

**MODELING ECG USING POLYGONAL FUNCTION METHOD**

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Spectral analysis usually remains one of the most common analytical approaches to determine diagnostically significant characteristics of electrical activity signals of organs and systems of the body [1, 2]. Regarding the cardiovascular system, the electrocardiogram (ECG) is an integral indicator of the system state and is widely used in diagnostics and research. The paper shows the results of using a polygonal function (PF) as an ECG signal model, which allows simplifying the calculation of the signal spectrum and further revealing the possibilities of correlation analysis.

A polygonal function is a special case of piecewise-analytical functions consisting of a linear combination of linear functions [3]

$$f_k(t) = a_k \cdot t + b_k, \quad (1)$$

where  $a_k, b_k$  – real constants. The graph of the polygonal function is a family of  $N$  straight-line segments inclined to the abscissa at an angle determined by  $a_k$ . At the junction points of the segments  $t_k$ , the function  $f(t)$  undergoes a first-order discontinuity. To calculate the transform of the polygonal function, we introduce the notation for function jumps at these points [3]

$$\Delta f(t_k) = (a_k \cdot t_k + b_k) - (a_{k-1} \cdot t_k + b_{k-1}) = h_k, \quad (2)$$

$$\Delta f'(t_k) = a_k - a_{k-1} = d_k. \quad (3)$$

It follows that  $h_1 = a_1 \cdot t_1 + b_1$ , since for  $t < \tau_0$   $f(t) = 0$ . Thus,  $f(\tau_0) = b_1$ . At the endpoint of the function definition

$$h_{m+1} = -(a_m \cdot t_{m+1} + b_m), \quad (4)$$

$$d_{m+1} = -a_m.$$

These parameters are further used in the calculation of spectral density. For modeling a synthetic (artificial) ECG signal, in this case, materials [4] are used. The paper proposes a dynamic model that allows generating synthetic ECG signals close to real ones. In addition, differential equations are used to model the characteristic features of the ECG signal (P wave, QRS complex, and T wave). This model is suitable for testing signal processing algorithms since it allows obtaining controlled variations of the ECG. The developed model can be used to analyze pathological conditions such as arrhythmias. The Python programming

language is used for differential equation calculations. The modeling result is shown in Figure 1.

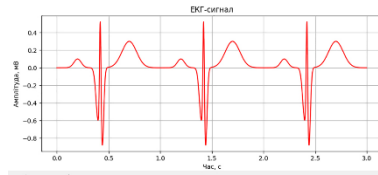


Figure 1 – Visualization of the synthetic (artificial) ECG signal

To represent the synthetic ECG signal using a polygonal rectangular function (PRF), signal value quantization is used, described by the equations from [3], as well as segmentation of the signal into constant-level segments (the signal level remains unchanged on each segment). The Python programming language is used for equation calculations and rectangular function generation. The modeling result is shown in Figure 2(a, b).

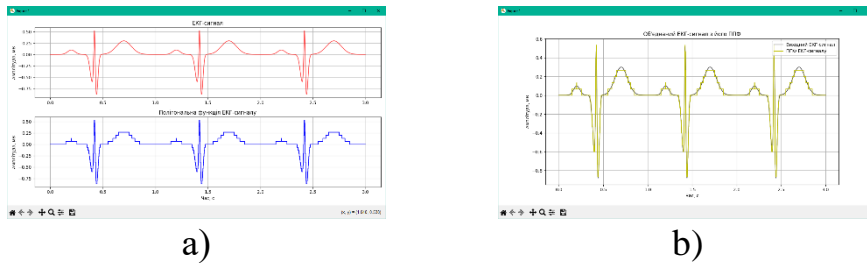


Figure 2 – Visualization of the polygonal rectangular function of the ECG signal

To represent the derivative of the polygonal rectangular function of the ECG signal, a difference ratio between neighboring signal values is used. In Python, this is implemented using  $np.diff(qs[0])$ , multiplied by the sampling frequency  $f_s$  to obtain correct measurement units. The derivative of the polygonal rectangular function consists of negative and positive pulses occurring at level jumps. The amplitude of these pulses is proportional to the difference in signal levels. The modeling result is shown in Figure 3(a, b).

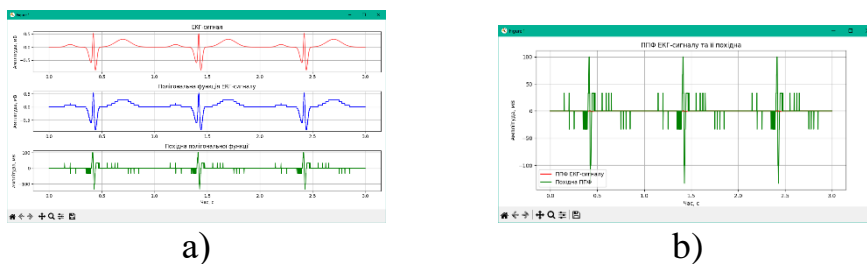


Figure 3 – Visualization of the derivative PRF of the ECG signal

After obtaining the derivative of the polygonal rectangular function, it is necessary to calculate the spectral density of the ECG signal. To obtain the spectral density, [3] is used. The spectral density of the distorted pulse decreases, meaning that linear distortions are accompanied by a narrowing of the signal spectrum. The quantitative comparison of spectrum construction equals 0.85. The

spectral function has a pedal-like character, which is a property of finite video pulses. In Python, the spectral density of the ECG signal is calculated using `np.fft.fft`, which performs a Fast Fourier Transform (FFT), converting the time-domain signal `ecg_signal` into the frequency-domain representation, `np.fft.fftfreq(N, d=1/fs)`, which returns an array of frequencies corresponding to the Fourier transform components ( $f^s$  – sampling frequency (500 Hz),  $d=1/fs$  – time step), `np.fft.fftshift()`, which centers the zero frequency in the middle of the graph, and `np.abs(spectrum)`, which takes the modulus of spectral coefficients corresponding to the spectrum amplitude. The calculation results are shown in Figure 4(a, b).

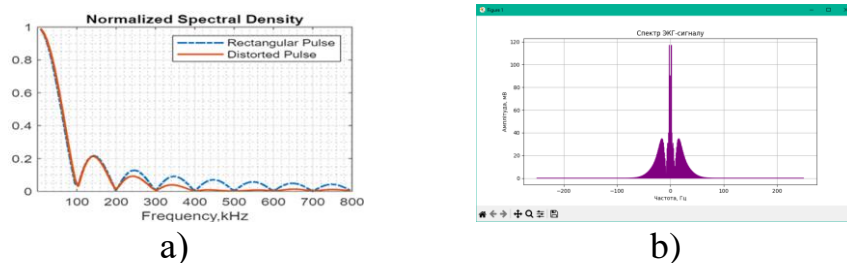


Figure 4 – Calculation results: a) – signal spectrum according to [3],  
b) – signal spectrum calculated using Python

The paper demonstrates the feasibility of using a polygonal function in modeling the bioelectrical activity signal of the heart of the ECG type. The model calculation is performed, and the spectrum calculation result is presented.

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