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GEOMETRIC PRINCIPLES OF CONSTRUCTING 3D MODELS

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The article examines geometric principles of three-dimensional modeling in computer graphics. It focuses on polygonal representation, topological analysis, and subdivision methods for creating smooth and detailed surfaces. Constructive solid geometry allows combining simple primitives into complex objects. Spatial transformations enable controlling position, orientation, and scale. These approaches ensure efficient rendering and high computational performance. The article provides an overview of fundamental techniques used in modern 3D modeling workflows.

Contemporary computer graphics and digital media technologies use mathematical methods to describe three-dimensional objects. Geometric modelling is a key component in the creation of scenes for games, virtual reality, animation and design, determining the shape, surface structure and spatial relationships of model elements.

The objective of this work is to examine the geometric principles of three-dimensional modelling in computer graphics, analyse methods of their mathematical representation, and study the characteristics of polygonal modelling and surface smoothing algorithms used in the creation of modern digital objects.

Polygonal modelling is the most common method of representing 3D geometry in computer graphics, in which the surface of an object is described by a polygonal mesh of vertices, edges and faces that form a discrete approximation of a continuous surface [1]. In mathematical terms, a polygonal model is described by the structure:

$$M = (V, E, F), \quad (1)$$

where V – is the set of vertices; E – is the set of edges; F – is the set of faces that form the surface of a three-dimensional object.

Each i vertex is defined by coordinates in three-dimensional space.

$$v_i = (x_i, y_i, z_i). \quad (2)$$

which defines its position in the Cartesian coordinate system.

The edges connect pairs of vertices, and the faces are formed by closed contours of edges and can be triangular, quadrangular, or polygonal in shape. Such a representation allows complex surfaces to be approximated using a set of polygons, which is why polygonal meshes are widely used in computer graphics for visualising and editing three-dimensional models.

An important characteristic of a three-dimensional model is not only its geometry, but also its topological structure, which determines how the elements of the polygonal mesh are connected. The topology of a model describes the relationships between vertices, edges, and faces, regardless of the specific coordinates of points in space. It is topology that determines whether a surface is closed or contains holes, as well as how the elements of the surface interact with each other. One of the basic topological invariants used in geometric modelling is the Euler characteristic, which is defined by the ratio

$$\chi = V - E + F, \tag{3}$$

where V, E, F – denote the number of vertices, edges and faces of a polygonal mesh, respectively [2].

For closed surfaces, the Euler characteristic allows us to determine the number of topological holes or genera of the surface. For example, for a surface that is topologically equivalent to a sphere, the following equality holds:

$$\chi = 2. \tag{4}$$

The use of topological invariants allows to check the correctness of the construction of three-dimensional models and detect errors in the structure of the polygonal mesh.

An important component of the geometric description of the surface is the determination of polygon normals. A surface normal is a vector perpendicular to the plane of the polygon, which is used to calculate the lighting and reflection of an object in a scene. For a triangular face with vertices v_1, v_2, v_3 the normal can be calculated using the vector resultant:

$$n = (v_2 - v_1) \times (v_3 - v_1). \tag{5}$$

The resulting vector is normalized

$$\bar{n} = \frac{n}{|n|} \tag{6}$$

and used in lighting models to determine the angle at which light falls on the surface [3].

More complex models often use vertex normals, which are defined as the average of the normals of neighbouring polygons [4]. This allows for the creation of a smooth surface effect even when using polygonal geometry.

Due to the discrete nature of polygonal models, their surfaces may appear jagged. To smooth them, surface subdivision methods are employed, which recursively divide polygons into smaller ones with recalculated vertex coordinates, gradually approximating a smooth shape.

One of the most well-known algorithms of this type is the Catmull–Clark method, which was proposed in 1978 as a generalisation of B-spline surfaces for polygonal meshes of arbitrary topology. The algorithm involves creating new vertices for each face, edge, and vertex of the initial mesh [5]. The new vertex position is calculated using the formula

$$V' = \frac{F + 2R + (n-3)V}{n}, \tag{7}$$

where F – is the average value of the centers of adjacent faces; R – is the average value of the midpoints of the edges; V – is the initial vertex, and n is the number of adjacent edges.

After each iteration of the algorithm, the number of polygons in the mesh increases, gradually forming a smoother surface.

Geometric modelling also uses constructive solid geometry, where complex objects are created by combining, intersecting, and subtracting simple primitives. This method is commonly used in CAD and procedural modelling [2].

Consequently, modern three-dimensional modelling is based on a combination of polygonal surface representation, topological analysis and smoothing algorithms, which allows the creation of complex objects with high detail and computational efficiency. Further development in this field is linked to the improvement of modelling algorithms, procedural methods, and the integration of artificial intelligence to automate the creation of three-dimensional geometry.

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