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On the construction of covering shapes for the analysis of economic dynamics

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Abstract: Getting more information from the available statistical data involves the use of different methods and approaches to the processing of such data. This is especially important from the point of view of weakly formalized processes, phenomena and objects, which include data on economic dynamics. At the same time an important element of extracting additional information about the studied processes, phenomena or objects is their visualization. This suggests the application feasibility of the visual analytics methodology, where the construction of covering figures on the set of points that describe processes, phenomena or objects of economic dynamics should be pointed out. Based on this, the paper suggests the procedure for the construction of the covering shape as a rectangle. Possible application methods for the proposed procedure in the analysis of economic dynamics tasks are described.

Keywords: analysis, covering shapes, data visualization, economic dynamics, the regression line, phase portrait, visual analytics.

Introduction:

Presentation of the analyzed data as visual images and applying to such newly formed structures of special purpose processing procedures have been recently widely used in various fields of research. Moreover, among these fields of study the processes of economic dynamics hold special place. This is foremost due to the fact that the study of the processes of economic dynamics refers to weakly formalized problems due to the presence of a number of subjective factors that affect the possibility of constructing adequate formalized models.

At the same time the constant variability of processes describing different aspects of economic dynamics, isolated usage of the existing mathematical tool for constructing adequate models and the necessity to extract additional knowledge from the existing data, determines the appropriateness to consider innovative approaches and techniques to obtain acceptable results in the study of various economic processes and phenomena. In this aspect, in particular, the work relating to the development of new approaches to statistical analysis of banks operation (such as Dobrovolskaya and Lyashenko, 2013; Sahoo and Mandal, 2011) should be noted, where positional parameters of the studied banks regarding the effectiveness of various banking operations are considered as visual images. In the works of Rey, Murray and Anselin (2011), Krempel and Plümper (2003) income volume spatial and time changes that correspond to different regional clusters are considered as visual images that reflect certain aspects of economic dynamics.

In general, the visualization of the processes of economic dynamics relates to such line of research as visual analytics (Savikhin, Maciejewski and Ebert, 2008). The main objective of this line of research in the analysis of economic dynamics is to justify the adoption of the most appropriate management decisions. However, the appropriate use of visual analytics as a tool for the analysis of economic dynamics and decision-making provides for the necessity to choose the method of visual data representation of economic dynamics, and at the same time to choose the method of further processing of such visualization. This is what explains the choice of the subject matter in this study.

Review of Literature:

Usually, for analysis of economic dynamics time-series data is used that allows not only indentifying the specific tendencies, but also exploring the nature of changes inherent in the analyzed dynamic processes. An example of this kind of research can be the works of North (1994), Hamilton (1990), Korotayev and Tsire (2010), Gali and Gertler (1999). In particular, the mentioned papers discuss the feasibility and availability of using different data analysis tools presented in the form of time series, in order to study such trends in the study of economic dynamics as analysis of:

- general equilibrium between supply and demand (North, 1994) ;
- changes of price ranges in the cost of resources (Hamilton, 1990) ;
- cycles of economic development (Korotayev and Tsire, 2010);
- inflations and inertia on decisions made (Gali and Gertler, 1999).

To study economic dynamics chaos theory methods are also used. In particular the use of phase analysis methodology should be noted that allows taking into account the inequality in the development of economic processes in the relationship between separate time intervals for the data under study (Rosser, 1999).

It should be noted that each of the above mentioned tools for economic dynamics analysis is based on the specific time series value in the certain point of time. In other words, the description and formalization of economic dynamics can be based on the existing set of points that define such dynamics over time. Proceeding from this fact, the methodology of constructing covering shapes on a group of points characterizing such dynamics should also be considered as one of the tools for economic dynamics analysis.

Construction of covering shaes allows introducing new features for the analysis of economic dynamics, which are based on the analysis of the geometric properties of the constructed covering shapes and can be used for further interpretation of economic dynamics.

At the same time, for example, Křivánek (1990) studies the application of the methodology of covering shapes construction with the purpose to visualize hierarchical clustering of objects of different nature. Cheng, Yan and Han (2008) ground the importance of considering methodology of covering shapes construction in geographic information systems and computer graphics. Huebner, Ruthotto and Kragic (2008) suggest the appropriateness of using methodology of covering shapes construction in robotics.

The methodology for constructing covering shapes on the set of given points is based on (Křivánek, 1990; Post, 1981; Papadias and Theodoridis, 1997):

- analysis of topology structure formed by a plurality of analyzed pixels;
- solution of a certain optimization task, as a rule, based on the condition of minimizing the area of covering shape or minimizing the error in defining the area of covering shape when constructing it.

Time required for construction of covering shapes is considered important in some applications that focus on real-time in the use of methodology for constructing covering shapes (Cheng, Yan and Han, 2008; Huebner, Ruthotto and Kragic, 2008).

The most complete description of the methodology of covering shapes can be found, for example, in the following papers:

- O'Rourke, Aggarwal, Maddila and Baldwin, M. (1986), which grounds the possibility of constructing minimum covering shapes for the number of predefined points with the triangle is the covering shape.While further building of covering minimum triangle is the basis for constructing a minimum covering rectangle shape;
- Aggarwal, Chang and Yap (1985), that suggests an algorithm for constructing minimum covering triangles taking into account the minimization of the time spent on the construction of such shapes;
- Silverman and Titterington (1980), Moshtagh (2005), which studies algorithms for constructing minimum covering ellipses.

Thus, the geometry of constructing covering shape can be different. The selection of the form for constructing covering shape is mostly determined by the specific of the problem under study (Edelsbrunner, Kirkpatrick and Seidel, 1983).

Formalization of the decision on the construction of covering shapes on the set of given points is regarded primarily as a convex programming problem (Post, 1981). At the same time, based on the spatial topology of the given points, the construction of covering shapes can be based on solving the task of the nearest neighbor search for undefined initial search point (Hjaltson and Samet, 1995). Nevertheless, it should be noted that the selection of the method for constructing covering shape is also largely determined by the specifics of the problem of the overall study. At the same time, as noted by Alt, Aichholzer and Rote (1994), the construction of the covering shape may be based on the selection of the reference points, the topology of the set of these points, which allows taking into account the overall specific character of the original task solved.

Thus, all the above mentioned determines in the end the significance of the suggested research subject and the necessity to study it in two directions:

- grounding the method or approach to obtain initial points with the purpose of constructing covering shape;
- developing an algorithm to construct covering shape taking into account the specific character of the task of economic dynamics analysis.

Grounding the approach to obtain initial points with the purpose of constructing covering shape:
As noted earlier, it is also appropriate to apply the methodology of phase portraits referring to the methods of nonlinear dynamics along with the traditional forms of studying the processes as time series in the tasks of economic dynamics analysis. Using these methods allows first of all obtaining the basic characteristics of the functioning dynamics of the economic system or object being studied, determining the presence of cyclic behavior in their evolutionary development.This is important when studying the topology and relationship between the points obtained taking into account the relationship between separate time intervals for the input data under study.

At the same time, the application of nonlinear dynamics methods based on the methodology of the phase portraits contributes to both the identification of determinated chaotic component in the dynamics of the statistical data and to the definition of irregularities in the development of economic processes, and, hence, can be formalized basis for taking management decisions.

If we study discrete indicators in the task of economic dynamics analysis for some research process, phenomenon or object, then in the phase space of dimension 2 using grid coordinates, the phase portrait of the relevant statistical data series in its simplest form is defined as the set of points (Kuzemin, Lyashenko and Korovyakovskii, 2013):

$$\Phi(CHR) = \{(g(r_i), g(r_{i+1}))\}, i = \overline{1, t-1}, \qquad (1)$$

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where CHR – some statistics, which corresponds to a process, phenomenon or object under study, that is summarized in the form of certain parameters (for example, in terms of bank it is the volume of loans, raised deposits, interest accrued, etc.; in terms of the insurance market - volumes of assets, equity capital, insurance premiums, etc.);

r_i – immediate values of CHR statistics that are defined by different parameters of the test process, phenomenon or object in the task of economic dynamics analysis (volumes of loans, raised deposits, interest accrued, equity capital, insurance premiums, etc.) at specified time intervals $i = \overline{1,t}$, where t - the length of the required series CHR;

$g(...)$ – comparison function between the starting points of statistical data, which ultimately determines $\Phi(...)$. In its simplest form, this function compares between neighbor points of the original series of statistics;

$\Phi(...)$ – functional dependence of the conversion of the statistical data series CHR into the phase portrait

More complex construction of phase portrait for analyzing the tasks of economic dynamics is taking account of the first and second derivative of the original statistical data series (Kuzemin, Lyashenko and Korovyakovskii, 2013). For example, if there is a certain number of statistical values describing the economic dynamics of the process, phenomenon or object under study in regular time intervals, then its first and second derivatives can be found by the following formulas:

$$g'(q_j) = g(r_{i+1}) - g(r_i), \qquad (2)$$

$$g''(d_c) = g'(q_{j+1}) - g'(q_j), \qquad (3)$$

where $g'(q_j)$ – series values of the first-order derivative from basic series $g(r_i)$, $j = \overline{1,t-1}$;

$g''(d_c)$ – series values of the first-order derivative from basic series $g(r_i)$, $c = \overline{1,t-2}$.

Then the required phase portrait is defined as a set of points:

$$\tilde{\Phi}(CHR) = \{(g'(q_j)), (q''(d_c))\}, \; j = c = \overline{1,t-2}. \qquad (4)$$

Fig. 1, as an example, presents the time series and phase portrait for some of the studied process of economic dynamics, where the corresponding transition is determined taking into account the model of constructing the phase portrait of the formula (1).

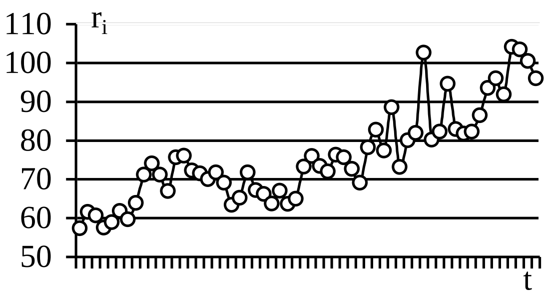

Fig. 1a. Time series

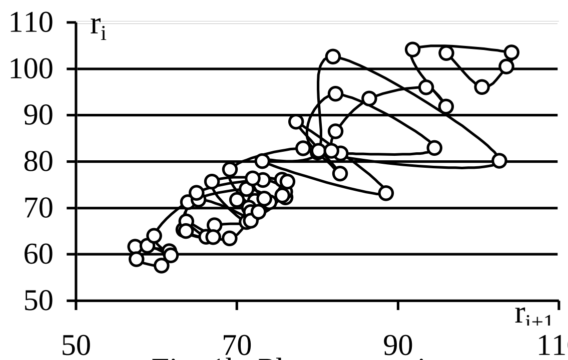

Fig. 1b. Phase portrait

Constructing covering shape taking into account the specific character of the task of economic dynamics analysis:

With the purpose to formalize the general procedure for constructing covering shape on the set of given points (in the space of dimension 2), reflecting the essence of the studied economic dynamics, we will represented the formed phase portrait in the form of a set M of points $\{(x, y)\}$. Then the task is to construct covering shape for the set of points M where x - corresponds to the values of the phase portrait r_{i+1}, and y - corresponds to the values of the phase portrait r_i .

Covering the problem of economic dynamics analysis, special attention should be paid to the fact that it is important to consider the direction of development of the appropriate dynamics. At the same time, the direction of the economic dynamics development is retained from the time series to the phase portrait of the test process, phenomenon or object (see Fig. 1). Consequently, as a reference topology of the studied set of points of the economic dynamics being researched one can choose the development vector of such dynamics. In the formalized view such vector can be characterized by the line of regression, built on a set of points studied, reflecting the essence of the analyzed economic dynamics. In the simplest case, including the further construction of covering shapes for the set of points, a linear regression equation can be considered to construct the regression line. This allows constructing covering shape as a rectangle, based on which you can then construct, for example, covering shape in the form of an ellipse. At the same time such ellipse can be inscribed or circumscribed for the constructed covering rectangle based on additional conditions of the economic dynamics analysis tasks. Then the general construction procedure of rectangle covering shape on the defined set of points in dimensionality space 2 can be represented as the following sequence of steps.

First step. For the range M consisting of the points $\{(x, y)\}$ defined on the basis of constructing the phase portrait for the original time series CHR the regression line is determined on the basis of the corresponding linear regression equation:

$$y = kx + b, \qquad (5)$$

where k, b – the parameters of the regression line. The statistical evaluation of such regression line can be generally considered as statistical evaluations for constructing coverings shape as a rectangle.

As a result of construction of the regression line the set M is divided into two subsets. One of these subsets contains points that are above the line of the regression equation, the other subset contains points that are below the line of the regression equation.

Second step. Two subsets $M1$ and $M2$ are being defined, into which the set M is divided using the line of regression constructed on the basis of the regression equation by the formula (5). This dividing is based on the following condition:

$$\begin{cases} (x^*, y^*) \in M1, \; y^* > y = kx^* + b \\ (x^*, y^*) \in M2, \; y^* < y = kx^* + b \end{cases}, (x^*, y^*) \in M. \qquad (6)$$

Third step. Among the points of subsets $M1$ and $M2$ the points are defined which are furthest from the regression line. For this the regression equation for the formula (5) is represented as:

$$-kx + y - b = 0. \qquad (7)$$

Then, the distance from some point $(x^*, y^*) \in M1$ or $(x^*, y^*) \in M2$ to the line of regression is defined by the formula:

$$d = \left|A \cdot x^* + B \cdot y^* + C\right| / \sqrt{A^2 + B^2}\ , \qquad (8)$$

where $A = -k; B = 1; C = -b$.

Among the set of distance values for different points of subsets $M1$ and $M2$ we find the points for which the corresponding distance values d_1 (for $M1$ subset of points) and d_2 (for subset of points $M2$) are maximal. We get two (the so-called grid) points $(x_1^*, y_1^*) \in M1$ and $(x_2^*, y_2^*) \in M2$ which define end points for the construction of lines of the unknown covering rectangle parallel to the line of regression. The total value of d_1 and d_2 define one of the linear dimensions of the required covering rectangle.

Fourth step. We find all the projections of points from the set M to the line of regression. To do this, we drop a perpendicular from each point of M set with respect to the line of regression using the following relationship:

$$y - y^* = -\frac{1}{k} \cdot (x - x^*),\ (x^*, y^*) \in M\ . \qquad (9)$$

Hereafter, in order to find the projections of the points from M set on the line of regression, we find points of intersection of the corresponding perpendiculars and the line of regression based on the solution of the following system:

$$\begin{cases} y = kx + b \\ y - y^* = -\frac{1}{k} \cdot (x - x^*) \end{cases},\ (x^*, y^*) \in M\ . \qquad (10)$$

As a result, we get some set N consisting of the points that are projections of the points from M set on the line of regression.

Fifth step. We find (the so-called grid) points with minimum and maximum values of x coordinates from N set of points. Points found in this way are two end points to construct the required covering rectangle for M set of points. Therefore, another linear dimension of the unknown covering rectangle can be found using the following formula:

$$b = \sqrt{(x_{\max} - x_{\min})^2 + (y_{\max} - y_{\min})^2},\ (x_{\min}, y_{\min}) \in N, (x_{\max}, y_{\max}) \in N\ . \qquad (11)$$

It can be noted that the covering rectangle constructed based on the above suggested procedure on basis of regression line as reference topology of the studied set is the rectangle with minimum surface. Such a conclusion is primarily based on the procedure of constructing covering rectangle. In particular, it suggests the possibility of considering the variability of the chaotic component in the dynamics of the statistics under study based on the geometry of the covering shape. At the same time the variability of the chaotic component from the economic dynamics analysis can reflect the risks found in the processes, phenomena and objects under study.

Conclusions:

Thus, the paper not only proves the importance of using visual analytics methodology in the form of construction of covering shapes economic dynamics analysis tasks, but also suggests certain algorithm for constructing covering shapes as a rectangle. Basis for constructing covering shape as a rectangle for the subsequent analysis of the economic dynamics is determined by the solution of the problem of optimizing the selection of (grid) points from a set of predefined ones regarding to its reference topology. As a reference topology of the set of points to solve the tasks economic dynamics analysis it is suggested to use the line of regression, which is constructed on the basis of the linear regression equation.

It is also noted that the proposed procedure to construct covering shape of the given set of points suggests the possibility of transition to the construction of other covering shapes such as an ellipse.

In general, the suggested procedure to the construction of covering shape for the given set of points allows implementing visual analytics methodology for the tasks of economic dynamics analysis, as well as gives the ability to consider the variability of the chaotic component in the dynamics of the statistics under study based on the geometry of the covering shape.

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