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Abstract — Using of linearized systems describing the crossed–fields devices’ operation, taking into account space charge fields, is studied. It allowed simplifying significantly the processes study in crossed-fields devices. It was shown that one of spectrum components coincides with the fundamental mode of magnetron oscillations.

I. Introduction

Modern science deals with the analysis of complex phenomena and, simplifying them, later studies their rules. Now scientists’ attention is directed at complex systems.

Such systems consist of simple elements. Such systems’ properties are known and system behavior is not a sum of its components. Such behaviour is named as a cooperative or synergetic effect.

The wideband noise which exceeds flicker noise 5 – 6 times as much, presents in crossed-field devices. It was shown that one of spectrum components coincides with the fundamental mode of magnetron oscillations.

II. Main Part

We observed and analyzed motion equations for two types of crossed–field devices: a magnetron diode and a magnetron.

For magnetron diode a potential distribution is described by the following expression

\[ U(s) = U_a \ln \frac{s}{s_a}, \]

where \( U_a \) – anode potential; \( s_a = r_f r_c \). For magnetron a potential distribution is described by [5].

Using above mentioned expressions for potential distribution in crossed–field devices we had described motion equations for magnetron diode as

\[
\begin{align*}
\frac{d^2 s}{dt^2} + \left( 1 - \frac{d^2 \phi}{dt^2} \right) \frac{d \phi}{dt} &= \frac{\beta}{s} \left( 1 - 2 \ln \frac{s}{s_a} \sum_{n=1}^{\infty} a_n \cos 2n \phi \cos Nn \theta \right) \\
\frac{d^2 \phi}{dt^2} + \frac{d \phi}{dt} \left( \frac{d^2 \phi}{dt^2} \right) &= \frac{2 \beta}{s} \ln \frac{s}{s_a} \sum_{n=1}^{\infty} a_n \sin 2n \phi \sin Nn \theta
\end{align*}
\]

and for magnetron

\[
\begin{align*}
\frac{d^2 s}{dt^2} + \left( 1 - \frac{d^2 \phi}{dt^2} \right) \frac{d \phi}{dt} &= \frac{\beta}{s} \left( \frac{N n \theta}{\pi} \ln \frac{s}{s_a} \sin \frac{s}{s_a} + \ln s \right) \\
\frac{d^2 \phi}{dt^2} + \frac{d \phi}{dt} \left( \frac{d^2 \phi}{dt^2} \right) &= \frac{2 \beta}{s} \ln \frac{s}{s_a} \sum_{n=1}^{\infty} a_n \sin 2n \phi \sin Nn \theta
\end{align*}
\]

where

\[ \beta = \frac{\eta U_a}{\pi \ln s}, \]

\[ a_n = \frac{\sin N n \theta}{(N n \theta + \sin 2 N n \theta) \left( \sin s a \right) + \pi s a}. \]

Considering the relation \( r_s/r_c < 2 \) for modern magnetron-type devices, the dimensionless radius \( s \) can be represented as \( s = 1 + \lambda \).

In this case using the linearization method for motion equations (1) we obtain

\[ \frac{d^2 x}{dt^2} = b - (1 + b) x \]

and motion equations (2)
\[
\frac{d^2 x}{dt^2} = \gamma - \left(1 + \frac{\gamma}{\omega}\right)x \\
\frac{d\varphi}{dt} = \gamma \cdot (1 - \frac{\gamma}{\omega})x
\]

Taking into account the Brillouin flow we have the following form for \(b\):

\[
b = \frac{\eta}{(\omega_b r_c^3)} \left[ U_x - \frac{\omega_b r_c^3}{2h} \left( s_x + \frac{1}{9s_x^2} \right) \frac{5\omega_b r_c^3}{9h} \ln s_x \right] + \frac{\omega_b r_c^3}{2h} \left( 1 - \frac{1}{3s_x^2} \right)
\]

and for \(\gamma\):

\[
\gamma = \frac{\eta}{(\omega_b r_c^3)} \left[ U_x - \frac{\omega_b r_c^3}{2h} \left( s_x + \frac{1}{9s_x^2} \right) \frac{5\omega_b r_c^3}{9h} \ln s_x \right] + \frac{\omega_b r_c^3}{2h} \left( 1 - \frac{1}{3s_x^2} \right)
\]

Solutions of equations (3) can be found in analytical form. Thus, for magnetron diodes we have the following:

\[
x(t) = \frac{b}{1 + b} \left(1 - \cos \sqrt{1 + bt}\right)
\]
\[
\phi(t) = \frac{b}{1 + b} \left(t - \sin \sqrt{1 + bt} \div \sqrt{1 + bt}\right)
\]

These solutions show that the charged particle in crossed–fields in magnetron diodes takes part in two kinds of motion: oscillation motion in radial direction and rotation in azimuthal direction.

We have solutions of equations (4) for magnetrons:

\[
x(t) = \frac{\beta}{1 + \beta} \left(1 - \cos \sqrt{1 + \beta t}\right)
\]
\[
\phi(t) = \frac{\beta}{1 + \beta} \left(t - \sin \sqrt{1 + \beta t} \div \sqrt{1 + \beta t}\right)
\]

This solution is similar to the interpretation that obtained for magnetron diodes.

These solutions show that the charged particle in magnetron diodes in crossed fields takes part in two kinds of motion: oscillation motion in radial direction and rotation in azimuthal direction.

III. The Research Results

To estimate the excited oscillations we must build an “energetic” spectrum. Here we studied \(K_c\)-band magnetron.

Spectral components were shown on Fig. 1, where a magnetron diode is shown on Fig. 1a) and a magnetron on Fig. 1b).

On these figures we can see fundamental and second cycloid harmonics and an undefined component.

For magnetron diodes the second cycloid harmonic coincides with a fundamental mode of \(K_c\)-band magnetron.

For magnetron the undefined component coincides with the fundamental mode of \(K_c\)-band magnetron.

IV. Conclusions

It is shown that the use of the linearized approach for the study of charged particles’ motion in crossed electric and magnetic fields taking into account the Brillouin flow allowed simplifying the solution of such problems and estimating the rotational motion’s frequency.

Thus using the proposed approach we can analytically investigate behavior of nonlinear dynamical systems “a magnetron diode” and “a magnetron” and prove existence of oscillation and rotation types of motion. It will allow improving the theory of analytic investigation of crossed–field systems.

\[\text{Fig. 1. Energetic spectrum:} \quad a - \text{magnetron diode}; \quad b - \text{magnetron}\]

For magnetron diode model the second harmonic of cycloid frequency coincides with fundamental mode of \(K_c\)-band magnetron oscillations. For magnetron model the undefined component coincides with fundamental mode of \(K_c\)-band magnetron oscillations. Accuracy for magnetron diode is 0.5% and for magnetron is 2.3%.

In future such investigations will be made for magnetrons in other bands.

V. References


