Signal flow graph models and alternative gain formula for multiprobe microwave multimeter

The article was shown and proved that the construction of the model in the form of signal flow graph for gain determination does not necessarily use the rule of non touching loop (the Mason rule) for defining gain from generator to sensors, and it can be successfully replaced by the matrix computation.

Key words: multiprobe microwave multimeter, signal flow graph, non touching loop rule, gain.

Introduction

The article refers to the microwave measurement technique domain. When formulating the problem as a contradiction, it is difficult to set aside one of them. For example, there are contradiction: 1) the measurement technique develops slower that just technique; 2) power measurement accuracy at many times worse than the frequency measurement accuracy due to the mismatch error; 3) improvement at the expense of hardware has reached the limit, but the processing improvement has reserves; 4) which one analytical or graph-analytical models of microwave devices is better still is question; 5) naming of the same thing are different in foreign western (six-port reflectometers) and domestic(microwave multimeter or analyzer) publications; 6) role of the computer in measurement is main or dependent; 7) quadratic or linear models of sensors of microwave multimeter is more appropriate; 8) the rigorous mathematical models or engineering practical simplification; 9) electrodynamics or microwave circuit theory for model building; 10) traditional or innovative approach; 11) choose as signal flow graph reduction method: algebraic or topology or matrix; 12) different calculation amount for small or large number of sensors, 13) discrete and continuous presentation of microwave measurement system for defining of modulus and phase of termination reflection coefficient and passing, incident, reflected power, with improved efficiency and accuracy, optimize this information-measuring systems, its metrological providing. Among the practical applications, for example, during microwave heating and drying, measurement results are used for dynamic matching of microwave tract termination.

Analysis of recent researches and publications that taken as basis in the author research shows signal flow graph representation as topological model of multiprobe microwave multimeter still widely used in modern publications [1 – 6] started a long time ago in 1953 (signal flow graph and non touching loop rule of Mason) [7 – 8]. Since than it stays almost unchangeable, because signal flow graph model is very fit. It consists of cascade connection of part that corresponds to slightly modified scattering matrix of sensors entwined with parts that corresponds to waveguide that located between sensors, when it is necessary to research inner connection between sensors like in case of reflection between neighbor sensors. If unnecessary to research inner connection microwave block with sensors whole is represented by a six-port signal flow graph. We consider first of this two models in the article. It has advantages such as simpler representation than electrodynamics representation, it can be easily upgrade if add more sensors. But the method of calculation of gain unlike the model itself have disadvantages. For example, the bulkiness of non touching loop formula grows as number of sensors grows, authors [8] point on 800 loops for six-port reflectometer, i.e. four sensors. Other gap is that loops that have order above the second are excludes from engineering calculations which may be source of uncertainty.

Recent publications is relating mainly to automation of calculation [9 – 11], because of great amount of computation. From where one can take a good idea of system representation through reverse transition from signal flow graph to matrix representation for computer input.

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Highlight of outstanding aspects of the problem, which the article is dedicated. From analysis sees necessity of creating of new method for gain computation which is alternative to non touching loop rule.

So objectives are development of the theoretical basis and signal flow graph model, and creating new method for signal flow graph gain computation. Than simulating this new method to find the weaknesses and shortcomings, for what define the criteria for comparing (like amount of calculation and accuracy), compare new method with known non touching loop method and to identify constraints, show the conditions under which the method is applicable and gives good results in comparison with analogues, and where method should be applied carefully or in combination with other methods or not applied at all.

**Signal flow graph and non touching loop rule**

A signal flow graph is a network of directed branches which connect at nodes. Branch \( jk \) originates at node \( j \) and terminates upon node \( k \), the direction from \( j \) to \( k \) being indicated by an arrowhead on the branch. Each branch \( jk \) has associated with it a quantity called the branch gain \( g_{jk} \).

The various node signals are related by the associated equations (1). The graph shown in fig.1 has

\[
\begin{align*}
ax_1 + dx_1 &= x_2 \\
b x_2 + f x_3 &= x_3 \\
ec x_3 + cx_4 &= x_4 \\
g x_4 + hx_5 &= x_5 \\
\end{align*}
\]

We shall need certain definitions. A source is a node having only outgoing branches (node 1 in Fig. 3). A sink is a node having only incoming branches. A path is any continuous succession of branches traversed in the indicated branch directions. A forward path is a path from source to sink along which no node is encountered more than once (abch, aeh, aefg, Fig. 1).

A feedback loop is a path that forms a closed cycle along which each node is encountered once per cycle (bd, cf, def, but not bcfd, in Fig. 1). A path gain is the product of the branch gains along that path. The loop gain of a feedback loop is the product of the gains of the branches forming that loop. The gain of a flow graph is the signal appearing at the sink per unit signal applied at the source. Only one source and one sink need be considered, since sources are superposable and sinks are independent of each other.

The form suggests that we call \( \Delta \) the determinant of the graph (3), and call \( \Delta_k \) cofactor of forward path \( k \) [7].

![Fig. 1. A simple signal flow graph](image)

The general expression for graph gain may be written as

\[
G = \frac{\sum G_k \Delta_k}{\Delta},
\]

where \( G_k \) – gain of the \( k \)th forward pass,

\[
\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} + \sum_m P_{m3};
\]

\( P_{mr} \) – gain product of \( r \) non touching loops;

\( \Delta_k \) – the value of \( \Delta \) for that part of the graph not touching the \( k \)th forward path.

For signal flow graph fig.1 we get gain

\[
x_5 = \frac{abch + aeh + aefg + abg}{1 - bd - cf - def} \cdot x_1.
\]

We propose to leave model as mentioned forward only write them down as matrix

\[
\begin{bmatrix}
\frac{1}{a} & -\frac{d}{a} & 0 & 0 \\
b & -\frac{a}{f} & 0 & 0 \\
e & c & -1 & 0 \\
g & h & -1 & 0
\end{bmatrix}
\begin{bmatrix}
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
= \begin{bmatrix}
x_1 \\
0 \\
0 \\
0
\end{bmatrix}
\]

To express \( x_5 \) through \( x_1 \) formally calculate inverse system matrix in MathCad.
As it could be seen gain is

\[-(b \cdot c \cdot h + b \cdot g + e_1 \cdot h + e_1 \cdot f \cdot g) \cdot \frac{a}{(-1 + c \cdot f + b \cdot d + e_1 \cdot d \cdot f)}\]

The gain expression is identical to results used with non touching loop rule. So we can use our method in the multiprobe microwave multimeter signal computation.

Model and alternative gain finding for multiprobe microwave multimeter

The first approach is application of non touching loop rule. Result according to fig. 2 is

\[x_5 = \frac{e \cdot y \cdot (1 - e^2 \cdot p \cdot \Gamma_n) + e^2 \cdot t \cdot y}{1 - e^2 \cdot p \cdot \Gamma - e^2 \cdot p \cdot \Gamma_n - e^4 \cdot t^2 \cdot \Gamma \cdot \Gamma_n}, \quad (6)\]

Where \(x_i\) is the value of the signal at the nodal points, nodal points 2 and 5 are sensors itself, \(y\) – transformation coefficient from microwave input to low frequency output, \(p\) – reflection between sensors coefficient, which is the ratio between the power output of the previous sensor and the microwave input of the following sensor, \(t\) – transfer coefficient between the input microwave and a microwave output of the unique sensor, \(e\) – transmit ion coefficient of waveguide part between sensors \(e = e^{-i\theta}\), \(\theta\) – phase distance between sensors, \(\theta = \frac{2\pi}{\lambda_w}\); \(1\) – physical distance between sensors, \(\lambda_w\) – wavelength in the waveguide, \(\Gamma\) – generator reflection coefficient, \(\Gamma_n\) – termination reflection coefficient, \(P_{inc}\) – incident power. The coefficients \(y, p, t\) is the slightly transformed elements of the scattering matrix.

![Fig. 2. Signal flow graph of microwave multimeter with one sensor](image)

The second approach consists in elaboration of matrix (7) of microwave unit with one sensor on the basis of same fig. 2.

\[\begin{align*}
x_1 &= x_2 \Gamma e^2 + P_{inc} e \\
x_2 &= x_1 p + x_3 t \\
x_3 &= x_4 e^2 \Gamma_n^2 \\
x_4 &= x_1 t + x_4 p \\
x_5 &= x_1 y + x_3 y
\end{align*}\]

Matrix form of equation system (7) is

\[\begin{bmatrix}
\frac{1}{\Gamma} - \Gamma e & 0 & 0 & 0 & x_1 \\
p & 1 & - t & 0 & 0 & x_2 \\
0 & 0 & 1 & - e^2 \Gamma_n & 0 & x_3 \\
t & 0 & - p & 1 & 0 & x_4 \\
y & 0 & - y & 0 & 1 & x_5
\end{bmatrix} = \begin{bmatrix}
P_{inc} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}. \quad (8)\]

As calculation in Mathcad in symbolic form impossible. There are conditions of numerical calculation

\[-\frac{\pi i}{y} \quad y := 1 \quad e_1 := e^3 \quad \Gamma := 0.01 \quad \Gamma_n := 0.2 \quad p := 0.01t := 0.5\]

Results of first approach (6) mainly coincide with results of second approach (8). This is first traditional approach (6) results

\[x_5 = \frac{e_1 \cdot y \cdot (1 - e_1^2 \cdot p \cdot \Gamma_n) + e_1^3 \cdot t \cdot y}{1 - e_1^2 \cdot p \cdot \Gamma - e_1^2 \cdot p \cdot \Gamma_n - e_1^4 \cdot t^2 \cdot \Gamma \cdot \Gamma_n}
\]

\[x_5 = -0.398 - 0.864i\]

According to proposed second approach (8) it is necessary to solve system of equation, which can fulfill by different methods. Our choice is calculation through inverse matrix.

Then sensor signal multiple on complex conjugate as real sensor have quadratic function of transformation. In first approach (6) sensor signal equal 0.905, in second approach (8) is 0.851 what shows that results have uncertainty which one is more precise is the aim of future research.
Continuing on case of two sensor on the microwave multimeter one gets such expression (fig.4), i.e. known method of non touching loops gives

\[
x_2 = \frac{e \cdot y \cdot (l - p^2 - e^2 - p \cdot \Gamma_n \cdot e^4 - p \cdot \Gamma_n \cdot e^4 \cdot t^2) + e^2 \cdot t^4 \cdot \Gamma_n \cdot y + e^2 \cdot t \cdot p \cdot (l - e^2 - p) \cdot \Gamma_n \cdot y}{(l - \Gamma \cdot e^2 - p - e^2 \cdot p - e^2 \cdot \Gamma_n - p - \Gamma_n \cdot e^4 \cdot t^2 \cdot p - \Gamma - e^4 \cdot t^2 \cdot \Gamma_n)},
\]

(9)

\[
x_5 = \frac{e^2 \cdot t \cdot y \cdot (l - p \cdot e^2 \cdot \Gamma_n) + e^2 \cdot t^2 \cdot \Gamma_n \cdot y}{(l - \Gamma \cdot e^2 - p - e^2 \cdot p - e^2 \cdot \Gamma_n - p - \Gamma_n \cdot e^4 \cdot t^2 \cdot p - \Gamma - e^4 \cdot t^2 \cdot \Gamma_n)}.
\]

(10)

The matrix below shows the matrix representation corresponding to fig. 4 as proposed in the article.
Modelling in the MathCad two sensor microwave block also shows similarity of results from expression (9), (10) with results from equation system (11) for $x_2$ and $x_5$ what corresponds sensor signals in (11) but with uncertainty too.

**Conclusion**

As signal flow graph model of multiprobe microwave multimeter sensors block includes sensor mutual reflection, it allows to consider influence of sensor number on the accuracy. In the future it is planned to analyze results in the wide range of parameter variance. Now it is possible to state that our approach much faster than traditional. Its accuracy is sufficient.

Number of sensors can be more than two. In this case difficulties arise because of limitation on matrix size in the MathCad to 10x10 dimension. Like prospect is considered difficulties arise because of limitation on matrix size in the range of parameter variance. Now it is possible to state that our approach much faster than traditional. Its accuracy is sufficient.

One more advantage of proposed model is that it allows to account on sensor own reflection and simultaneously is less clumsy than non touching loops rule. In the future, the proposed model can be used to optimize the number of sensors in the microwave multiprobe multimeter at the stage of its design.

**REFERENCES**

ІНФОРМАЦІЙНО-КЕРУЮЧІ СИСТЕМИ НА ЗАЛІЗНИЧНОМУ ТРАНСПОРТІ

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