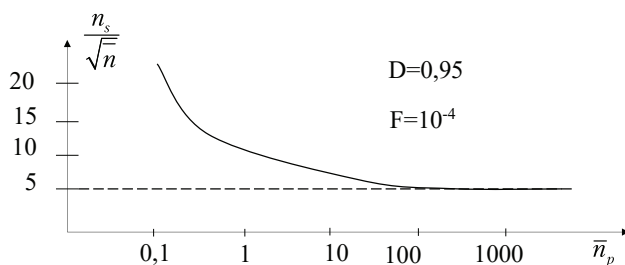


Fig. 4. Dependence of the value of the signal-to-noise ratio on the noise component

The physical meaning of the transition to the classical case is that when $\bar{n}_p > \bar{n}_s$ the impulses that the interference forms at the exit merge with each other. The implementation of the obstacle takes the form of a characteristic noise track. In this case, for reliable detection of the signal, it is sufficient to ensure its excess over the interference fluctuations, and not over its constant component.

7. Discussion of the results on the use of signal description models in the calculation of system efficiency indicators

Based on the description of the process of formation of signals from optoelectronic systems, the influence of external and internal factors on the output signals was considered. The following is indicated.

When applying the model, which is based on the wave theory of light, taking into account the wave properties of optical radiation allows one to take into account the statistical characteristics of random changes in the complex amplitude of the field by adding a random phase function to the wavefront, is the result of wave interference from all microsources.

When applying a model based on the corpuscular theory of light, taking into account the corpuscular properties of optical fields makes it possible to take into account changes in the statistical characteristics of signals that occur when the energy of optical signals decreases. In this situation, the corpuscular structure of the signals plays an essential role. The description of optical fields by continuous functions, which is characteristic of the wave theory and the use of continuous probability distributions in the description of the statistical characteristics of signals can become a source of errors in estimates of the performance indicators of systems.

In the engineering practice of modern optoelectronic systems based on optimal detection methods, both for direct (incoherent) and heterodyne (coherent) detection, there are statistical properties of real signals and noise at the output of a photodetector. Changes in the variance of the output stream, expressions (30) and (32), play a special role in detecting low energy signals.

The proposed description of the statistical features of the initial signals showed that when the energy properties of the signal and noise components change, the distribution laws of the output signals, expressions (26), (27), (29), change. The statistical distribution of signals at the output of the photodetector at a constant intensity is described by Poisson's law (26). When random fluctuations of the signal component are taken into account using the corpuscular model, the variance in the output signal, expression (32), can increase, which in turn can affect the accuracy of evaluating the performance indicators of systems.

The analysis of the efficiency of systems using various methods for describing optical fields showed the possibility of using the criterion of the signal-to-noise ratio at fixed characteristics of signal detection (Fig. 4). However, a comparative analysis of the obtained expressions showed that when using the corpuscular description, expression (33), in contrast to the wave description, expression (34), the signal-to-noise ratio has a finite value, even if there is a decrease or in the complete absence of interference. The final value of the value of the signal-to-noise ratio is due to taking into account, in the corpuscular description, the intrinsic noise of the signal component, which is a consequence of its corpuscular (discrete in time) structure.

Between the application of corpuscular and wave models, it is possible to determine the value of the energy of the signal and noise components. When receiving "strong" signals, the energy of which is sufficient to manifest the wave structure of optical radiation, it is possible to use the wave description of the signals without significant errors. When describing "weak" signals, for which the corpuscular structure is out-

standing, it is necessary to use the corpuscular model. This approach makes it possible to more accurately assess the performance indicators of systems taking into account changes in the statistical properties of optical signals. Taking into account the operating conditions of optoelectronic systems requires additional calculations of the parameters of signals and interference in each specific case.

Promising for further research are areas related to the development of highly efficient algorithms for detecting signals in a wide energy range, taking into account changes in the statistical features of the output signals of optoelectronic systems.

8. Conclusions

1. Input signals can be described in terms of wave and corpuscular models, since optical signals exhibit both the properties of a wave and a corpuscle. The wave theory allows

one to describe the processes of diffraction and interference. Since the energy of a quantum in the optical range is large enough, the interaction of the field with matter can also be described using the corpuscular theory, taking into account absorption processes.

2. Analysis of methods for describing output signals taking into account statistical properties showed that when describing output signals of optoelectronic systems that register signals with different properties, Poisson and Gaussian distributions are used. Signals with constant flow intensity have all the properties of the Poisson law. The invariance of Poisson flows provides a description of an additive mixture of signal and background flows using Poisson flow.

3. Calculations of the efficiency of optoelectronic systems according to the criterion of the signal-to-noise ratio based on the corpuscular and wave description of signals have shown the feasibility of its application under the condition $\bar{n}_p > 100$ and stabilization of the statistical properties of signal to background flows.

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