FORECAST PROPERTIES ANALYSIS OF BRAUN'S MODEL IN EXTENDED AREA OF INNER PARAMETER

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Among the key functions of systems for controlling social and economic processes are forecasting and process planning. Proper usage of predictive models, a clear understanding of their internal workings, and a knowledge of the limits of model adequacy are the necessary conditions for quality and well-grounded managerial decisions, and consequently, for effective management as a whole.

R. Brown suggested his predictive model or exponential smoothing model in the late '50s of the last century. His concept was to use the exponential average value of a stationary time series:

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + ... + \alpha(1-\alpha)^{n-1} A_{t-n} = \sum_{i=1}^{n} \alpha(1-\alpha)^{i-1} A_{t-i}, \quad (1)$$

for short-term forecasts, where F_t is forecast at point of time t (exponential mean), $A_{t-1}, A_{t-2}, ..., A_{t-n}$ are series values at respective time points, n is time series length, α is smoothing factor (a constant).

Practical application of Brown's model requires solving the model parametric setting problem, i.e. substantiate the choice of smoothing factor α. Many publications have dealt with the problem of choosing this Brown's model factor; however, to date there is no single approach to this.

The classical range of admissible values of the smoothing factor is the interval $\alpha \in [0, 1]$. This range is logically conditioned by the necessity to ensure convergence of the series of weight coefficients in formula (1)

$$\{a_k\}_{k=1}^{n} = \alpha, \ \alpha(1-\alpha), \ ..., \ \alpha(1-\alpha)^{n-1} \quad (2)$$

to unit.

At the turn of the Millennium, S.G. Svetun'kov in his studies demonstrated that the classical range $\alpha \in [0, 1]$ could be extended to $\alpha \in [0, 2]$ without violating the condition of convergence of weight coefficients series (2) to unit. In this case, series (2) changes from a fixed-sign one in the interval $\alpha \in [0, 1]$ to a variable-sign one in the interval $\alpha \in (1, 2]$.

Let set K_c be a classical admissible set, set K_{out} be an out-of-limit admissible set, and set $K_{ext} = K_c \cup K_{out}$ be an extended admissible set of smoothing factor α:

$$K_c = \{\alpha: \ 0 \le \alpha \le 1\}, \ K_{out} = \{\alpha: \ 1 < \alpha \le 2\}, \ K_{ext} = \{\alpha: \ 0 \le \alpha \le 2\}. \quad (3)$$

Let us investigate the behaviour of the sum of series (2) with an increasing number of its terms n on extended set K_{ext} of smoothing factor α:

$$S_n = 1-(1-\alpha)^n. \tag{4}$$

Fig. 1 shows dependence $S_n(\alpha, n)$ according to (4).

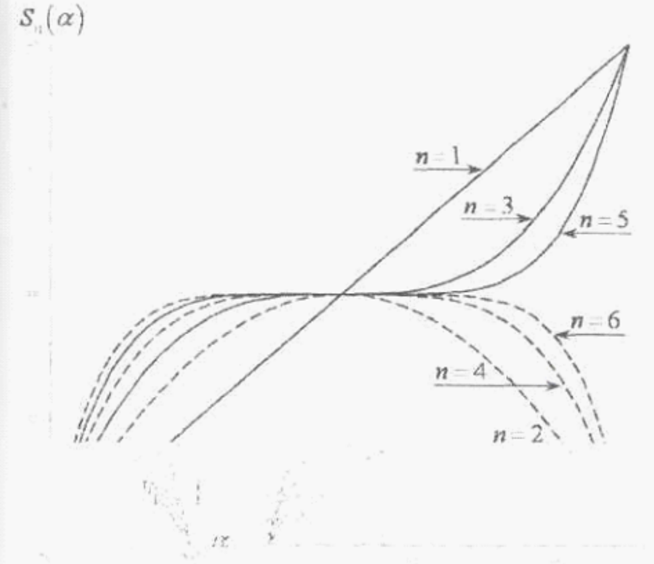

Fig. 1 – Sum of series of Brown's model weight coefficients vs. smoothing factor α and number of series elements n on extended set K_{ext}

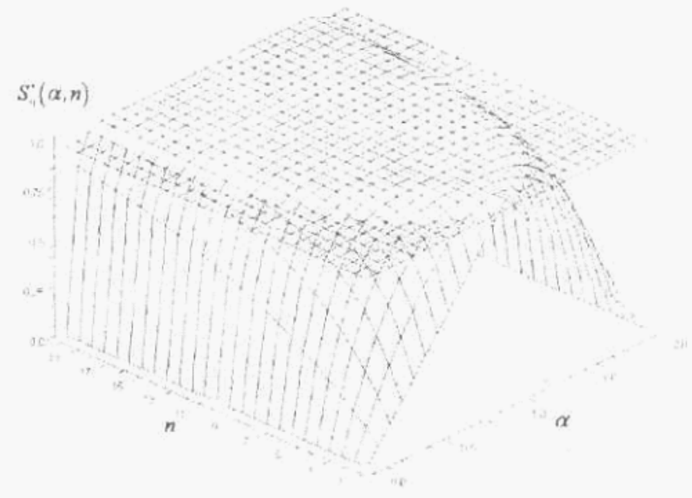

Fig. 2 – Transformed sum of Brown's model weight coefficients vs. smoothing factor α and number of series elements n on extended set K_{ext}

From Fig. 1, it is obvious that the sum of coefficients in (1) is not equal to unit in all cases. This means that Brown's model uses strictly speaking not the exponential average as a forecast, but the exponential weighted value of the initial series.

The closeness of the forecast to the exponential average can be evaluated analytically. For this, let us transform dependence (4) by mirror imaging a group of growing branches with respect to a unit level.

Fig. 2 shows dependence $S'_n = 1-|1-\alpha|^n$. Fig. 2, besides showing dependence $S'_n(\alpha,n)$, shows a plane at level $1-0.01\lambda = 0.95$, where λ is measure of closeness to the exponential average value. It intercepts the domain of parameters in plane (α,n), within which the predictive value is close to the exponential average one by less than λ percent. The boundaries of this domain can be found from relationship (Fig. 3).

$$1-(0.01\lambda)^{1/n} \le \alpha \le 1+(0.01\lambda)^{1/n} \quad . \tag{5}$$

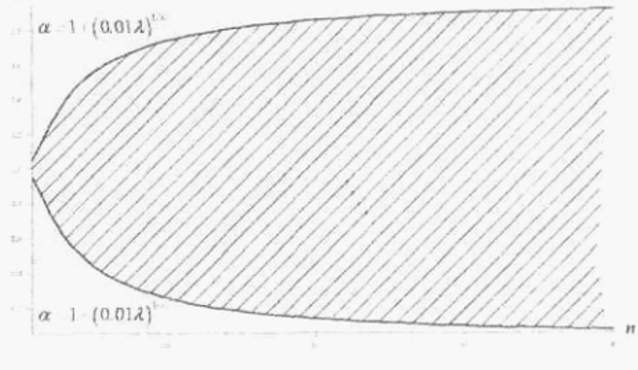

Fig. 3 – Domain in plane of factors (α,n) ensuring closeness of Brown's model forecast to the average exponential value of n series elements by less than λ percent ($\lambda = 5$)