The layout problem is a part of computational geometry that has rich applications in garment industry, sheet metal cutting, furniture making, shoe manufacturing, glass industry shipbuilding industry, etc. The common task in these areas is to arrange a set of shapes of specified shapes and sizes within a given sheet (strip) of material (textile, wood, metal, glass etc.). To minimize waste one wants to arrange shapes as close to each other as possible.

The problems are NP-hard, and as a result solution methodologies predominantly utilize heuristics and nearly all practical algorithms deal with shapes which are approximated by polygons (see tutorial [1] and references therein). The most popular and most frequently cited tool in the modern literature on the Cutting and Packing is the No-Fit Polygon, it is designed to work for polygonal objects without rotations. A notable exception being [2], which allows circular shapes, but they cannot be freely rotated. Tools of packing of rotated polygons is considered in [3]. In [4] an extended local search algorithm (ELS) for the irregular strip packing problem is discussed. Objects are approximated by polygons and can be free rotated. The local search algorithm is used to minimize the overlap. The tabu search algorithm is used to avoid local minima, and a compact algorithm is presented to improve the result.

Due to the extreme complexity of the analytical description of the relationship between geometric objects, bounded by circular arcs and lines segments, only a few papers devoted to placement of arbitrary shaped objects.

We present the layout problems in a formal mathematical manner. In the paper we deal with objects of very general shape and we characterize their arrangements by means of special phi-functions. As a convex domain $\Omega$ we consider a nonempty intersection of finite number of convex polygons and circles (in particular: a rectangle, a convex polygon and a circle).

The concept the phi-functions is a highly convenient for practical solution of the layout problem. In particular, we take advantage of phi-functions [5] to develop more efficient algorithms.

Our principal goal is to present here a generator of mathematical models of layout problems using the phi-function technique and demonstrate practical benefits of our algorithms.

We consider layout problems in the following basic formulation. Basic layout problem (BLP). Place a set of objects $T_i$, $i \in \{1, 2, ..., N\} = I_N$ within a convex domain $\Omega$ of variable sizes, so that the given restrictions on the placement of the objects are
fulfilled and the area of $\Omega$ reaches the minimal value. We assume, that each object $T_i$ is two-dimensional phi-object, bounded by line segments and circular arcs. We allow here free rotations and translations of objects. The restrictions include: containment of objects into a container, non-overlapping of objects, given minimal allowable distances between objects, prohibited areas, rotation constraints, and other specific technological restrictions (e.g. a given allowable ranges of rotation angles). A multiplicity of shapes of objects as well as a variety of restrictions creates a wide spectrum of subsequent problems of the basic layout problem. Our intention is to present each of the subsequent problems as a nonlinear programming problem. To this aim we provide a generation of a solution space for the class of problems based on phi-functions technique. We provide an exact NLP-model and develop the mathematical model generator. In order to solve the problem, we propose the algorithm which works very fast and uses multistart method for a set of feasible starting points. For each starting point we apply special algorithm to search for locally optimal solutions. The algorithm involves of the following procedures:

1) generation of a number of starting points from feasible region of the BLP problem, employing the starting point algorithm. The algorithm also allows us to fill holes of composed objects by smaller objects;

2) search for a local minimum of the BLP problem based on our solution tree technique and employing the algorithm of Local Optimisation Reduction Algorithm for each starting point;

3) choice of the best of local minima obtained at the second step as an approximation to the global solution of the BLP problem.

We develop special solver for layout problems which uses the core representation of inequalities in a symbol form and provides exact calculation of Jacobian and Hessian matrixes.

We give a number of examples to demonstrate the effectiveness of our methodology for rectangular domain. For local optimisation in our programs we use IPOPT, which is available at an open access noncommercial software depository (https://projects.coin-or.org/Ipopt). The comparison was carried out with the results given in [2]. The results have been improved.