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MATHEMATICS BEHIND SIGNAL ANALYSIS

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This work is devoted to investigation of how does Fourier Transform works and what purpose of it. Also was mentioned two types of transformation, Discrete Fourier Transform and Fast Fourier Transform and their specific uses. Finally, some of the most common applications of the Fourier Transform in fields such as signal processing, image processing, communication systems, and quantum mechanics will be discussed. The Fourier Transform is a fundamental mathematical tool that has revolutionized the way we analyze and understand signals, and its applications continue to expand to new fields and technologies so it is important to understand the way it works and what benefits it might bring.

The Fourier Transform is a mathematical technique that converts a time-domain signal into a frequency-domain representation which means, it decomposes a complex signal into its constituent frequencies, allowing to analyze and understand its behavior more easily.

To understand how the Fourier Transform works, let us consider a simple example. Suppose we have a signal that represents the sound produced by a musical instrument. When we plot this signal on a time-domain graph, we can see how the sound wave varies over time. However, this representation does not provide us with any information about the frequencies that make up the sound.

The Fourier Transform solves this problem by breaking down the signal into a series of sinusoidal waves of varying frequencies. These waves are then represented on a frequency-domain graph, where we can see the amplitude of each frequency component. This representation provides us with a much clearer understanding of the underlying signal.

The Fourier Transform is based on a complex mathematical formula known as the Fourier Transform equation. The equation defines how a time-domain signal can be transformed into a frequency-domain representation [1]. The equation is as follows:

$$F(\omega) = \int \quad f(t)e^{(-i\omega t)}dt$$

where F(ω) is the Fourier Transform of the signal, f(t) is the time-domain signal, ω is the frequency, and e is the mathematical constant Euler's number.

In simpler terms, the equation states that the Fourier Transform of a signal is the sum of all the frequencies in the signal, weighted by their respective amplitudes.

There are two types of Fourier Transforms - the Discrete Fourier Transform (DFT) and the Fast Fourier Transform (FFT). The DFT is used to transform discrete signals, while the FFT is used to transform continuous signals. The FFT is a more efficient version of the DFT and is widely used in digital signal processing for example in digital microphones.

The Fourier Transform has numerous applications in various fields. Some of its most common applications include:

Signal processing: The Fourier Transform is used extensively in signal processing to analyze and filter signals. For example, FFT can be used to analyze sound waves, filter out unwanted noise, and compress digital audio and video signals.

Image processing: FFT is also used in image processing to analyze the frequency domain characteristics of an image. This can be useful for image enhancement, filtering, and compression.

Communication systems: The Fourier Transform is used to encode and decode signals in communication systems.

Quantum mechanics: The Fourier Transform is used to solve quantum mechanics equations and analyze quantum systems.

Looking towards the future, signal processing is expected to continue advancing rapidly with the help of technologies like artificial intelligence and machine learning. These technologies can be used to develop more sophisticated algorithms and models for analyzing and processing signals, leading to improved performance and accuracy in various applications. Additionally, the increasing use of sensors and Internet of Things (IoT) devices is expected to generate vast amounts of data, which will require more efficient and automated signal processing techniques. As such, the Fourier Transform and other signal processing tools are likely to remain crucial in advancing technology and improving our understanding of the world around us.

List of used sources:

1. Katznelson, Yitzhak (1976). An Introduction to Harmonic Analysis. Dover.