Evolution of an Airy pulse energy flow induced by a dielectric plane boundary

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***Abstract*—Transformation of energy flows of an electromagnetic Airy pulse by a plane boundary of a dielectric is considered. It is shown that a problem formulation is equivalent to a case of a permittivity sharp change in a half space at zero moment of time. It allows to use the resolvent method for solution of the Volterra integral equation describing the phenomenon. Explicit expressions for transmitted and reflected pulses are obtained and energy flows are analyzed.**

Keywords—electromagnetic Airy pulse; dielectric inhomogeneity; Volterra integral equation

I. INTRODUCTION

Phenomena of interactions of electromagnetic pulses with the inhomogeneous medium cause very wide range of problems in the electrodynamics. It may be radar-related problems, electromagnetic signals transformation technology applications, laser technology, optical phenomena and their applications etc. Special interest is turned on the Airy pulses because of their extraordinary properties in paraxial approximation, such as non-diffractive propagation, self-acceleration and self-healing [1]. Intensive theoretical and experimental researches of these pulses are topical either for perspective development [2], or already implemented ones [3]. Using an Airy pulse as an initial pulse in a phenomenon of electromagnetic signal interaction with a plane boundary of two media is caused by two reasons. First, this impulse seems to be very topical in a paraxial approximation considering its properties. Second, it has an asymmetrical form with the exponentially fading front and the oscillating tail, fading out at the infinity. And, third, its form allows investigating the inhomogeneous medium interaction process starting at a fixed moment of time, unlike a plane harmonic wave.

The research of the electromagnetic waves interaction with the inhomogeneous and nonstationary medium requires an adequate and realizable mathematical modeling, which is optimal for the problem. In the given paper such an approach is based on the Volterra integral equations method, which is capable of physical clarity and descriptive completeness. The latter consists of, besides the other pros, the automatic involving of both the initial and the boundary conditions and the versatility regarding the free term of the equation (the initial pulse).

II. THE VOLTERRA INTEGRAL EQUATION METHOD

A. Statement of a problem

Transformation of electromagnetic pulses at a plane boundary between two dielectric half-spaces characterized by dielectric indexes n and n_1 is described by the Volterra integral equation method [4]. An electromagnetic pulse propagates along the normal to the boundary from the half-space with the index n to the one with n_1. In this case a boundary-initial value problem is described by the integral equation for the pulse electric field

$$E(t,x) = E_0(t,x) - \frac{v^2 - v_1^2}{2v_1^2 v}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty} dt' \int_0^{\infty} dx' \theta\left(t - t' - \frac{|x - x'|}{v}\right) E(t',x') \quad (1)$$

where

$$E_0(t,x) = I_0 \mathrm{Ai}(-t/T + x/vT) \quad (1)$$

is the initial pulse generated by an extrinsic source, $\mathrm{Ai}(x)$ is the Airy function, $v = c/n$, $v_1 = c/n_1$, c is the light velocity in vacuum, $\theta(x)$ is the Heaviside unit function, and T is a value of a time scale.

Such statement of a problem is equivalent to a problem about interaction of a pulse with a boundary which begins to exist at zero moment of time. The initial pulse in the form of the Airy function is just adequate to this statement.

B. Algorythm for solution of the problem by the resolvent method

As equation (2) is the Volterra integral equation its solution can be found by the resolvent method [4, 5]. In this case it can be used the resolvent obtained for a problem of formation of a half-space at zero moment of time, i.e. for a problem on a temporal half-axis $0 \le t < \infty$:

$$\hat{R} = \theta(x)(\hat{R}_1 + \hat{R}_2)\theta(x') \quad (3)$$

The first part of this resolvent describes the initial-value part of the problem

$$\left\langle \mathbf{x} \middle| \hat{R}_1 \middle| \mathbf{x}' \right\rangle = \frac{v_1^2 - v^2}{2v^2 v_1}\theta(x)\frac{\partial^2}{\partial t^2}\theta\left(t - t' - \frac{|x - x'|}{v_1}\right)\theta(x'). \quad (4)$$

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The second part is conditioned by presence of a medium boundary

$$\left\langle \mathbf{x}\left|\hat{R}_2\right|\mathbf{x}'\right\rangle = \frac{v_1^2 - v^2}{2v^2 v_1}\theta(x)R\frac{\partial^2}{\partial t^2}\theta\left(t - t' - \frac{x + x'}{v_1}\right)\theta(x'), \qquad (5)$$

Here, $R = (v_1 - v)/(v_1 + v)$ is the reflection coefficient.

Using this resolvent the electric field of the pulse transmitted through the boundary is given by the formula

$$\begin{aligned} E_{Tr} &= E_0(t,x) + \theta(x)\int_{-\infty}^{+\infty} dt' \int_0^{+\infty} dx' \left\{\left\langle \mathbf{x}\left|\hat{R}_1\right|\mathbf{x}'\right\rangle + \left\langle \mathbf{x}\left|\hat{R}_2\right|\mathbf{x}'\right\rangle\right\} E_0(t',x') = \\ & E_0(t,x) + \theta(x)\frac{v_1^2 - v^2}{2v^2 v_1}\frac{\partial^2}{\partial t^2}\left\{\int_{-\infty}^{+\infty} dt' \int_0^{\infty} dx' \theta\left(t - t' - \frac{|x - x'|}{v_1}\right)E_0(t',x') + \right. \qquad (6) \\ & \left. \int_{-\infty}^{+\infty} dt' \int_0^{+\infty} dx' R\theta\left(t - t' - \frac{x + x'}{v_1}\right)E_0(t',x')\right\} \end{aligned}$$

The integration region defined by the inequality $t - t' - |x - x'|/v_1 > 0$, following from the first part of the resolvent $\hat{R}_1$, is shown in Fig. 1 by vertical hatching. The region $t - t' - (x + x')/v_1 > 0$ following from the second part of the resolvent $\hat{R}_2$ is shown by the horizontal hatching. The initial pulse is located in the region $x' < vt'$ and the trajectory of its time-spatial boundary is shown by the dashed blue line. The structure of the initial Airy pulse provides a zero field value to the left of this line.

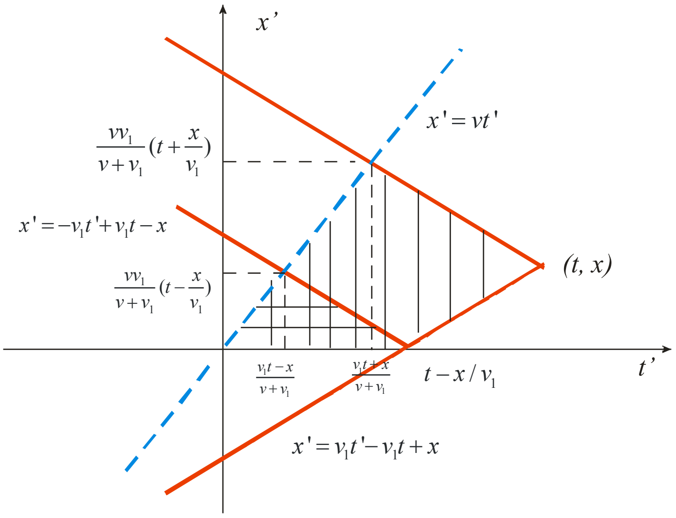


Fig. 1. The integration regions in the formula (6).

It is convenient to execute calculations in the formula (6) using a Fourier representation of the Airy pulse

$$E_0(t,x) = I_0 \mathrm{Ai}(-t/T + x/vT) = \frac{I_0 T}{2\pi}\int_{-\infty}^{+\infty} e^{i\frac{(\omega T)^3}{3} - i\omega t + i\omega x/vT} d\omega \qquad (7)$$

As a result we obtain the field in the region of a transmitted pulse $x > 0$:

$$E_T = E_0 - I_0 \mathrm{Ai}(-t/T + x/vT) + E_T(t,x) \qquad (8)$$

The second term turns off the initial pulse in accordance with the Ewald-Oseen extinction theorem [6]. The third term is represented the pulse transmitted through the boundary

$$E_{Tr}(t,x) = \frac{2v_1}{v_1 + v}\theta(x) I_0 \mathrm{Ai}(-t/T + x/v_1 T) \qquad (9)$$

The propagation velocity of this pulse is determined by the new wave velocity v_1 determined by the dielectric index value behind the boundary.

The obtained transmitted pulse (9) allows to get the reflected pulse via integration in the formula (2) by substituting $E(t,x)$ for the expression (9)

$$E_R(t,x) = E_0(t,x) - \frac{v_1^2 - v^2}{2v^2 v_1}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{+\infty} dt' \int_0^{x} dx' \theta\left(t - t' - \frac{x' - x}{v}\right)E_{Tr}(t',x') \qquad (10)$$

As a result, we obtain

$$E_R = \frac{v_1 - v}{v_1 + v} I_0 \mathrm{Ai}(-t/T - x/vT) \qquad (11)$$

The transmittance coefficient $2v_1/(v_1 + v)$ in (9) and the reflection coefficient $(v_1 - v)/(v_1 + v)$ in (11) for the secondary Airy pulses coincide with corresponding coefficients for a plane harmonic wave. The form of the transmitted pulse repeats the initial pulse form but with another propagation velocity. The form of the reflected pulse is the same as in the initial one but with the opposite direction of propagation.

The magnetic field can be found from Maxwell's equation $B(t,x) = -\int_0^t \frac{\partial E(t',x)}{\partial x} dt'$, that allows to get the energy flow $P(t,x) = E(t,x)B(t,x)$. It is convenient to consider an averaged value of the energy flow over some time interval T_0.

$$\bar{P}(t,x) = \frac{1}{T_0}\int_t^{t+T_0} P(t',x)dt', \qquad (12)$$

So, the averaged energy flows are:

for the transmitted pulse

$$\bar{P}_{Tr}(x) = \left(\frac{2v_1}{v_1 + v}\right)^2 I_0^{\,2} \frac{1}{T_0}\int_t^{t+T_0} [\mathrm{Ai}(-t/T + x/v_1 T)]^2 dt, \qquad (13)$$

and for the reflected one

$$\bar{P}_R(x) = -\left(\frac{v_1 - v}{v_1 + v}\right)^2 I_0^{\,2} \frac{1}{T_0}\int_t^{t+T_0} [\mathrm{Ai}(-t/T - x/vT)]^2 dt. \qquad (14)$$

III. ENERGY FLOWS IN TRANSMITTED AND REFLECTED PULSES

Evolution of the energy flows given by the expressions (13) – (14) at the different points and on both sides of the boundary is shown in Fig. 2-5.

The energy flow of the initial Airy pulse is illustrated by Fig. 2.

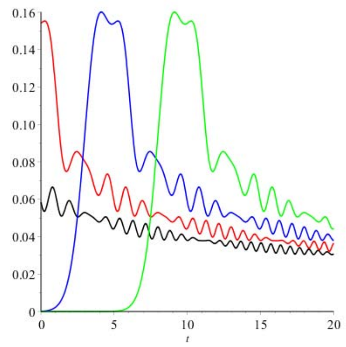
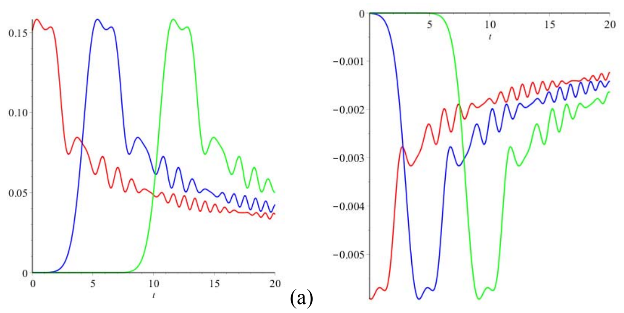


Fig.2. The evolution of the initial Airy pulse energy at various points: $x / vT = -5$ (black) $x / vT = 0$ (red), $x / vT = 5$ (blue), $x / vT = 10$ (green)

The movement of the initial pulse main lobe is quite visual in Fig.2, as the energy maximum is reached at the different space points and time moments. The pulse tail movement is almost insensible to a position of the observation point and the energy flow within it asymptotically tends with the time $t \to \infty$ to the same value at all points.

The transmitted and the reflected pulses evolution is shown in Fig.3-4 for two refraction indexes. The transfer from the less optical dense medium, $v_1 / v = 0.8$, to the more optical dense one is shown in Fig. 3. The transmitted pulse evolution at the different points is shown in Fig. 3a and for the reflected one in Fig. 3b.

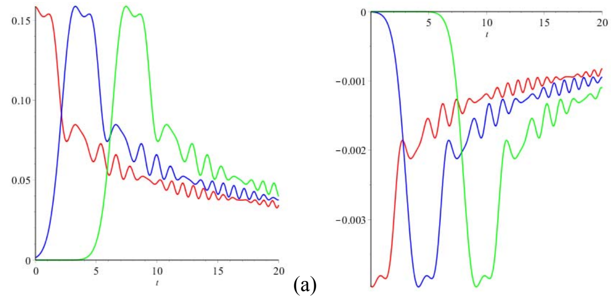


(a) (b)

Fig.3. The evolution of an energy flow at various points for $v_1 / v = 0.8$: the transmitted (a) and the reflected (b) Airy pulses ($x / vT = 1$ (red), $x / vT = 5$ (blue), $x / vT = 10$ (green)).

As the pulse velocity in more optical dense medium is less than in the initial pulse the maximum of the transmitted pulse main lobe is reached later in comparison with the reflected pulse which velocity is equal to the initial pulse velocity. The energy flow in the pulse tail tends asymptotically with $t \to \infty$ to the same value at various points.

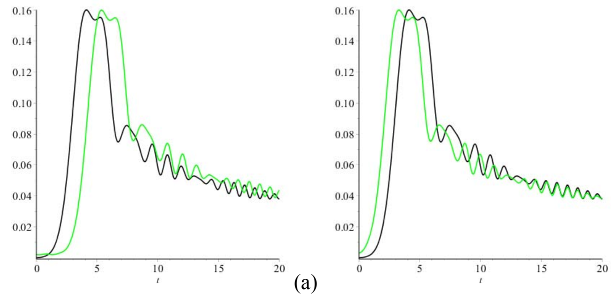
When the initial pulse comes to the less optical dense medium the maximum of the main lobe is reached earlier, Fig, 4a, than in the previous case. The energy flow in the pulse tails have the same asymptotical behavior with $t \to \infty$ at various points, Fig. 4a,b.



(a) (b)

Fig.4. The evolution of the energy flow at various points for $v_1 / v = 1.2$: the transmitted (a) and the reflected (b) Airy pulses ($x / vT = 1$ (red), $x / vT = 5$ (blue), $x / vT = 10$ (green))

The balance between the energy flows of the initial pulse $\bar{P}_0(t,x)$ (black) and the diffracted ones $\bar{P}_{Tr}(t,x) - \bar{P}_R(t,x)$ (green) is demonstrated in Fig.5. Different velocities of the transmitted pulses determine a delay in the more optical dense medium and a lead in the opposite case.



(a) (b)

Fig.5. The balance between the energy flows at the point $x / vT = 5$: a) $v_1 / v = 0.8$, b) $v_1 / v = 1.2$.

IV. CONCLUSION

Analysis of evolution of the initial Airy pulse induced by a plane boundary between two dielectrics shows that the energy flows in the transmitted and reflected pulses are characterized by a slow-down of the main pulse lobe in the more optical dense medium and converge asymptotically with time to the same value at the different points for both transmitted and reflected pulses.

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