Changing of an Airy Pulse Form due to Re-Reflections Inside a Dielectric Layer

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***Abstract*— A problem of Airy pulse transformation by a plane-parallel layer is considered. A problem formulation is equivalent to a case of permittivity sharp change in a dielectric layer at zero moment of time. It allows to use the resolvent method for solution of the Volterra integral equation describing this phenomenon. It is shown a virtual structure of a pulse field inside the layer. Influence of a number of pulse re-reflections from the layer boundaries on a form of a primary pulse is analyzed.**

Keywords*— *Airy pulse; layer; re-realection, electromagnetic field

I. Introduction

Intensive theoretical and experimental investigations of Airy beams are motivated by their unusual features (non-diffractive propagation, accelerating motion, and self-healing). A solution to the Schrodinger equation in the form of a non-spreading accelerating Airy wave function found by Berry and Balazs in 1979 [1] inspired Siviloglou and Christodoulides to put forward the concept of electromagnetic accelerated Airy beams [2, 3]. These seminal publications with theoretical formulations based on the paraxial approximation of the wave equation and experimental confirmation were followed by many works on the Airy beam properties, for example [4]. Nonparaxial Airy beams are studied in less degree than paraxial ones [5]. One important feature of the Airy pulse in paraxial approximation is absence of delay when any disturbance propagates with infinity velocity and there are not cause-and-effect relations. To take in account a finite velocity of disturbance propagation one has to consider an exact, nonparaxial, description of a phenomenon. Here, apart from such a description, we use another special feature of the Airy pulse, its asymmetry. Asymmetrical form of the Airy pulse is characterized by an exponentially fading front and oscillating "tail", decaying at infinity. It allows investigating the inhomogeneous medium interaction process starting at a fixed moment of time and to connect a boundary value problem and an initial one. In the given paper the optimal approach for the solving problem is based on the Volterra integral equations method [6], which is capable of physical clarity and descriptive completeness. The latter consists of, besides the other pros, the automatic involving of both the initial and the boundary conditions and the versatility regarding the free term of the equation (the initial pulse). We consider an electromagnetic Airy pulse $E_0(t,x)$ as an initial one that falls normally on a plane dielectric layer $0<x<a$ with the permittivity ε_1.

II. Solution of a Problem

A. The integral equation method

The Volterra integral equation describing the evolution of the electromagnetic field inside and outside the layer has the form

$$E(t,x)=E_0(t,x)-W\frac{\partial^2}{\partial t^2}\int_0^{\infty}dt'\int_0^{a}dx'\theta\left(t-t'-\frac{|x-x'|}{v}\right)E(t',x') \quad (1)$$

where $W=\dfrac{(1-v^2/v_1^2)v_1}{2v^2}$, $v=c/\sqrt{\varepsilon}$ is a wave velocity in the medium with the permittivity ε, $v_1=c/\sqrt{\varepsilon_1}$, c is the light velocity in vacuum and $\theta(t,x)$ - is the Heaviside unit step-function. The free term corresponds to a primary pulse which has the form of the Airy function

$$E_0(t,x)=\mathrm{Ai}(-t+\frac{x}{vT})=\frac{T}{2\pi}\int_{-\infty}^{+\infty}e^{i\frac{(\omega T)^3}{3}-i\omega t+i\omega x/vT}d\omega \quad (2)$$

where T is a temporal scale. It is very important that the Airy pulse begins to interact with the layer at zero moment of time. Its behavior and spatial distribution is shown in Fig.1.

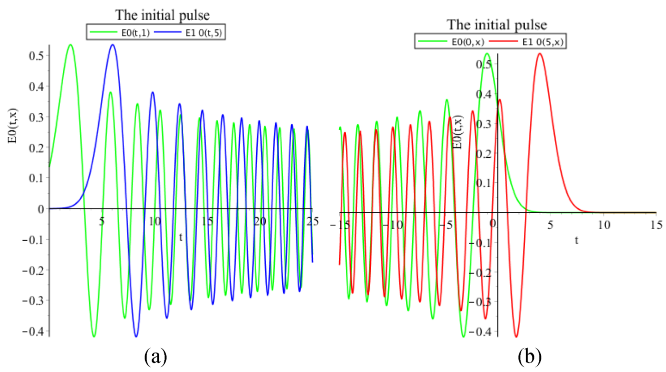

Fig. 1. The temporal behaviour of the initial Airy pulse at various points: (a) x/vT=1 (green) and x/vT=5 (blue). (b) The positions of the initial Airy pulse at various moments: t/T=0 (green) and t/T=5 (red). The layer thickness is supposed to be equal to L/vT=10.

B. The calculation of the field inside the layer

The solution to equation (1) is the field inside the layer and it can be found by the means of the resolvent [6]:

$$\hat{R} = \chi(x)(\hat{R}_1 + \hat{R}_{mn})\chi(x') \tag{3}$$

The characteristic functions of this region determined as $\chi(x) = \theta(x)\theta(a-x)$ is equal to one inside the layer and to zero outside it.

The electric field inside the layer is calculated by virtue of the resolvent (3) and is expressed by the formula

$$\begin{aligned} E(t,x) &= E_0(t,x) + \\ &+ W\chi(x)\frac{\partial^2}{\partial t^2}\int_0^\infty dt'\int_0^a dx'\theta\left(t - t' - \frac{|x-x'|}{v_1}\right)E_0(t',x') + \\ &+ W\chi(x)\frac{\partial^2}{\partial t^2}\int_0^\infty dt'\int_0^a dx'\left(\sum_{k=1}^{m} R^k\theta\left[t-t'-s_k^{(+)}(x,x')\right]E_0(t',x') + \right. \\ &\left. + \sum_{k=1}^{n} R^k\theta\left[t-t'+s_k^{(-)}(x,x')\right]\right) = E_0 + E_{00} + E_{mn} \end{aligned} \tag{4}$$

Here $s_k^{(\pm)}(x,x') = \frac{1}{2v_1}\{2x + (-1)^k\left(a - 2x'\right) \pm (2k \mp 1)a\}$.

The resolvent in a zone that is independent on the layer boundary influence is the same as in the case of an unbounded medium and is given by the term $E_{00}(t,x)$. Outside this zone the resolvent is constructed with taking into account re-reflections of the signal from the layer boundaries as it is shown in Fig.2. Result of this re-reflections is division of the whole stripe determined by the layer characteristic function $\chi(x)$ on zones labelled with two indexes m, n that is shown in Fig. 2. The first index of the zone number indicates how many times the pulse has been reflected from the lower boundary of the layer and the second one is related with the upper boundary. Weight contribution of every reflection of the pulse is equal to $R = (v - v_1)/(v + v_1)$.

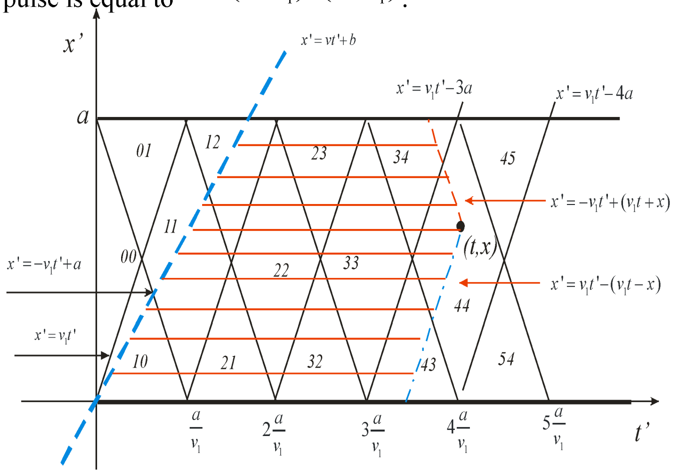

Fig. 2. Time-spatial zones corresponding to re-reflections of the pulse from each boundary of the layer after beginning of the pulse layer interaction.

For distinctness let the observation point (t,x) be located in the zone 44, Fig.2. The initial pulse is located in the region bounded by the time-spatial trajectory shown in Fig.2 by the dashed blue line. A zone that is independent on the layer boundary influence is determined by the inequality $t - t' - |x - x'|/v_1 > 0$ and is shown in Fig. 2 by the red hatching.

Using the Fourier representation of the Airy pulse (2) the main component calculated by the formula (4) is given by the expression

$$\begin{aligned} E_{00}(t,x) &= \frac{v_1 + v}{2v}\mathrm{Ai}\left(-\frac{t}{T} + \frac{x}{v_1 T}\right) + \\ &\frac{v_1 - v}{2v}\mathrm{Ai}\left(-\frac{t}{T} - \frac{x}{v_1 T} + a\frac{v_1 + v}{v_1 v}\right) \end{aligned} \tag{5}$$

The pulse propagation in this case is determined by the new velocity v_1 corresponding to the permittivity inside the layer.

One pulse reflection from the upper boundary is described by the first term in the first sum in (4), $k = 1$. It is determined by the field in the region shown in Fig. 3 by red hatching.

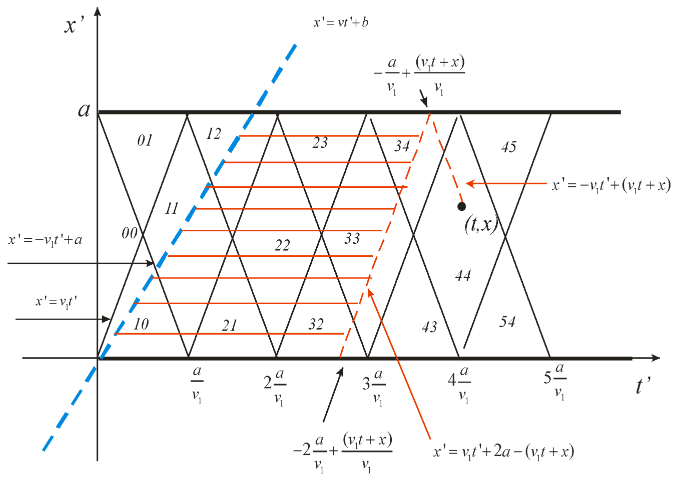

Fig. 3. The integration region for the inner field with one re-reflection from the upper boundary of the layer.

This one reflection from the upper boundary of the layer is calculated by the integral over this region and gives the contribution to the field caused by one reflection from the upper boundary with the weight R. A result consists of two waves propagating with the same velocity but in different directions:

$$\begin{aligned} E_{10}(t,x) &= E_{00}(t,x) + \\ &+ R\frac{v_1 + v}{v_1 v}\left(\mathrm{Ai}\left(-\frac{t}{T} - \frac{x}{v_1 T} + a\frac{v_1 + v}{v_1 v}\right) - \mathrm{Ai}\left(-\frac{t}{T} + \frac{x}{v_1 T} + \frac{2a}{v_1}\right)\right) \end{aligned} \tag{6}$$

The first term of the second sum in (4), $k=1$, gives also the contribution with the weight R, but as the result of the pulse reflection from the lower layer boundary and also consists of two types of waves which propagate in the same direction

$$E_{01}(t,x) = E_{00}(t,x) + R\frac{v_1-v}{v_1 v}\left(\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+a\frac{v_1+v}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T})\right) \quad (7)$$

Contributions of one re-reflection from the both layer boundaries has a form

$$E_{11}(t,x) = E_{00}(t,x) + R\frac{v_1-v}{2v}\left(\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+a\frac{v_1+v}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}+x\frac{2v+v_1}{v_1 v T})\right) + R\frac{v_1+v}{2v}\left(\mathrm{Ai}(-\frac{t}{T}-x\frac{2v-v_1}{v_1 v T}+\frac{2a}{v_1}) - \mathrm{Ai}(-\frac{t}{T}-\frac{x}{vT}+\frac{2a}{v_1})\right) \quad (8)$$

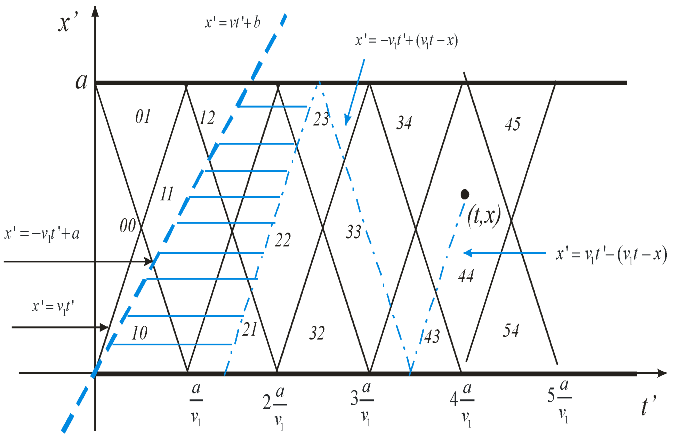

Fig. 4. The integration region for the inner field with two re-reflections from the upper boundary of the layer.

The second term of the first sum in (4), $k=2$, gives the contribution with the weight R^2 and it is the result of the two pulse reflections from the upper layer boundary and one pulse reflection from the lower boundary. Taking into account these re-reflections the field inside the layer is represented by the formula:

$$E_{21}(t,x) = E_{00}(t,x) + R\frac{v_1+v}{v_1 v}\left(\mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1 T}+a\frac{v_1+v}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+\frac{2a}{v_1})\right) + R^2\frac{v_1+v}{v_1 v}\left(\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+\frac{a}{v}) - \mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+\frac{a}{v_1})\right) \quad (9)$$

The second term of the second sum in (4), $k=2$, also gives the contribution with the weight R^2 as the result of the one pulse reflections from the upper layer boundary and two pulse reflection from the lower boundary. So, the inner field in this case is

$$E_{12}(t,x) = E_{00}(t,x) + R\frac{v_1-v}{v_1 v}\left(\mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+a\frac{v_1+v}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T})\right) + R^2\frac{v_1-v}{v_1 v}\left(\mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1 T}+a\frac{v_1+3v}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1 T}+\frac{2a}{v_1})\right) \quad (10)$$

The field inside the layer in the case of two re-reflections is represented by the formula

$$E_{22}(t,x) = E_{00}(t,x) + E_{11}(t,x) + R^2\frac{v_1+v}{2v}\left(\mathrm{Ai}(-\frac{t}{T}+\frac{x}{vT}+\frac{a}{v_1}) - \mathrm{Ai}(-\frac{t}{T}+\frac{x}{v_1 T}+\frac{a}{v_1})\right) + R^2\frac{v_1-v}{2v}\left(\mathrm{Ai}(-\frac{t}{T}-\frac{x}{v_1 T}+a\frac{3v+v_1}{v_1 v}) - \mathrm{Ai}(-\frac{t}{T}-\frac{x}{vT}+\frac{2a}{v_1})\right) \quad (11)$$

The expressions (8) and (11) show more complicated structure of the inner field as a result of re-reflections from the layer boundaries. The field inside the layer in this case consists of waves of the same type. But they propagate with the new velocity and have different contributions depending on the layer thickness.

III. RESULTS AND DISSCUSSION

Numerical comparison of an initial Airy pulse and the inner pulse in the layer is shown in Fig.5 - Fig.7. We consider two cases of the medium parameters: a) a less transparent layer when $v_1/v = 0.5$ and b) more transparent layer when $v_1/v = 1.5$. The normalized layer width $L=(b-a)/vT$ are used. The structure of a pulse inside the layer depending on the number of re-reflections from layer boundaries and layer thickness is shown also.

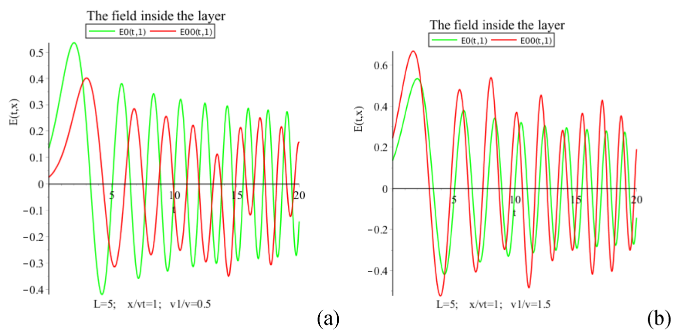

Fig. 5. Airy pulse evolution with re-reflections from the boundaries of the layer: at the less optical dense medium (left) and at the more optical dense medium (right)

It is shown in Fig. 5 that near the lower boundary the inner field without re-reflections has practically the same form as the incident wave. The pulse inside the layer propagates with a new velocity: at the less optical dense medium it propagates faster than an initial pulse, at the more optical dense medium – vice versa. The pulse magnitude at the more optical dense medium is greater than in the medium where the reflection coefficient is smaller.

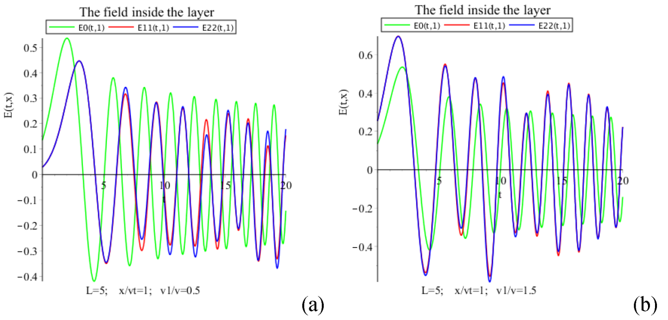

Fig. 6. Airy pulse evolution with the one and two re-reflections from the boundaries of the thin layer: at the less optical dense medium (left) and at the more optical dense medium (right)

Fig. 6 illustrates the influence of re-reflections from the layer boundaries in a thin layer on a pulse form. Re-reflections of the pulse from the boundaries decrease the modulation amplitude in case of a less dense medium. The main contribution to the inner field gives the term with the weight R (blue in Fig. 8). Terms that give contribution with weight R^2 have insignificant effect on the form of the inner pulse. It is consequent of that the coefficient R is less than 1. All terms of two sums in the formula (4), which are proportional to higher powers of the coefficient R (including the second power), give very small contributions to the field. A typical form of the Airy pulse is violated and appears "chaotic" fluctuations at the more optical dense medium Fig. 6(b).

The evolution of Airy pulse inside the thick layer is shown in Fig. 7. In both cases of the medium parameters the inner pulse has the same form as the incident one but a different amplitude compared with the initial pulse.

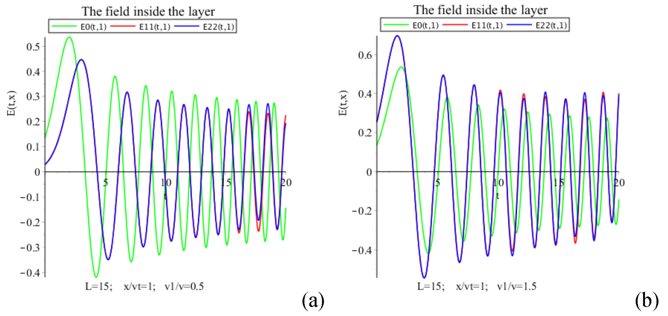

Fig. 7. Airy pulse evolution with the one and two re-reflections from the boundaries of the thick layer: at the less optical dense medium (left) and at the more optical dense medium (right)

Calculation of the transformed pulse form inside the layer shows that it differs from the form of the initial one in the case of the thin layer only. Nearer location of the layer boundaries causes the appearance of a signal modulation. With increasing the layer thickness the inner field practically coincides with the initial pulse by form and size.

IV. CONCLUSION

It can be said in conclusion that impinging of the Airy pulse on the dielectric layer gives a complex picture of re-reflections of secondary Airy pulses with amplitudes tending to zero as $R < 1$. Solution of this problem and its analysis show that the form of the inner pulse has more exact form compared to the undisturbed Airy pulse for a thick layer in the less optical dense medium. The main contribution to the field gives a terms of two sums in formula (4), which are proportional to the first power of the coefficient R.

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