Does a Diocotron Effect Exist in Magnetrons?

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***Abstract*—In this article magnetron model has been developed for explanation why the electron beam had wavelike form before oscilation stage. As well known in crossed-field devices electrons move by cycloid-like trajectories. In accordance with the diocotron effect electron beam form has wavelike form because interaction between RF wave and electron beam. But at preoscillation stage we had the same electron beam form in magnetron both planar and cylindrical. Such form we explained by space periodical distribution of an electrostatic potential distribution.**

Keywords*—*magnetron, electrostatic potential distribution, electron beam, wavelike form, diocotron effect.

I. INTRODUCTION

The crossed-field devices such as magnetron diode, magnetron, magnetron amplifier etc. were used in industry, science, medicine and domestic [1].

A magnetron is one of the first and the most wide propagating microwave oscillators. Electrons move in such oscillators as magnetrons occur under the action of crossed static electrical and magnetic fields and interact with RF electromagnetic fields [2].

The dominant interaction in such a crossed-field device is the wave particle interaction (diocotron). This interaction drives the classical Brillouin flow nonlinearly unstable in the presence of high-frequency (RF) waves, propagating in the slow-wave structures that this devices contain [3].

These devices have been analyzed with PIC (particle in cell) codes and guiding-centre theory [3].

Once the above operating density profile has been achieved, the RF wave grows until it saturates. Now it can no longer be considered to be infinitesimal, and, owing to nonlinear terms in the cold-fluid equations, the main RF wave will beat against any other RF waves (noise) in the divece, which will then genegate all possible harmonics and beat waves. To study this, we need, and shall develop here, the small-signal response for these noise modes, their dispersion characteristics, their modal profiles, and their energy characteristics. We then allow these waves to beat against the main RF wave (the pump), and determine their nonlinear growth characteristics [3].

Thus form of an electron beam has wavelike one.

But when we have space periodical electrostatic field the form of electron beam formed as wavelike.

This article purpose is to show how formed sinelike electron beam without diocotron effect.

II. BASIC EQUATIONS

Here we considered two configuration of magnetron systems: planar and cylindrical.

A. Motion equation for planar crossed-field devices

The geometry and configuration of a ‘planar magnetron’ diagram is shown in Fig. 1. A planar geometry is preferred for analytical treatment because of its simplicity. At the bottom is the cathode, which is an electron-emitting matirial. It is located in cartesian coordinates (x, y)-plane and at $z = 0$. We take the normal magnetic field to be along the z axis, so that the electron drift velocity will be in the positive x direction. The electron frequency is $\Omega = \dfrac{eB_0}{mc}$, where B_0 is the magnetic field. The anode is located at the distance d above the cathode. On the anode is a slow-wave structure (magnetron), which is simply a waveguide, along which the RF wave will propagate. If the anode is plain such device is magnetron diode.

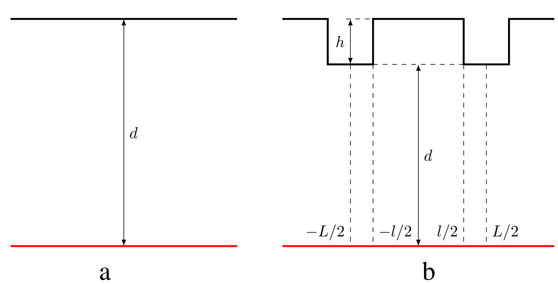

Fig. 1. Interaction space. a – planar magnetron diode; b – planar magnetron

In general case electron motion in cartesian coordinate may be wrote as

$$\frac{d^2x}{dt^2} = \eta E_x + \omega_H \frac{dy}{dt} \quad (1)$$

$$\frac{d^2y}{dt^2} = \eta E_y + \omega_H \frac{dx}{dt} \quad (2)$$

with initial conditions

$$x(0) = 0$$

$$\left.\frac{dx}{dt}\right|_{t=0}$$

$$y(0) = 0$$

$$\left.\frac{dy}{dt}\right|_{t=0}$$

For planar magnetron diode these equations take on the appearance

$$\frac{d^2x}{dt^2} = \omega_H \frac{dy}{dt} \tag{3}$$

$$\frac{d^2y}{dt^2} = \frac{\eta U_a}{d} + \omega_H \frac{dx}{dt} \tag{4}$$

From (3) we obtained

$$\frac{dx}{dt} = \omega_H y + C \tag{5}$$

Using initial conditions we obtained an indefinite constant $C : C = 0$.

Substituting expression (5) into equation (4) we obtained such differential equation

$$\frac{d^2y}{dt^2} = \frac{\eta U_a}{d} + \omega_H^2 y$$

Using initial conditions we obtained this equation solution

$$y = \frac{\eta U_a}{\omega_H^2}(1 - \cos \omega_H t) \tag{6}$$

Substituting expression (6) into equation (5) we obtained

$$x = \frac{\eta U_a}{\omega_H^2}(\omega_H t - \sin \omega_H t).$$

Thus we obtained

$$\begin{cases} x = \dfrac{\eta U_a}{\omega_H^2}(\omega_H t - \sin \omega_H t) \\ y = \dfrac{\eta U_a}{\omega_H^2}(1 - \cos \omega_H t) \end{cases}$$

Electron trajectories in planar magnetron diode are cycloides. It is well known fact.

In planar magnerton electrostatic potential distribution has such expression

$$u(x,y) = A_0 \left(y - 2h \sum_{n=1}^{\infty} A_n \cos \frac{2\pi n x}{L} \sinh \frac{ny}{D} \right)$$

this implies that electrical field density are [4]

$$E_x = \frac{4\pi h}{L} A_0 \sum_{n=1}^{\infty} n A_n \sin \frac{2\pi n x}{L} \sinh \frac{ny}{D}$$
$$E_y = A_0 \left(1 - \frac{4h}{D} \sum_{n=1}^{\infty} n A_n \cos \frac{2\pi n x}{L} \cosh \frac{ny}{D} \right),$$

where

$$A_0 = \frac{U_a}{D + \dfrac{lh}{L}}$$

$$A_n = \frac{sin \dfrac{\pi n l}{L}}{\left(\dfrac{\pi n l}{L} + \sin \dfrac{2\pi n l}{L} \right) \left(\sinh \dfrac{n(h+D)}{D} - \sinh n \right) + \pi \sinh n}.$$

B. Motion equation for cylindrical crossed-field devices

Real such devices are usually cylindrical.

Here we described two forms of cylindrical crossed-field systems: the simplest form — cylindrical magnetron diode (fig. 2a) and cylindrical magnetron (fig. 2b). These contructions consist of two coaxial cylinders (magnetron diode) and instead of the outer cylinder placed electrode with complex configuration (magnetron).

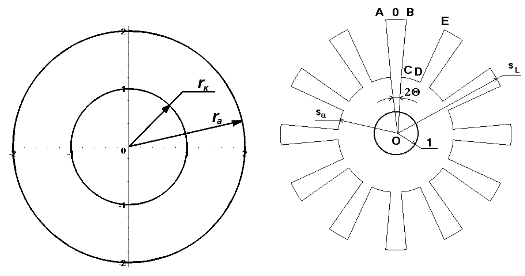

Fig. 2. Interaction space. a – magnetron diode; b – magnetron

The motion equations of charged particles in crossed-field to cylindrical constructions were described in polar coordinates (s, φ). It is Cauchy problem defined system equations [5]

In polar coordinates (s, φ) the motion equations is Cauchy problem and described such equations

$$\frac{d^2s}{dt^2} - s\left(\frac{d\varphi}{dt}\right)^2 = \eta \left(E_s + Bs\frac{d\varphi}{dt} \right) \tag{7}$$

$$s\frac{d^2\varphi}{dt^2} + 2\frac{ds}{dt}\frac{d\varphi}{dt} = \eta \left(E_\varphi - B\frac{ds}{dt} \right), \tag{8}$$

where s — dimensionless radius r/r_c;

$\eta = 1.76 \cdot 10^{11} C/kg$ — the specific charge of electron;

B — axial magnetic field density;

E_s — electrostatic field density along the radial coordinate;

E_φ — electrostatic field density in azimuthal coordinate.

with initial conditions

$$s(0) = 1; \quad \left.\frac{ds}{dt}\right|_{t=0} = 0; \tag{9}$$

$$\varphi(0) = 0; \quad \left.\frac{d\varphi}{dt}\right|_{t=0} = 0. \tag{10}$$

In magetron diode (fig. 2a) cathode radius is equal 1, anode radius denoted as s_a which used to solve Laplace equation.

Electrostatic field density in magnetron diode was defined as

$$E_s = \frac{\eta}{r_k^2 \omega_H^2} \frac{U_a}{s \ln s_a} \tag{11}$$
$$E_\phi = 0.$$

The above mentioned system (7) and (8) for a magnetron diode was transformed to

$$\frac{d^2 s}{dt^2} = -\frac{s}{4} + \frac{b}{s} + \frac{1}{s^3} \tag{12}$$

$$\frac{d\varphi}{dt} = \frac{1}{2}\left(1 - \frac{1}{s^2}\right), \tag{13}$$

where $b = \dfrac{\eta U_a}{\omega_H^2 \ln s_a}$

In magetron (fig. 2b) cathode radius is equal 1, anode radius denoted as s_a, wane radius denoted as s_L, 2θ is resonator central angle, $ABCDE0$ are points which used to solve Laplace equation.

A magnetron electrostatic field density from (7) and (8) was defined as $\vec{E} = -gradU$.

An electrostatic field distribution for complex electrodes' configuration was described in [6], [7] and shown in (14)

$$U(s,\varphi) = \frac{U_a}{\ln s_a + \frac{N\theta}{\pi}\ln\frac{s_L}{s_a}} \left(\ln s - 2\ln\frac{s_L}{s_a}\sum_{n=1}^{\infty} a_n sirs^{nN}\cos nN\varphi\right), \tag{14}$$

where

$a_n = \dfrac{\sin nN\theta}{(nN\theta + \sin 2nN\theta)(sirs_L^{nN} - sirs_a^{nN}) + \pi sirs_a^{nN}}$;

$sirx = \dfrac{x + x^{-1}}{2}$.

From (14) we can obtain the coordinate components of electrostatic field density

$$E_s = \frac{A}{s}\left(1 - 2N\ln\frac{s_L}{s_a}\sum_{n=1}^{\infty} na_n cors^{Nn}\cos Nn\varphi\right)$$

$$E_\varphi = -\frac{2AN}{s}\ln\frac{s_L}{s_a}\sum_{n=1}^{\infty} na_n sirs^{Nn}\sin Nn\varphi$$

where $A = \dfrac{\eta U_a}{\left(\frac{N\theta}{\pi}\ln\frac{s_L}{s_a} + \ln s_a\right) r_c^2 \omega_H^2}$.

The above mentioned system (7) and (8) for a magnetron was transformed to

$$\frac{d^2 s}{dt^2} + \left(1 - \frac{d\varphi}{dt}\right)\frac{d\varphi}{dt}s = E_s \tag{15}$$

$$\frac{d^2\varphi}{dt^2} + \frac{1}{s}\frac{ds}{dt}\left(2\frac{d\varphi}{dt} - 1\right) = E_\varphi. \tag{16}$$

III. Results

Electron motion trajectory was described by (3) and (4) was shown in fig. 3.

Unfortunately for equation system (1), (2), (12), (13), (15) and (16) we can't obtained analytical solution. The solution obtained by numerical fourth order Runge-Kutta method.

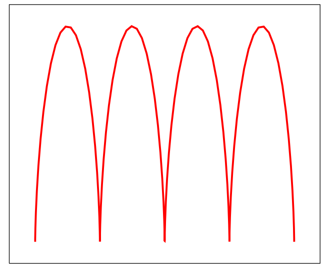

Fig. 3. Electron motion trajectory in planar magnetron diode

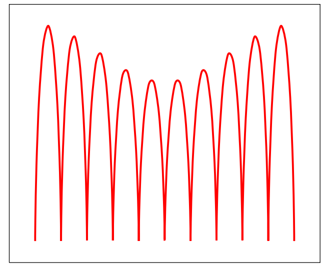

Fig. 4. Electron motion trajectory in planar magnetron

Motion trajectories obtained by numerical method for planar magnerton was shown in fig. 4.

Motion trajectories obtained by numerical method for cylindrical magnerton diode was shown in fig. 5.

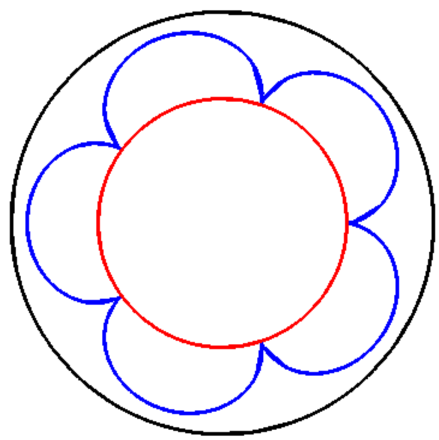

Fig. 5. Electron motion trajectory in cylindrical magnetron diode

These trajectories were not epicycloid.

Motion trajectories obtained by numerical method for cylindrical magnerton was shown in fig. 6.

IV. Discussion

Comparising images in fig. 3 and 4 we an see during when we accounted field distribution that inherent slow-wave structure (fig. 3b), detect presence of modulation charged particles beam. It looks like a manifestation of diocotron effect [8] and yields to issential influence on system evalution.

Concerning cylindrical constructions they repeated those planar conditions but a manifestation of diocotron effect is

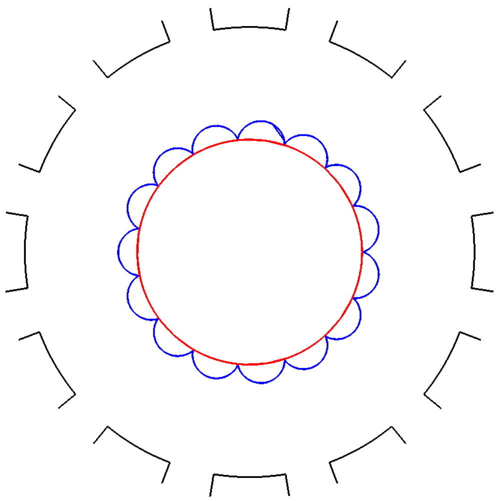

Fig. 6. Electron motion trajectory in cylindrical magnetron diode

less noticeable (fig. 5 and 6).

V. Conclusion

Proposed magnetron model has been explaned why the electron beam had wavelike form before oscilation stage. As well known in crossed-field devices electrons move by cycloid-like trajectories. It was compared motions' trajectories of four constructions crossed-field devices. In planar magnerton diode trajectories transform to rectilinear electron beam. In planar magnerton trajectories transform to wavelike form due to space-periodical electrostatic field distribution. In cylidrical magetron diode trajectories transform to annular electron beam. In cylidrical magnerton trajectories transform to wavelike form due to space-periodical electrostatic field distribution. Although this transformation is less noticeable than in planar magnerton.

Thus space-periodical electrostatic field distribution lead to a manifestation of diocotron effect at pre-oscillation stage in crossed-field devices.

References

[1] O. Nikitenko, V. Volovenko "Changed particles motion in crossed-field system under dissipation" International Conference Modern Problems of radio Engineering, Telecommunications and Computer Science. Proceeding of the International Conference TCSET 2002. Lviv Polytechnic National University. February 18-23, 2002. Lviv-Slavske, Ukraine, 2002. P. 59-60.

[2] O. Nikitenko "Has a magnetron abnormal current" The Fourth International Conference on Information and Telecommunication Technologies and Radio Electronics (UkrMiCo'2019) Odesa doi: 10.1109/UkrMiCo47782.2019.9165516.

[3] D.J. Kaup, J.O. El-Reedy, G.E. Thomas "Parametric interactions inside a magnetron" Journal Plasma Physics, vol. 64, part 4, 2000, pp. 489 – 506

[4] I. Moshchenko, O. Nikitenko, N. Shtefan "Why an anode current in magnetron iss not abnormal" The Fifth International Conference on Information and Telecommunication Technologies and Radio Electronics (UkrMiCo'2021) Kyiv, 2021, pp. 267270

[5] T. Krokhmal, O. Nikitenko "Feature of charged particles trajectories in crossed electrical and magnetic fields" Scientific Works of the Faculty of Physics and Mathematics of Donbas State Pedagogical Uuniversity, 2020 11, pp. 59 72 https://doi.org/10.31865/2413-26672415-3079112021234838 in Ukrainian

[6] O.M. Nikitenko "Distribution of electrostatic potential in crossedfield system with complex electrodes' configuration," Journal of Microwaves and Optoelectronics, Vol. 2, No. 2, December 2000, pp. 1 9

[7] A.G. Shein, S.S. Bulantsev "Calculation of electrostatic field distribution in planar comb system," News of VolgSTU No 3(51), 2009, pp. 44 48

[8] D.J. Kaup, G.E. Thomas "Chaotic instabilities and density profiles in a crossedfield electron vacuum devices" SPIE, 2000, pp. 54 64.