ATOMIC FUNCTIONS IN CHOI-WILLIAMS ANALYSIS OF THE ULTRAWIDEBAND SIGNALS

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Abstract

Atomic functions (AF) and Kravchenko-Rvachev's window functions based on them are proposed to be applied for the Choi-Williams analysis of the ultrawideband (UWB) signals. New class of the non-linear Choi-Williams-Rvachev transforms (ChWRT) is considered. The advantages and disadvantages of such transforms are discussed. The abilities of the ChWRT at the sample of the UWB signal model analysis are demonstrated.

Keywords: Atomic functions, Kravchenko-Rvachev's window functions, non-linear integral transforms, Choi-Williams analysis, ultrawideband signals.

1. INTRODUCTION

The usage of the new signal types, in particular, of the UWB signals requires the new mathematical method application [1-4].

Different types of wavelet transform [5], atomic functions [6], adaptive Fourier transform [7], non-linear transforms from Cohen's class, in particular, Wigner and Choi-Williams transforms (ChWT) [8] were made a good showing during the analysis of such signals [9, 10].

The combination of the AF advantages with the ChWT abilities seems to be useful and advisable for the UWB signal digital processing.

2. THE BASIC RELATIONS

2.1. THE CHOI-WILLIAMS TRANSFORM TYPES

The ChWT called by authors as Exponential Distribution has been proposed by H. L. Choi and W. J. Williams in 1989 [11]. This non-linear transform with Gaussian kernel function refers to the transforms from Cohen's class. The Choi-Williams transform of the signal $s(t)$ is given by

$$P_{CW}[s(t)] \equiv P_{CW}f(\tau,\omega) = \sqrt{\frac{\sigma}{4\pi}}\int_{-\infty}^{\infty}\frac{\exp(-i\omega t)}{|t|}\times$$
$$\times\int_{-\infty}^{\infty}\exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right)s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)dudt,$$

where symbol "*" denotes the complex conjugation operation, σ $(\sigma > 0)$ is the scaling factor. It was shown [11] that if σ is aspiring to infinity the ChWT changes into the Wigner transform (WiT).

All transforms from Cohen's class, one hand, have good time-frequency properties comparing with other transforms as their basic advantage, but other hand, they have interfering cross terms for multicomponent signals as their basic disadvantage. These cross-terms cause redundancy in the information, and they may obscure the true energy distribution over time and frequency. The exponential kernel function application allows to reduce the cross terms with presents of relatively sharp resolution of auto-terms describing the signal in time and frequency domains. The authors of [11] assert that in this case the ChWT is more effective than smoothed WT. Furthermore, with the control parameter σ, the researcher can adjust the resolution of the auto-terms and the effects of cross terms according to the characteristics of the signal to be analyzed.

Next steps for the ChWT ability improvement [12] were the pseudo Choi-Williams transform (PChWT) given by

$$P_{PCW}f(\tau,\omega;h) = \sqrt{\frac{\sigma}{4\pi}}\int_{-\infty}^{\infty}h(t)\frac{\exp(-i\omega t)}{|t|}\times$$
$$\times\int_{-\infty}^{\infty}\exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right)s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)dudt, \quad (1)$$

and the smoothed pseudo Choi-Williams transform (SPChWT) given by

$$P_{SPCW}f(\tau,\omega;h,g) = \sqrt{\frac{\sigma}{4\pi}}\int_{-\infty}^{\infty}h(t)\frac{\exp(-i\omega t)}{|t|}\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(v-u)\exp\left(\frac{-(v-\tau)^2\sigma}{4t^2}\right)\times \quad (2)$$
$$\times s\left(v+\frac{t}{2}\right)s^*\left(v-\frac{t}{2}\right)dvdudt,$$

where $h(t)$ is the frequency smoothing window in time domain, $g(t)$ is the time smoothing window. The PChWT and the SPChWT allow more effective reduce the cross-terms for the multicomponent signals that the ChWT was allowing.

2.2. KRAVCHENKO-RVACHEV'S WINDOWS

There are many Kravchenko-Rvachev's window functions based on the AF [6]. Only four from them, as the example, given by

$$w_1(t) = \mathrm{up}(t), \tag{3}$$

$$w_2(t) = \mathrm{fup}_1(3t/2)/\mathrm{fup}_1(0), \tag{4}$$

$$w_3(t) = \mathrm{h}_{3/2}(t), \tag{5}$$

$$w_4(t) = \Xi_2(t)/\Xi_2(0), \tag{6}$$

where $\mathrm{up}(t)$, $\mathrm{fup}_1(t)$, $\mathrm{h}_{3/2}(t)$ and $\Xi_2(t)$ are the AF, are used in this work.

All AF and Kravchenko-Rvachev's window functions have good localization both in time domain and in frequency domain [6]. They have been successfully applied for new basis creation in Wigner analysis [13] and in wavelet analysis [14].

3. THE CHOI-WILLIAMS-RVACHEV TRANSFORM CLASS

The usage of the AF and Kravchenko-Rvachev's window functions for the non-linear digital signal processing is appears to be advisable and very promising. This idea allows combining the unique potentials of the AF with good abilities of the ChWT.

One from the possible ways of this idea realization is the simultaneous usage of the Kravchenko-Rvachev's window functions and the PChWT. Let's place the relations (3) – (6) in the equation (1) and, therefore, the new transform class, called as pseudo Choi-Williams-Rvachev transforms (PChWRT) are given by

$$P_{PCW} f(\tau,\omega;\mathrm{up}) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} \mathrm{up}(t)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty} \exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right) s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)du dt,$$

$$P_{PCW} f(\tau,\omega;\mathrm{fup}_1) = \sqrt{\frac{\sigma}{4\pi}}\frac{1}{\mathrm{fup}_1(0)} \times$$
$$\times \int_{-\infty}^{\infty} \mathrm{fup}_1(3t/2)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty} \exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right) s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)du dt,$$

$$P_{PCW} f(\tau,\omega;\mathrm{h}_{3/2}) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} \mathrm{h}_{3/2}(t)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty} \exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right) s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)du dt,$$

$$P_{PCW} f(\tau,\omega;\Xi_2) = \sqrt{\frac{\sigma}{4\pi}}\frac{1}{\Xi_2(0)} \times$$
$$\times \int_{-\infty}^{\infty} \Xi_2(t)\frac{\exp(-i\omega t)}{|t|} \int_{-\infty}^{\infty} \exp\left(\frac{-(u-\tau)^2\sigma}{4t^2}\right) \times$$
$$\times s\left(u+\frac{t}{2}\right)s^*\left(u-\frac{t}{2}\right)du dt.$$

Another possible way is application of the Kravchenko-Rvachev's window functions and the SPChWT. Substituting (3) – (6) into (2), we obtain, so called, the smoothed pseudo Choi-Williams-Rvachev transforms (SPChWRT), given by

$$P_{SPCW} f(\tau,\omega;\mathrm{up},\mathrm{up}) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} \mathrm{up}(t)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{up}(v-u)\exp\left(\frac{-(v-\tau)^2\sigma}{4t^2}\right) \times$$
$$\times s\left(v+\frac{t}{2}\right)s^*\left(v-\frac{t}{2}\right)dv du dt,$$

$$P_{SPCW} f(\tau,\omega;\mathrm{fup}_1,\mathrm{fup}_1) = \sqrt{\frac{\sigma}{4\pi}}\frac{1}{\mathrm{fup}_1^2(0)} \times$$
$$\times \int_{-\infty}^{\infty} \mathrm{fup}_1(3t/2)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{fup}_1(3t/2)(v-u)\exp\left(\frac{-(v-\tau)^2\sigma}{4t^2}\right) \times$$
$$\times s\left(v+\frac{t}{2}\right)s^*\left(v-\frac{t}{2}\right)dv du dt,$$

$$P_{SPCW} f(\tau,\omega;\mathrm{h}_{\frac{3}{2}},\mathrm{h}_{\frac{3}{2}}) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} \mathrm{h}_{\frac{3}{2}}(t)\frac{\exp(-i\omega t)}{|t|} \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{h}_{\frac{3}{2}}(v-u)\exp\left(\frac{-(v-\tau)^2\sigma}{4t^2}\right) \times$$
$$\times s\left(v+\frac{t}{2}\right)s^*\left(v-\frac{t}{2}\right)dv du dt,$$

$$P_{SPCW} f(\tau,\omega;\Xi_2,\Xi_2) = \sqrt{\frac{\sigma}{4\pi}}\frac{1}{\Xi_2^2(0)} \int_{-\infty}^{\infty} \Xi_2(t) \times$$
$$\times \frac{\exp(-i\omega t)}{|t|} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Xi_2(v-u)\exp\left(\frac{-(v-\tau)^2\sigma}{4t^2}\right) \times$$
$$\times s\left(v+\frac{t}{2}\right)s^*\left(v-\frac{t}{2}\right)dv du dt.$$

It is taken to mean that any pairs of the Kravchenko-Rvachev's window functions (not only same) in the PChWRT and the SPChWRT can be used.

4. UWB SIGNAL ANALYSIS

For the demonstration of the PChWRT and the SPChWRT abilities we use such simple UWB signal model in time domain (fig. 1, a):

$$s_2(t) = 2s_1(4t - \tau_s/2) + s_1(t - 5\tau_s/4) +$$

$$+s_1(t-7\tau_s/2)+2s_1(4t-19\tau_s/4) \qquad (7)$$

where $s_1(t)$ is given with $N=2$ by

$$s_1(t)=(-1)^n A_0(1-|2t/\tau_s-1|)\sin(2\pi nt/\tau_s)\Theta(t),$$

A_0 is the signal amplitude, τ_s is the signal duration in time domain, $\Theta(t)=\eta(t/\tau_s)-\eta(t/\tau_s-1)$, $\eta(t)$ is Heavyside function, n is natural number, corresponding with the UWB signal sidelobe number N by the relation $N=2n$. The spectral density functions (SDF) for the ChWT (fig. 1, b), the PChWRT (fig. 1, c) and the SPChWRT (fig. 1, d) with $\sigma=10$ were calculated. For the PChWRT and the SPChWRT construction the window function (3) with width equal to $\tau_0/4$, where τ_0 is the signal (7) duration in time domain, was applied. The SDF of the Fourier spectrogram (FS) (fig. 1, e) with application of the Hamming window with same width was built. From the analysis shown at the fig. 1 it follows that the ChWT result is quite poor as long as the cross-terms have been weakly reduced (fig. 1, b). The results shown by the PChWRT (fig. 1, c) and, especially, by the SPChWRT (fig. 1, d) are much better. One hand, the effective cross-term reduction and, other hand, the simultaneous quite good time-frequency resolution of the PChWRT and SPChWRT (much better than for FS SDF (fig. 1, e)) are observed.

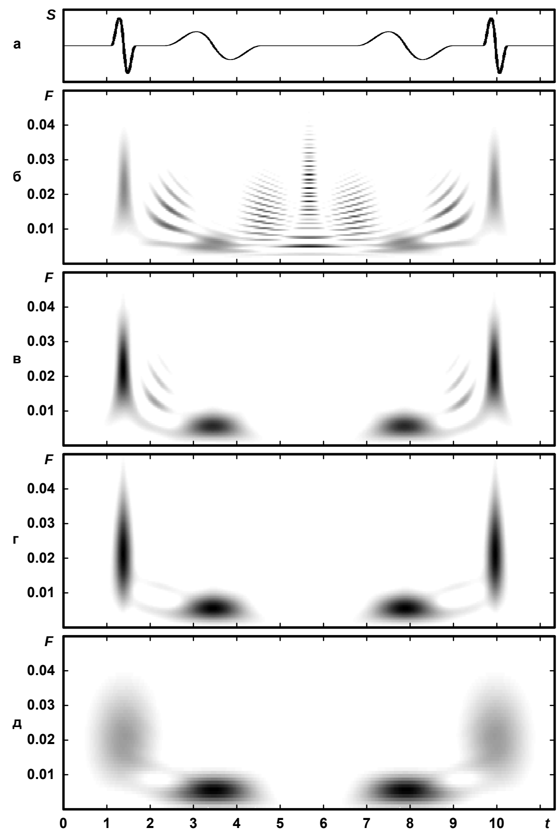

Fig. 1. Analysis of the model UWB signal (7): a – the signal in time domain, b – ChWT SDF ($\sigma=10$), c – PChWRT SDF($\sigma=10$), d – SPChWRT SDF ($\sigma=10$), e – FS SDF.

CONCLUSIONS

- The new class of the non-linear Choi-Williams-Rvachev transforms including the PChWRT and the SPChWRT is proposed.
- The transforms of this class are shown to combine the advantages of the non-linear ChWT with ones of the AF and Kravchenko-Rvachev's window functions based on then.
- The PChWRT and the SPChWRT are proposed to be used for the analysis of the UWB signals.

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