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Diffraction of the Plane Electromagnetic Wave on the Structure Incorporating Two Coaxial Unclosed Cones*

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ABSTRACT: The paper is devoted to the investigation of plane wave diffraction on the 3D composite unclosed conical structure consisting of two cones with longitudinal slots. The rigorous analytic-numerical approach, based on the Kontorovich-Lebedev integral transform and the semi-inversion method, has been developed and applied to solve this problem. On the basis of the numeral solution the influence of the longitudinal slot on the field forming in space is analyzed. The obtained results could be applied to the designing and development of scanning antennae and devices of measuring technique.

INTRODUCTION

Unclosed conical and biconical structures find applications not only for antenna engineering but also for radar, as far as radar reflectors with certain scattering properties can be designed on their bases [1]. It is known that the directional diagram of the scattering antenna array element should be monodirectional and close to cardioid one to radiate powerful superwideband pulses [2,3]. The superwideband antenna based on a solid cone or a bicone is inadequate to the requirement of unidirectionality of the directional diagram. The use of a reflector for designing a cardioid antenna pattern makes considerably worse the impedance characteristics of the conical structure and increases the over-all dimensions [2]. One of way to solve this problem is the substitute of the continuous surface for the unclosed one. In this connection the study of the problem of plane electromagnetic wave diffraction on composite unclosed conical structures is of special interest. Particularly, the results of the investigations allow us to study the behavior of the field near the vertex of the

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cone with longitudinal slots; that can be used for diagnosing the cracks on the conical surface [4].

The purpose of this paper is to study the problem of plane electromagnetic wave diffraction on the structure incorporating of two coaxial circular perfect conducting cones with periodical longitudinal slots as well as to study its scattering properties.

THE PROBLEM DEFINITION

Let a plane electromagnetic wave be incident upon the structure Σ along the axis of the considered surface.The structure Σ (Fig. 1) consists of two semi-infinite circular perfect conducting thin cones Σ_1 and Σ_2 possessing the slots ($\Sigma = \Sigma_1 \cup \Sigma_2$), which periodically cut along the generatrices N and with the common vertex. The opening angle of the cone Σ_j ($j = 1,2$) is designated by $2\gamma_j$; d_j is the width of the slots; $l = 2\pi / N$ is the period of the structure. The width of the slots and the period are the angular values equaling the values of the dihedral angles which are formed by the planes passed through the axis of the structure and the edges of adjoining conical sectors. The cones Σ_j are determined by the equations $\theta = \gamma j$ in the introduced coordinate frame r, θ, φ with the origin in the vertex of the plane. For the sake of definiteness, we consider the incident wave as a E-polarized one, field of which varies in time under the harmonic law $e^{-i\omega t}$:

$$\vec{E}^{(0)} = (E_x^{(0)}, 0, 0), \quad \vec{H}^{(0)} = (0, H_y^{(0)}, 0), \quad E_x^{(0)} = e^{ikz}, \quad H_y^{(0)} = \frac{1}{w} e^{ikz}$$

where $k = \omega\sqrt{\varepsilon\mu}$ is the wave number; ε and μ are the parameters of the homogeneous and isotropic media with the wave impedance w; the conical surface is located inside this media. The electromagnetic field $\vec{E}, \vec{H}$ in space with the conical structure Σ satisfies the Maxwell equations, the boundary condition of transformation of the electric field tangential component on the conical sectors as well as the radiation condition and the boundedness condition of the energy. The electrodynamic problem has a unique solution [5] in such a definition. By virtue of linearity of the diffraction problem under consideration, we represent the desired field $\vec{E}, \vec{H}$ in the following form

$$\vec{E} = \vec{E}^{(0)} + \vec{E}^{(1)}, \tag{2}$$

$$\vec{H} = \vec{H}^{(0)} + \vec{H}^{(1)}, \tag{3}$$

where the field $\vec{E}^{(1)}, \vec{H}^{(1)}$, is stipulated by the presence of the conical structure (the diffracted field). It is convenient to use the electric ($\upsilon^{(1)}(r,\theta,\varphi)$) and magnetic ($\upsilon^{(2)}(r,\theta,\varphi)$) Debye potentials when solving electrodynamic problems in the spherical coordinate frame.

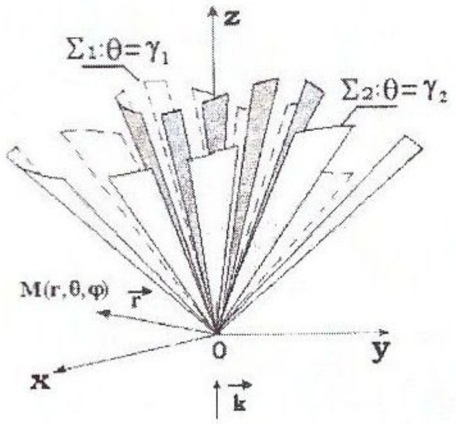

FIGURE 2.

The electromagnetic field components is expressed by the Debye potentials $\upsilon^{(x)}$ in the following form [6]:

$$E_r = \left(\frac{\partial^2}{\partial r^2} + k^2\right)\left(r\upsilon^{(1)}\right),$$

$$H_r = \left(\frac{\partial^2}{\partial r^2} + k^2\right)\left(r\upsilon^{(2)}\right)$$

$$E_\theta = \frac{1}{r}\frac{\partial^2}{\partial r \partial\theta}\left(r\upsilon^{(1)}\right) + \frac{ikw}{\sin\theta}\frac{\partial}{\partial\varphi}\upsilon^{(2)},$$

$$H_\theta = -\frac{ik}{w\sin\theta}\frac{\partial}{\partial\varphi}\upsilon^{(1)} + \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}(r\upsilon^{(2)}) \tag{4}$$

$$E_\varphi = \frac{1}{r\sin\theta}\frac{\partial^2}{\partial r\partial\varphi}\left(r\upsilon^{(1)}\right) - ikw\frac{\partial}{\partial\theta}\upsilon^{(2)}$$

$$H_\varphi = \frac{ik}{w}\frac{\partial}{\partial\theta}\upsilon^{(1),\varpi} + \frac{1}{r\sin\theta}\frac{\partial^2}{\partial r\partial\varphi}(r\upsilon^{(2)})$$

The desired potential $\upsilon^{(\chi)}$ is the solution of the first ($\chi=1$) or second ($\chi=2$) external boundary-value problem of the Helmholtz equation for the conical geometry [7]. In accordance with the structure of the electromagnetic field (2) and (3), we write down the potentials $\upsilon^{(\chi)}$ ($\chi=1,2$) in the form

$$\upsilon^{(\chi)} = \upsilon_0^{(\chi)} + \upsilon_1^{(\chi)} \tag{5}$$

where the potentials $\upsilon_1^{(\chi)}$ correspond to the field $\vec{E}^{(1)}, \vec{H}^{(1)}$ and the potentials $\upsilon_0^{(\chi)}$ relate to the plane wave field $\vec{E}^{(0)}, \vec{H}^{(0)}$ (1), in this case

$$\upsilon_0^{(\chi)} = -\frac{1}{(w)^{1-\chi}}\frac{\sin\left(\frac{\pi}{2}\chi-\varphi\right)}{k^2 r\sin\theta}\left(\cos kr + i\cos\theta\sin kr - e^{ikr\cos\theta}\right).$$

To solve the problem, we use the Kontorovich-Lebedev integral transformation in the following form

$$\Psi(r) = -\frac{1}{2}\int_0^{+\infty}\tau sh\pi\tau e^{-\frac{\pi\tau}{2}}\tilde{\Psi}(r)\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}d\tau,$$

$$\tilde{\Psi}(r) = \int_0^{+\infty}\Psi(r)e^{-\frac{\pi\tau}{2}}\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}dr,$$

where $H_{i\tau}^{(1)}(kr)$ is the Hankel function of the first kind. We represent $\upsilon_1^{(\chi)}$ in the form

$$\upsilon_1^{(\chi)} = \upsilon_{1,dvr}^{(\chi)} + \upsilon_{1,cnv}^{(\chi)}. \tag{6}$$

$$\upsilon_{1,dvr}^{(\chi),E} = -\frac{1}{2}\int_0^{+\infty}\tau sh\pi\tau e^{-\frac{\pi\tau}{2}}\hat{\upsilon}_{1,dvr}^{(\chi),E}\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}d\tau,$$

$$\hat{\upsilon}^{(s),E}_{1,dvr} = -\sum_{m=-1;1} \tilde{a}^{(s),E}_{m\tau}\tilde{b}^{(s)}_{m\tau}\tilde{U}^{(s)}_{m\tau},$$

$$\hat{\upsilon}^{(s),E}_{dvr,cnv} = -\frac{i}{2}\frac{\sin kr}{k^2 r}\frac{d^{s-1}}{d\gamma_2^{s-1}}\left(tg\frac{\gamma_2}{2}\right)\sum_{m=-1;1}\left(-\frac{i}{w}\frac{|m|}{m}\right)^{s-1}\tilde{\tilde{U}}^{(s)}_{m},$$

where $\tilde{b}^{(s)}_{m\tau} = \frac{d^{s-1}}{d\gamma_2^{s-1}} P^{m}_{-1/2+i\tau}(\cos\gamma_2)$. By virtue of periodicity of the solution to the problem on the azimuth coordinate φ, the unknown functions $\tilde{U}^{(\chi)}_{m\tau}(\theta,\varphi)$ and $\tilde{\tilde{U}}^{(\chi)}_{m}(\theta,\varphi)$ are expanded in the Fourier series, coefficients of which $y^{(\chi)}_{m,n}$ is the solution of two coupled sets of linear algebraic equations of the second kind of the Fredholm type in the following form:

$$Y^{(\chi)} = A^{(\chi)}Y^{(\chi)} + B^{(\chi)} \tag{7}$$

In the case of the cone with longitudinal slots and a internal continuous conical shield as well as a single cone with longitudinal slots the coupled sets (7) transforms into one set of linear equations of the second kind, solution of which can be obtained (in view of compactness of the matrix operator) either by the reduction method (for arbitrary parameters of the problem) or for a semitransparent cone by the method of successive approximations (the matrix operator is compressive).

THE ANALYTICAL SOLUTION FOR A SEMITRANSPARENT CONE WITH AN INTERNAL SOLID SHIELD

We consider the problem of the plane wave diffraction on a cone with the longitudinal slots Σ_2 and the insert in the form of a continuous conical shield Σ_1 when the number of slots is large and their width d_2 is comparable with the period l under condition of the limit existence.

$$W_\chi = \lim_{\substack{N\to+\infty \\ d_2/l\to 2-\chi}} \left[-\frac{1}{N}\ln\sin\left(\frac{\pi}{2}(\chi - d_2/l)\right)\right] > 0 \tag{8}$$

The conical surface which is determined by the limit (8) has the property to pass through partly and to reflect partly the incident field. The structures with such properties are named semitransparent [8] and are particular cases of anisotropic conducting surfaces [9]. In the case of the semitransparent cone Σ_2

a) $N >> 1$, $(l-d_2)/l << 1$, $W_2 = 0$,

$$W_1 = \lim_{\substack{N\to\infty \\ d_2/l\to 1}} \left[-\frac{1}{N} \ln \cos \frac{\pi d_2}{2l} \right] > 0 \tag{9}$$

In this case the electric Debye potential $\upsilon_1^{(1)}$ (5) and (6) can be represented in the following form

$$\upsilon_1^{(1)} = \upsilon_1^{(1),sol\Sigma_1} + \upsilon_1^{(1),\Sigma_{1,2}} \tag{10}$$

where $\upsilon_1^{(1),sol\Sigma_1}$ corresponds to the potential at the plane wave diffraction (1) on the single (in the absence of Σ_2) solid cone Σ_1 [7] and the item $\upsilon_1^{(1),\Sigma_{1,2}}$ takes into account the presence of the cone Σ_1 and the semitransparent cone Σ_2. In this case

$$\begin{gathered} \upsilon_1^{(1),\Sigma_{1,2}} = \cos\varphi \int_0^{+\infty} \varpi_{i\tau} \frac{H_{i\tau}^{(1)}(k\tau)}{\sqrt{r}} \times \\ \times A_{i\tau} \frac{1 - C_{i\tau}^{(1),-1}(\gamma_1, \gamma_2)}{A_{i\tau} + 2W_1} \frac{P_{-1/2+i\tau}^{-1}(\cos\gamma_2)}{P_{-1/2+i\tau}^{-1}(-\cos\gamma_2)} P_{-1/2+i\tau}^{-1}(-\cos\theta) d\tau - \\ -itg^2(\gamma_2/2) \frac{\left(1 - tg^2(\gamma_1/2)/tg^2(\gamma_2/2)\right)^2}{1 - tg^2(\gamma_1/2)/tg^2(\gamma_2/2) + 2W_1} \times \\ \times \cos\varphi \cdot ctg\frac{\theta}{2} \frac{\sin kr}{k^2 r}, \gamma_2 < \theta < \pi \end{gathered} \tag{11}$$

$$C_{i\tau}^{(\chi),M}(\gamma_k, \gamma_j) = \frac{\dfrac{d^{\chi-1}}{d\gamma_k^{\chi-1}} P_{-1/2+i\tau}^{M}(\cos\gamma_k)}{\dfrac{d^{\chi-1}}{d\gamma_k^{\chi-1}} P_{-1/2+i\tau}^{M}(-\cos\gamma_k)} \frac{\dfrac{d^{\chi-1}}{d\gamma_j^{\chi-1}} P_{-1/2+i\tau}^{M}(-\cos\gamma_j)}{\dfrac{d^{\chi-1}}{d\gamma_j^{\chi-1}} P_{-1/2+i\tau}^{M}(\cos\gamma_j)},$$

$$A_{i\tau} = \frac{\pi(\tau^2+1/4)}{ch\pi\tau} P^{-1}_{-1/2+i\tau}(\cos\gamma_2) P^{-1}_{-1/2+i\tau}(-\cos\gamma_2)\left[1 - C^{(1),-1}_{i\tau}(\gamma_1,\gamma_2)\right],$$

$$\varpi_{i\tau} = -\frac{1}{k}\sqrt{\frac{\pi}{2k}} e^{i\frac{\pi}{4}} \tau th\pi\tau \cdot e^{-\frac{\pi\tau}{2}}.$$

The similar expressions for $\upsilon_1^{(1),\Sigma_{1,2}}$ are valid at $\gamma_1 < \theta < \gamma_2$. The limit passing to (11) at $W_1 \to 0$, that corresponds to the transformation of the semitransparent structure into the solid cone Σ_2, leads to the potential for the solid cone Σ_2 [10]. The magnetic potential $\upsilon_1^{(2)}$ is not under influence of slots and one is similar to the case of the solid cone. Tending γ_1 to zero (the internal cone Σ_1 vanishes) in the expressions (10) and (11), we get the Debye potential $\upsilon_1^{(1)}$ for the single semitransparent cone Σ_2:

$$\upsilon_1^{(1)} = \cos\varphi \int_0^{+\infty} \varpi_{i\tau} \frac{H^{(1)}_{i\tau}(k\tau)}{\sqrt{r}} \frac{\tau(\tau^2+1/4)}{\sigma_{i\tau}} P^{-1}_{-1/2+i\tau}(\cos\gamma_2) P^{-1}_{-1/2+i\tau}(-\cos\gamma_2) \times$$
$$\times P^{-1}_{-1/2+i\tau}(\cos\theta)d\tau - \frac{i}{k^2}\frac{\cos\varphi}{1+2W_1} tg\frac{\theta}{2}\frac{\sin kr}{r},\ 0<\theta<\gamma_2,$$

$$\upsilon_1^{(1)} = \cos\varphi \int_0^{+\infty} \varpi_{i\tau} \frac{H^{(1)}_{i\tau}(k\tau)}{\sqrt{r}} \frac{\tau(\tau^2+1/4)}{\sigma_{i\tau}} \left[P^{-1}_{-1/2+i\tau}(\cos\gamma_2)\right]^2 P^{-1}_{-1/2+i\tau}(-\cos\theta)d\tau -$$

$$-\frac{i}{k^2}\frac{\cos\varphi}{1+2W_1} tg^2\frac{\gamma_2}{2} ctg\frac{\theta}{2}\frac{\sin kr}{r},\ \gamma_2<\theta<\pi,$$

$$\sigma_{i\tau} = (\tau^2+1/4) P^{-1}_{-1/2+i\tau}(\cos\gamma_2) P^{-1}_{-1/2+i\tau}(-\cos\gamma_2) + \frac{2}{\pi} W_1 ch\pi\tau.$$

In the case of the single semitransparent cone (9) the magnetic potential for the diffracted field equals zero. It follows that the diffracted field

(a) is determined by only one potential $\upsilon_1^{(1)}$ in the case of the semitransparent cone. For this reason it has not a radial component of the magnetic field and is a TM-field;

(b) $N>>1,\ d_2/l<<1, W_1=0$. In this case the semitransparent cone is determined by the existence of the limit

$$W_1 = \lim_{\substack{N\to\infty \\ d_2/l\to 1}} \left[-\frac{1}{N} \ln \sin \frac{\pi d_2}{2l} \right] > 0 \tag{12}$$

and the presence of heterogeneities in the form of a great number of narrow slots has an influence only on the Debye potential $\upsilon_1^{(1)}$, structure of which can have a form

$$\upsilon_1^{(2)} = \upsilon_1^{(2),sol\Sigma_2} + \tilde{\upsilon}_1^{(2),\Sigma_{1,2}}, \tag{13}$$

where $\upsilon_1^{(2),sol\Sigma_2}$ is the magnetic Debye potential for the solid cone Σ_2 [10], the item $\tilde{\upsilon}_1^{(2),\Sigma_{1,2}}$ corresponds to the interaction of fields as a result of the presence of the cones Σ_1 and Σ_2. In this case

$$\tilde{\upsilon}_1^{(2),\Sigma_{1,2}} = -\frac{1}{w}\sin\varphi \int_0^{+\infty} \frac{H_{i\tau}^{(1)}(k\tau)}{\sqrt{r}} \hat{A}_{i\tau}\Omega_{i\tau}^{(2),-1} P_{-1/2+i\tau}^{-1}(-\cos\theta)d\tau -$$
$$-\frac{i}{2wW_2}\Omega(\gamma_1,\gamma_2,W_2)\sin\varphi ctg\frac{\theta}{2}\frac{\sin kr}{k^2 r},\ \gamma_2<\theta<\pi, \tag{14}$$

$$\Omega_{i\tau}^{(2),-1}(\gamma_1,\gamma_2,W_2) = \varpi_{i\tau}\frac{1-C_{i\tau}^{(2),-1}(\gamma_1,\gamma_2)}{\hat{A}_{i\tau}+2W_2}\frac{\frac{d}{d\gamma_2}P_{-1/2+i\tau}^{-1}(\cos\gamma_2)}{\frac{d}{d\gamma_2}P_{-1/2+i\tau}^{-1}(-\cos\gamma_2)},$$

$$\Omega(\gamma_1,\gamma_2,W_2) = \frac{tg^2(\gamma_2/2)-tg^2(\gamma_1/2)}{1-tg^2(\gamma_1/2)/tg^2(\gamma_2/2)+1/2W_2},$$

$$\hat{A}_{i\tau} = -\frac{ch\pi\tau}{\pi\sin^2\gamma_2(\tau^2+1/4)}\times$$
$$\times\frac{1}{\frac{d}{d\gamma_2}P_{-1/2+i\tau}^{-1}(\cos\gamma_2)\frac{d}{d\gamma_2}P_{-1/2+i\tau}^{-1}(-\cos\gamma_2)}\frac{1}{1-C_{i\tau}^{(2),-1}(\gamma_1,\gamma_2)}.$$

The similar presentation takes place for $0<\theta<\gamma_2$ as well. The expression for $\upsilon_1^{(2)}$ is convenient for taking into account the so-called correction field to the field of the solid cone Σ_2 at the expense of the heterogeneities in the form of a great number of narrow longitudinal slots and the presence of the cone Σ_1. To take the correction to the field of the solid cone Σ_1 into consideration due to the presence of the semitransparent cone Σ_2 and the interaction of the fields, it is convenient to use the following presentation:

$$\upsilon_1^{(2)}=\upsilon_1^{(2),sol\Sigma_1}+\upsilon_1^{(2),\Sigma_{1,2}} \tag{15}$$

where $\upsilon_1^{(2),sol\Sigma_1}$ is the magnetic Debye potential for the solid cone Σ_1 and

$$\upsilon_1^{(2),\Sigma_{1,2}}=\frac{2}{w}W_2\sin\varphi\int_0^{+\infty}\frac{H_{i\tau}^{(1)}(k\tau)}{\sqrt{r}}\Omega_{i\tau}^{(2),-1}P_{-1/2+i\tau}^{-1}(-\cos\theta)d\tau+$$
$$+\frac{i}{w}\Omega\left(1-\frac{tg^2(\gamma_1/2)}{tg^2(\gamma_2/2)}\right)\sin\varphi\cdot ctg\frac{\theta}{2}\frac{\sin kr}{k^2r},\ \gamma_2<\theta<\pi. \tag{16}$$

In this case the electric Debye potential of the semitransparent cone is not affected by the slots and it is the same as in the solid cone $\upsilon_1^{(1)}=\upsilon_1^{(1),sol\Sigma_2}$. Proceeding to limit in (14) and (16) at $W_2\to+\infty$ (the cone Σ_2 disappears) leads to the presentation for the magnetic Debye potential in the case of the solid cone. As a result of the limit in the expressions (15) and (16) at $\gamma_1\to 0$, we get the expression for the magnetic Debye potential for the single semitransparent cone Σ_2, which is determined by the existence of the limit (12).

THE NUMERICAL SOLUTION. THE FIELD IN THE WAVE ZONE AND NEARBY THE VERTEX

The analysis for the field distribution in the wave zone, where the diffracted field represents only the outgoing wave, has been carried out on the basis of the obtained numerical solution of the set (7) by the reduction method. The diffracted field can be represented as the sum of the field mirror-reflected from the cone surface and the field stipulated by the presence of the vertex. In the spatial region determined by the inequality $2\gamma_2<\theta$ the field of the mirror reflection is absent and the distribution of the diffracted field is characterized by only the field scattered by the common vertex of the conical structure Σ. The use of the asymptotic form of the potential (6) at $kr >> 1$ as well as the presentations for the field components (4) and the numerical solution (7) allows us to study numerically the distribution of the diffracted field in the wave zone. Figure 2 shows the field distribution diagrams in the wave zone ($\theta > 2\gamma_2$) in the presence of the single cone Σ_2 with one longitudinal slot (the axis of the slot coincides with the ray, $\varphi = 0^{\circ}$) $(\gamma_2 = \pi / 8)$ with the internal shield Σ_1 $(\gamma_1 = \pi / 10)$ and without it $(\gamma_1 = 0)$. The analysis of the diagrams has shown that we can correct the form of a diagram, changing the width of the slots and the opening angles of the cones. With the help of the single cone with a longitudinal slot, it is possible to obtain the spatial field distribution, diagram of which has a form of cardioid at the angular width of the slot, varying from 120° to 210°, whereupon it is shaped into ellipse. Inserting the solid cone in the internal domain of the cone with a longitudinal slot has an influence on the field distribution diagram in the wave zone and as a result the so-called hole being opposite the slot disappears. With broadening the slot, the influence of the internal solid shield reveals more and more and the diagram takes a form of ellipse.

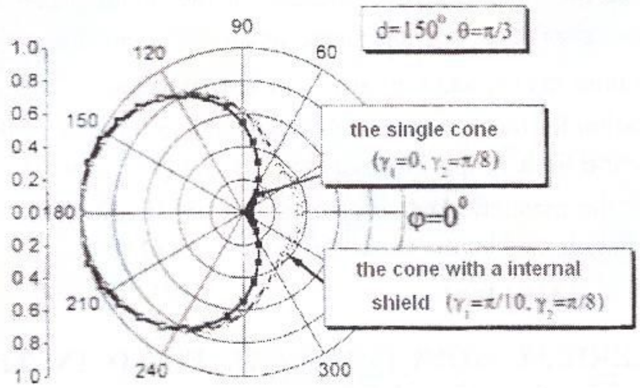

FIGURE 2.

The electric field near the vertex $(kr << 1)$ of the open-ended conical surface asymptotically behave in the following way:

$$\left|\vec{E}\right| \sim \left|kr\right|^{-1+\alpha},$$

where $\alpha = -1/2 + \zeta_0^{(1)}$, $\zeta_0^{(1)}$ is the least spectral value of the first boundary-value problem for the Debye electric potential $\upsilon^{(1)}$. In a similar manner, the magnetic field at the vertex of the conical surface is asymptotically determined in such a way:

$$\left|\vec{H}\right| \sim \left|kr\right|^{-1+\beta},$$

where $\beta = -1/2 + \zeta_0^{(2)}$, $\zeta_0^{(2)}$ is the least spectral value of the second boundary-value problem for the Debye magnetic potential $\upsilon^{(2)}$.

Thus, Figs. 3 and 4 show the curves of the dependence for the parameters α and β versus the width of the gap d_2^0, which describe the degree of field singularity for the particular case of the structure under consideration: the plane ($\gamma_2 = \pi/2$) with a longitudinal widened gap (Fig. 3) and the plane with a widened gap and a solid cone located above it $\gamma_1 = \pi/8$ (Fig. 4).

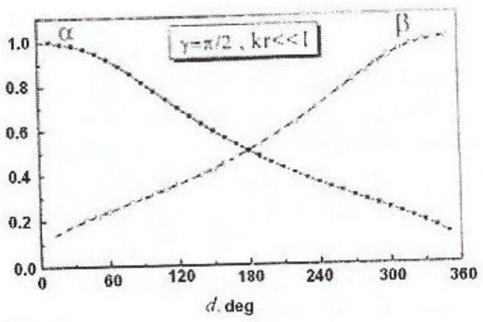

FIGURE 3.

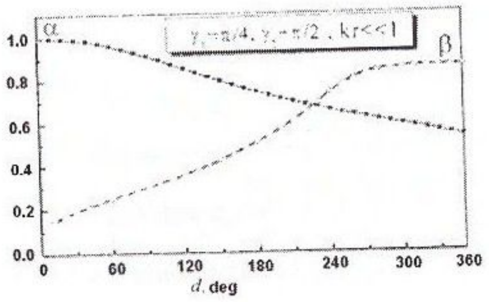

FIGURE 4.

Based on the obtained graphs we can conclude that the presence of the solid cone inside the cone with a gap, decreases the singularity of the electric field at the vertex and has a slight influence on the behavior of the magnetic field. When $d_2 = 180^{\circ}$ the plane with a cut transforms into a half-plane and the field singularities E and H get the known root singularity $|kr|^{-1/2}$ at the edge of the half-plane [5]. This fact is evidence of the validity of the obtained results.

CONCLUSIONS

In the present paper we have studied the three-dimensional problem of diffraction of plane electromagnetic waves on a compound open-ended conical structure which consists of two cones with longitudinal gaps. To solve this problem, we have developed the rigorous numerical-analytical method based on the use of the Kontorovich-Lebedev integral transformation in conjunction with the semi-inversion method. As a result of this, the original problem is reduced to the solution of a set of the Fredholm-type algebraic equations of second kind. In the case of a semitransparent cone with a bulk internal shield, we have obtained the analytical solution which allows us to study in a qualitative sense the influence of the heterogeneities formed by longitudinal gaps and an unbroken insertion, on the main scattering signatures as well as to investigate the spectrum of the boundary-value electrodynamic problem and the field structure. In terms of the numerical solution the influence of a longitudinal gap on the spatial forming of the field has been analyzed. It is shown that with the help of a cone having a longitudinal gap we can obtain a diagram of the spatial field distribution which has a form of cardioid. The analysis of the behavior of the field at the vertex of the conical surface testifies that the possibility exists of controlling the field singularity at the vertex by means of changing geometrical parameters of the structure under consideration. The obtained results can be useful in the design and development of scanning antennae and devices of measuring engineering.

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