SPECIFIC SUBSET EFFECTIVE OPTION IN TECHNOLOGY DESIGN DECISIONS

Annotation. The article deals with the theoretical aspects of effective allocation of subsets of the valid options sets in technology making design decisions. As a result of analysis of the current state of the problem revealed that due to the combinatorial nature of most tasks synthesis number of alternative solutions dramatically increases with the dimension of design problems. The vast majority of options is ineffective. They can be improved at the same time on all the quality parameters. This leads to the need to develop methods for the isolation procedures subsets of effective design solutions tailored to the features of the original sets, as the complexity of the requirements and the accuracy of the solution. To meet the challenges of various dimensions on convex and non-convex set of feasible options to choose the exact and approximate methods based on pair-wise analysis of the options, theorems Karlin and Gervmer. To reduce the time complexity problem solutions proposed methods of pre-allocate a plurality of approximate methods effective solutions “sector” and “segment”. According to the analysis method estimates the computational complexity as a function of the dimension of the original set of alternatives and the amount of local optimization criteria established that the selection of sets of effective solutions of approximate the original set of alternatives at high power always is appropriate. This can significantly reduce the complexity of solving the decision-making tasks without loss of effective alternatives. The analysis time complexity methods revealed that the most efficient for large-scale problems is to use a scheme based on a modified method “segment”. The results are recommended to be used in the procedures for multifactor solutions in the design and management systems. Their use will improve the degree of automation of processes.

Keywords: design technology; information technology; decision-making; the set of feasible solutions; optimization criterion; many compromises

Introduction

The effectiveness of man-made objects that are used in various spheres of human activity, is largely determined by the decisions taken in the course of their design [1-3]. The design process involves the iterative solution of a set of structural problems, topological, parametric, process optimization in the conditions of incomplete information for a variety of functional and cost indicators (performance criteria) [4-5]. Choosing the best solutions from a variety of effective only in the simplest cases can be the decision maker [6-10]. Because of the combinatorial nature of most tasks synthesis number of alternative solutions dramatically increases with the dimension of design problems. The vast majority of options is ineffective (dominated). Each of these options can be improved on the set of feasible solutions at the same time in all respects. There arises the problem of forming only efficient subset (unimprovable Pareto-optimal) design decisions constituting the plurality of compromises or selection of a subset on the created set of feasible embodiment [11-12]. In addition, for many contemporary design objects generated or selected subset of the effective options can be quite large, unsuitable for the final expert evaluation and selection. This leads to the need to reduce the set of effective options based on a programmed preference between quality indicators.

Literature review

As part of a systematic approach to the design of new equipment designed objects formalized representation of this process in the form of a logic of building design solution [13]:

$$T = \langle P, In, Res, DD, PD \rangle,$$  \hspace{1cm} (1)

where: $P = \{ P_i \}$, $i = 1, p$ – an ordered set of tasks (models) design; $In$ – a plurality of source data tasks; $Res$ – many limitations tasks; $DD$ – set of design decisions; $PD$ – display of the design procedure (method of solution), assigning to each pair $< In, Res >$ empty subset $< DD >$, $i = 1, p$.

From the viewpoint of information technology, each of the design challenges presented as input to the inverter output:

$$P_i : In_i \rightarrow Out_i, \ i = 1, p.$$  \hspace{1cm} (2)

The ordered set of problems in (1) is completely soluble, if for each of them there project procedures
The task of formalizing the goals of design automation systems in the simplest case reduces to the construction of the objective function $P(s)$ based on one or a plurality of indicators (local criteria) efficiency $k_i(s)$, $i=1,m$ taking real values on the set of alternatives $s \in S$. At the same time local criteria $k_i(s)$, $i=1,m$, usually, have different physical meaning, dimension, measurement interval, and are contradictory.

Problem of determining the universal set of alternatives $S^U$ it is based on the specificity of the original design goals.

The problem of determining the set of permissible alternatives $S \subseteq S^U$ it is to exclude from the universal set $S^U$ a subset of options $\tilde{S}$ not satisfying the constraints solved the problem of designing $S = S^U \setminus \tilde{S}$. It is required to determine the functional and cost performance options $s \in S^U$. The major means for estimating the local properties $k_i(s)$, $i=1,m$ options $s \in S^U$ are analytical and computer simulation. To obtain generalized assessments of the quality options $P(s)$ methods used expert and multivariate estimation based on local criteria of utility functions [2; 4; 17].

The problem of distinguishing subsets effective alternatives $S^E$ is excluded from the set of permissible $S$, dominated (suboptimal) alternatives belonging to the consent of the set $S^s$. The solution is called the effective $S^E \subseteq S$ (Pareto-optimal, best possible, non-dominated) if there is not a preferred solution $s \in S$, namely $s^o \succ s \ \forall s \in S$ [17].

Solution to the problem of ranking of alternatives is based on the paradigm of maximizing utility. To solve it, there are two approaches: ordinalist and cardinalistic [18]. When using ordinalistic approach ordering options made the decision maker. As part cardinalistic approach made the formation of a generalized criterion of efficiency and the reduction of the problem to solve optimization problems. It is assumed in both approaches, which each variant of the set of admissible $s \in S$ attributed some utility (value) $P(s)$ whose value is determined and the order [17]:

$$\forall s, v \in S : s \succ v \iff P(s) > P(v);$$

$$s \succ v \iff P(s) > P(v); \quad s \preceq v \iff P(s) \geq P(v).$$

Modern technology design of complex objects is iterative, involving alternating generation procedure for this analysis, select the best of them.

The essence of the problem of the decision seems logical statement “is required $s^o$” or formally $<s^o>$ (where $s^o$ – optimal design solution) [15]. In this case, for the problems are considered that the situation of decision-making $Sit$ determined accurately enough, since there is some uncertainty objectives and (or) the initial data (2). To go to the task of deciding the type $<sit, s^o>$ required decomposition of the problem and the solution of auxiliary problems of the form: “given $<sit, - >$ need $<sit, s^o>$” namely, $<sit, - >, <sit, s^o >=$ or “given $<s^o>$, need $<sit, s^o >=$”, namely $<sit, s^o >$.

Further detail of the decision-making task allows you to present it in the form of $<s, \Pi>$, where $S = \{s\}$ – a lot of options of design solutions (alternatives); $\Pi$ – principle of optimality [16]. As a solution to problems of the form $<s, \Pi>$ considered a subset $S^{\Pi} \subseteq S$, obtained based on the received principle of optimality $\Pi$. The optimality principle may be represented by a function selection $C^{\Pi}$, which compares subset $S^{\Pi} \subseteq S$ part $C^{\Pi}(S^{\Pi})$. Decision $S^{\Pi}$ the original problem is a lot of $C^{\Pi}(S)$.

Depending on the degree of certainty the situation of decision-making are distinguished: general problem (issue) decision $(S$ and $\Pi$ - unknown); task selection $(S$ known); general optimization problem $(S$ and $\Pi$ known).

The decision problems of the form $<s, \Pi>$ produced by forming a plurality of permissible alternatives $S$ with a further decision of the selection task. When forming a plurality of $S$ it assumed to be known a universal set of alternatives $S^U$. The task of forming a plurality of permissible alternatives $S$ considered as a problem of choice $<S^U, \Pi'>$ (where $\Pi'$ – principle of optimality) which expresses the conditions of admissibility of alternatives. The solution of the problem $S = C_{\Pi'}(S^U)$ it called the initial set of alternatives. In view of this problem decision can be reduced to solving the problems of two successive selections [16].

In the end, the decision-making process seems set of tasks [17]: the formalization of targets; determining a universal set of alternatives $S^U$; determining a plurality of acceptable alternatives $S \subseteq S^U$; allocating a subset of viable alternatives $S^E \subseteq S$; ranking alternatives $s \in S^E$; selecting the best alternative $s^o \in S^E$. 

The solution to the problem of forming $S^{\Pi}$ set of tasks 

$$PD_i, i = 1,p \ 	ext{and every solution is the only} \ [14]:$$

$$|PD_i(\langle In_i, Res_s >)| = 1, i = 1,p.$$  (3)
The task to choose the best alternative \( s^o \in S^E \) in the above specified conditions is reduced to externalization generalized utility functions:

\[
s^o = \arg \max_{s \in S^E} P(s),
\]

(4)

Choosing the best option \( s^o \in S^E \) it can be made the decision maker, or generated on the basis of the generalized criterion by solving the optimization problem of the form (4) methods of mathematical programming. In both cases it involves determining ratings \( k_i(s), i = \overline{1, m} \) all generated alternatives \( s \in S \) (where \( m \) – the number of local criteria evaluation and selection).

The vast majority of the known exact methods for solving design problems are non-polynomial time complexity. With their implementation requires the generation and analysis of a huge number of options \( n = Card(S) \). So the solution of problems of structural synthesis of the class centralized radial nodal structures using the brute force method of local extrema of objective analysis is required of the order \( Card(S) = 2^r \) options (where \( r \) – the number of components), a number of possible ring structures for nonsymmetrical matrices of values \( Card(S) = (r-1)! \) options. The problem arises of the generation and analysis only effective embodiments. In this version of the design solution \( S^E \in S^E \) it called the effective (non-dominated) if the set of admissible \( S \) there is no option \( s \in S \) for which would have the inequalities:

\[
k_i(s) \geq k_i(s^E), \text{ if } k_i(s) \to \max,
\]

(5)

\[
k_i(s) \leq k_i(s^E), \text{ if } k_i(s) \to \min,
\]

(6)

and at least one of them was strict.

Power subset effective radial nodal structures \( Card(S^E), S^E \subseteq S \) for \( r = 15 \div 40 \) it may range from a few percent to a few thousandths of a percent \( Card(S) \) [19].

Depending on the particular design problems using a variety of methods and algorithms of allocation of subsets of viable options \( S^E \subseteq S \hor{2} \): discrete choice gravimetric method [1; 3]; pairwise comparisons based on theorems Karlin and Germeyer [20]; evolutionary search based on genetic algorithms [21-23].

Methods discrete choice paired comparisons, and [3; 20] allow to correctly allocate subsets effective embodiments. However, due to high temporal complexity, these methods are applicable only to a relatively small set of feasible solutions.

Weighting methods, including methods based on theorems Germeyer and Karlin [1; 3; 20], have lower adjustable temporal complexity than accurate methods. However, they allow you to highlight incomplete subset \( S^E \). This method, built on the basis of theorem Karlin as the sector method is intended for convex set of feasible solutions [20]. By using genetic algorithms to solve problems multi objective optimization their effectiveness verified solution of two problems: the ability of the algorithm convergence to give Pareto optimal front (convergence problem) and to provide good distribution throughout the optimal solutions Pareto front (distribution problem).

One of the widely used solutions for the problems of formation of subsets of effective solutions (Pareto front) on ultra-large size admissible sets is a genetic algorithm with the nondominant sorting NSGA-II [24].

Its features include: a binary representation of the data can be used in conjunction with conventional genetic operators (crossing-point and point mutation); for continuous optimization problems with multiple objective functions it is recommended to use a realistic (decimal) representation of the data. The latter requires the use of specific genetic operators such as crossover and imitation binary polynomial mutation.

In [25] a method of reducing the number of target features based on the method of principal components. The basic idea is that if the two objective functions have a negative value of the correlation function, they are included in the conflict set and the data (matrix data) to analyze the Pareto front. To analyze this data set (goal functions) and its subsequent reduction using the method of principal components (eigenvectors of the correlation matrix). Choosing a higher eigenvalues vector components with the largest negative value, and with the largest positive value identifies two conflicting objective functions. Next, we study the eigenvectors that go beyond the senior eigenvector. Eigenvectors are selected such that their eigenvalues in the amount exceeding the threshold value. Then the idea to use this in any algorithm (for example, NSGA-II) procedures considered as an iterative process, and the resulting set of target functions reduce using correlation analysis. An iterative process stops when the current subset coincides with the subset that has developed in the previous iteration.

The disadvantage of this method is that it does not guarantee the preservation of the structure of domination.

The main drawback of evolutionary methods
implemented using genetic algorithms is the lack of checks or operators procedures that implement mechanisms to accelerate the convergence to the plurality of global optima. For example, the effectiveness of genetic algorithms NSGA-II/III family can be enhanced by including an operator implementing the method of principal components [25].

A review of publications on the issue of allocation of subsets of viable options in information technology making project decisions [26-27] shows that the existing mathematical models, methods, algorithms and procedures focused on specific types of permissible options sets are significantly different temporal complexity and accuracy of the solution.

The purpose of the article. Complex objects modern design technologies features show the accruing trend according to software universalization, dedicated to the identifying subsets of effective solutions task resolving. It allows you to create requirements for effective technology solutions of problems of formation and selection of subsets of non-dominated alternatives. With this in mind, the purpose of this article is to develop methods for the procedures for allocating subsets of effective design solutions tailored to the features of the original sets, as the complexity of the requirements and the accuracy of the solution.

Requirements for information technology

In the development of tools for allocation of subsets of viable options will take into account the characteristics of the design decision-making support technologies: close relationship problems formalizing objectives, definitions of the universal set of alternatives, the definition of reasonable alternatives, allocating a subset of efficient alternatives ranking alternatives, select the best alternative, which requires a common response; the combinatorial nature of the majority of its component tasks (subtasks); the need to solve large-scale problems; presence in the problem difficult to formalize factors; High dynamic uncertainty or source data; a wide range of conditions for solving problems.

Analysis of the above features of the problem allows us to formulate the requirements to be met by effective information decision support technologies:
– a close relationship and incomplete information certainty formalization purpose tasks, determining a universal set of alternatives, determine the set of feasible alternatives highlight subset effective alternatives, ranking and selection of the best alternative (1) of the initial data sets Ini and restrictions Re s, i = 1, p it causes the iterative nature of the methods and procedures to solve them. In this way, will be provided by the solubility of the complex decision-making tasks P = { Pi }, i = 1, p the inputs;
– high complexity exact solution methods (decision procedure) PD, i = 1, p (1) due to the combinatorial nature of certain tasks, and a wide range of conditions they require solutions in their decision to use multiple methods with different complexity and accuracy of the solution. This will ensure the solvability of the problems of acceptance of design decisions on resources;
– for the better use of experience of designers and accounting factors of Difficulty making processes of design decisions it is advisable to build on the interactive (man-machine) procedures. The process of finding the best solution in this case will consist of complementary automatic procedures and c involving intellectual synthesis system analysts and operators CASE-applying means and expert systems;
– at all stages it is advisable to use techniques that reduce the complexity of problem solving P = { Pi }, i = 1, p (1). For this purpose, they can be used various kinds of heuristics tailored tasks, solutions obtained by means of "quick” procedures, formal or expert estimates.

Methods for solving the problem. To solve the problem effective allocation of subsets of options within an adaptive design decision-making support technologies form the methods of the bank based on the convexity and non-convexity of the original sets, as the complexity of the requirements and the accuracy of the solution.

The problem of distinguishing subsets effective design options SE ⊆ S it is seen in the following formulation. Given a set of acceptable design solutions S = { s } each of which is defined by the values m local performance criteria k(s), i = 1, m. To be recovered from S = { s } a subset of alternatives SE ⊆ S for each of which the evaluation of local criteria do not satisfy the inequalities (5) and (6).

Paired comparison method. Combinatorial paired comparison method allows you to select subsets of viable options SE = { s } both convex and non-convex on the set of alternatives S. Its essence is as follows. Origin of the alternatives s ∈ S included in the set of effective SE. Each of the following options: veS is compared with each of the embodiments seSE (in the first step with a single one). If the next version veS best of each option SE at least one of the indicators k(s), i = 1, m, it is included in the SE. If some
variant $s \in S^E$ worse than the current version $v \in S$, it is excluded from $S^E$ and an option $v$ included in the subset $S^E$. After viewing all the alternatives $s \in S$ it will be allocated a subset of the effective options $S^E \subseteq S$. Thus the set of feasible design solutions will be divided into two disjoint subsets:

$$S = S^E \cup \bar{S}^E, \quad S^E \cap \bar{S}^E = \emptyset . \quad (7)$$

This method allows to obtain the exact solution of the problem, however, has a high time complexity.

**Method based on Karlin theorem.** A subset of the effective $S^E$ on a convex set of feasible alternatives $S$ based on theorem Karlin is by combining the embeddings $s^0_i$, $i = I, m$. Optimizing each of the local criteria $k_i(s)$, the decisions of parametric programming problem with respect to the parameters $28$:

$$\lambda_i \in \Lambda = \{ \lambda_i : \lambda_i > 0 \ \forall i = I, m, \sum_{i=1}^{m} \lambda_i = 1 \} . \quad (8)$$

$$s^0_i = \arg\max_{s \in S} \{ P(s) = \sum_{i=1}^{m} \lambda_i \xi_i(s) \} . \quad (9)$$

where $\xi_i(s)$, $i = I, m$ – normalized value or utility function $i$-th local criterion.

Usefulness (value) of the partial criteria values $k_i(s)$, $i = I, m$ are invited to express with their affiliation functions of fuzzy sets “best option”. These functions must satisfy a number of requirements $17$ and the dimensionless be monotonic; have a single interval changes (for example, from 0 to 1); be invariant to the form of the private extremum criterion (min or max); allow to realize both linear and non-linear, depending on local criteria values.

For a linear approximation of local criteria values assessments $k_i(s)$ we will use the value of the function:

$$\xi_i(s) = \overline{k}_i(s) - k_i^- = \frac{k_i(s) - k_i^-}{k_i^+ - k_i^-}, \quad i = I, m , \quad (10)$$

where $k_i(s)$, $k_i^+$, $k_i^-$ – accordingly, the importance of particular criteria for the option $s \in S$, the best and worst values of the criterion $k_i(s)$, $i = I, m$.

Function (10) requires minimal machining operations to calculate their values among known functions $29$.

For a more accurate non-linear ($S$- and Z-shaped) approximation criteria local count values will use the universal value function $30$:

$$\overline{a}(b_i + 1) \left( 1 - \frac{a}{b_i} \right) \left( b_i / \left( b_i + \frac{\overline{k}(s)}{\overline{k}_a} \right) \right), \quad 0 \leq \overline{k}(s) \leq \overline{k}_a;$$

$$\delta(s) = \overline{a} + (1 - \overline{a}) \left( b_i + 1 \right) \times \left( 1 - \frac{b_i}{b_i + \frac{\overline{k}(s) - \overline{k}_a}{\overline{k}_a - \overline{k}(s)}} \right), \quad \overline{k}_a < \overline{k}(s) \leq 1,$$

where: $\xi_i(s) = \overline{k}_i(s)$; $\overline{k}_a$, $\overline{a}$ – normalized coordinate values of point bonding, $0 \leq \overline{k}_a \leq 1$; $0 \leq \overline{a} \leq 1$; $b_1$, $b_2$ – factors that determine the form of the dependence on the initial and final sections of a function.

Function (11) has the best value of the complex index of “precision-complexity” to calculate its value among the known nonlinear functions $30$.

In practice, a reasonable amount of time to build a whole set of effective alternatives $S^E \subseteq S$ using a method based on Karlin theorem due to the difficulties in solving the problems of parametric programming (8) – (9) is possible only to a relatively small set of feasible solutions $S = \{ s \}$ $20$.

**Method based on Germeyer theorem.** A subset of the effective options $S^E \subseteq S$ on the basis of the theorem is Germeyer by combining $s^0_i$, $i = I, m$ optimizing each of the local criteria $k_i(s)$, $i = I, m$ the decisions of parametric programming problem with respect to the parameters $28$:

$$\lambda_i \in \Lambda = \{ \lambda_i : \lambda_i > 0 \ \forall i = I, m, \sum_{i=1}^{m} \lambda_i = 1 \} , \quad (12)$$

$$s^0_i = \arg\max_{s \in S} \{ P(s) = \min_i \lambda_i \xi_i(s) \} . \quad (13)$$

This method allows you to select subsets of viable options both convex and non-convex on the set of alternatives $S = \{ s \}$. In most cases, build a whole set of effective alternatives $S^E \subseteq S$ by a method based on the theorem Germeyer not possible due to the difficulty of solving problems of parametric programming (12) - (13) $20$.

**Methods for isolating rough sets effective solutions.** To reduce the time complexity of the methods considered are encouraged to use effective allocation of rough sets-making procedures (EARSM) $S'$. Must be fulfilled for such subsets requirement $S^E \subseteq S' \subseteq S$. To construct EARS are encouraged to use the methods of "sector" and "segment" $20$. For this purpose, the set of feasible solutions

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The method of “sector”. The essence of the basic method of "sector" to select a subset $S'_i \supseteq S^E$ on a convex set of feasible solutions $S = \{ s \}$ is as follows. On the set $S = \{ s \}$ it is optimized for each of the local criteria $k_i(s), i = 1, m$, with the result that identifies best solutions for each criterion:

$$s^o_i = \arg\max_{s \in S} k_i(s), i = 1, m,$$

and the corresponding values of other local criteria $k_j(s^o_i), j = 1, m, j \neq i$.

Then, the best value of the local criterion $k_i(s)$ equally $k_i^+ = k_i(s^o_i)$ and worst among the local criterion values $k_i(s)$ at the extremum of the other criteria are:

$$k_i^- = \max_j k_j(s^o_i), \text{ if } k_i(s) \to \min,$$

$$k_i^+ = \min_j k_j(s^o_i), \text{ if } k_i(s) \to \max.$$

The obtained value pairs $<k_i^+, k_i^->, i = 1, m$ displaying the approximate boundaries are set $S'_i \supseteq S^E$ space on local criteria $K(s) = \{ k_i(s) \}_{i=1}^m$. All variants of design solutions $s \in S$, for which conditions $k_i(s) \in [k_i^-, k_i^+], \forall i = 1, m$ included in EARSM $S'_i \supseteq S^E$. All other options fall into a subset obviously inefficient $S^E$.

Next, on the obtained EARSM $S'_i = \{ s \}$ implemented method of paired comparisons. The result will be allocated a subset of effective options $S^E \subseteq S$, whose correctness is the condition (7).

The degree of reduction of the set of options to be analyzed $\gamma = \text{Cond}(S) / \text{Cond}(S'_i)$ largely depends on the amount of local criteria $m$, particularly critical design objectives and methods used at the same time. In the case of uniform distribution of allowable characteristics for this separation method using subset $S'_i$ for a number of local criteria $m$ a reduction of the order $\gamma = 2^m$ time. This can significantly reduce the computational cost compared to the method of paired comparisons.

The method of “segment”. For a convex set of feasible solutions $S = \{ s \}$ also it proposed to use more sophisticated method “segment”, which allows to obtain EARSM much smaller size. Its essence is as follows. Pre converts local criteria values $k_i(s), i = 1, m$ in form of the utility values of the functions (10) or (11). Then, on the set of feasible solutions $S = \{ s \}$ identifies best solutions for each of the local criteria

$$s^o_i = \arg\max_{s \in S} k_i(s), i = 1, m.$$

Values obtained in this local criteria

$$k_{ij} = k_i(s^o_i), i, j = 1, m$$

define the extreme boundary point display set approximate $S'_2 \supseteq S^E$ space on local criteria $K(s) = \{ k_i(s) \}_{i=1}^m$. Construct plane $(m$-plane, hyperplane) extending through the end points (14) and a cut-off region of feasible solutions $S = \{ s \}$ an approximate set of effective solutions $S'_2 \supseteq S^E$ (Fig. 1):

$$\begin{vmatrix}
(\bar{k}_i(s) - \bar{k}_{11}) & \ldots & (\bar{k}_m(s) - \bar{k}_{m1}) \\
(\bar{k}_{12} - \bar{k}_{11}) & \ldots & (\bar{k}_{m2} - \bar{k}_{m1}) \\
\vdots & \ddots & \vdots \\
(\bar{k}_{1m} - \bar{k}_{11}) & \ldots & (\bar{k}_{mm} - \bar{k}_{m1})
\end{vmatrix} = 0.$$

Represent hyperplane equation (15) in the normal form:

$$F[a_1, \ldots, a_{m+1}, \bar{K}(s)] = a_1 \bar{k}_1(s) + a_2 \bar{k}_2(s) + \ldots + a_m \bar{k}_m(s) + a_{m+1} = 0,$$

where; $\bar{K}(s) = \{ \bar{k}_1(s), \bar{k}_2(s), \ldots, \bar{k}_m(s) \}$;

$$a_i, i = 1, m \text{ – hyperplane equation coefficients (15).}$$

For the separation points into subsets inefficient $S^E$ and EARSM $S^E_2$ we will determine their location relative to the plane (16).

For this we use the criterion of mutual
disposition of points $M_1(x_1,y_1,...,z_1)$ and $M_2(x_2,y_2,...,z_2)$ relative to the plane:

$$Ax + By + ... + Cz + D = 0,$$

wherein $A$, $B$, $C$, $D$ – the coefficients of the plane equation of the normal form.

Points $M_1(x_1,y_1,...,z_1)$ and $M_2(x_2,y_2,...,z_2)$ they are located on opposite sides of the plane, if the numbers

$$Ax_1 + By_1 + ... + Cz_1 + D,$$
$$Ax_2 + By_2 + ... + Cz_2 + D$$

have opposite signs. Lies in the plane if the corresponding number is zero.

As the first point will use the origin, i.e., $M_1(0,0,...,0)$. As a second corresponding to the value of the local criteria for point of use $M_2$ coordinates $M_2(\vec{k}_{i_1}, \vec{k}_{i_2},..., \vec{k}_{im})$, an alternative $s_i \in S$, namely $\vec{k}_i(s_j)$, $\vec{k}_2(s_j)$,..., $\vec{k}_m(s_j)$. Then a point lying in the field $S'_1$ must lie on the opposite side or plane (16) relative to the origin $M_1(0,0,...,0)$.

We define the ratio (16), the value $F(M_1)$ for the point of the origin. We calculate the value

$$F[\vec{K}(s_j)] = a_1\vec{k}_{i_1} + a_2\vec{k}_{i_2} + ... + a_m\vec{k}_{im} + a_{m+1},$$

to point $M_2$ with coordinates $(\vec{k}_{i_1}, \vec{k}_{i_2},..., \vec{k}_{im})$, corresponding to yet another embodiment $s_i \in S$. If the value $F[\vec{K}(s_j)] = 0$ or has a sign opposite to $F(M_1)$, we refer embodiment $s_i \in S$ set to the approximate $S'_2 \supseteq S^E$, otherwise set to inefficient $S^E$. For the problem is always $F(M_1) < 0$, and therefore, require the validation is performed only one condition $F[\vec{K}(s_j)] \geq 0$.

Next, on the obtained EARS $S'_2 = \{ s \}$ implemented method of paired comparisons. The result will be allocated a subset of effective options $S^E \subseteq S$, whose correctness is the condition (7).

The degree of reduction of the set of options to be analyzed $\gamma = Cond(S) / Cond(S'_2)$ also depends on the amount of local criteria $m$, particularly critical design objectives and methods used at the same time. Under the same conditions, the proposed method of isolating a subset $S'_2$ it gives a much more compact subset of design solutions options than the method of “sector” (Fig. 1).

For uniform distribution of origin of design solutions options in the space of local criteria for $m = 2$ we use as an estimate of the degree of reduction EARS $Card(S'_2)$ and $Card(S'_2)$ relations sector areas $S'_1$, segments $S'_2$ and $S'_3$. For a basic method “segment” degree of reduction $S'_2$ about $S'_1$ it is 2.74 times, and $S'_3$ regarding $S'_1$ about 11.03

![Fig. 1. The boundaries of subsets $S'_1$ and $S'_2$ on the convex set of alternatives in the space of normalized criteria $\vec{K}(s)$ and $\vec{K}_2(s)$](image-url)
To reduce the temporary complexity of the proposed modification of the “segment”. It assumes the definition of additional support options \( s'_{j}, s'_{2}, ..., s'_{r} \in S^{E} \) whose coordinates in the space of particular criteria \( k_{j}(s'_i) \), \( j=1,m \) define points, as can be more evenly distributed in \( S^{E} \). As the support may be selected embeddings \( s'_{1}, s'_{2}, ..., s'_{r} \in S^{E} \) which are solutions for parametric programming problems \( r \) points:

\[
\lambda_j \in \Lambda^+ = \{ \lambda_j : \lambda_j > 0 \ \forall j = \overline{1, m}, \ \sum \lambda_j = 1 \}, (18)
\]

\[
s' = \arg \max_{s \in S^*} \sum_{j} \lambda_j \bar{k}_j(s), \]  

where: \( \bar{k}_j(s) = [(k_j(s) - k_{j}^-) / (k_{j}^- - k_{j}^+)] \), \( j = \overline{1, m} \) - linear monotonic transformation of local criteria.

In particular, \( r=1 \) as the reference option is selected \( s' \in S^{E} \) whose coordinates in space criteria \( k_{j}(s'_i) \), \( j=1,m \) define a point lying closest to the center of the set \( S^{E} \) found regarding borders \( s' = \arg \max_{s \in S} \sum_{j} \bar{k}_j(s) \). We construct a plane passing through the different sets of boundary points \( s'_1, s'_2, ..., s'_r \in S^{E} \) coordinates \( k_{j} = k_{j}(s'_i) \), \( i, j = \overline{1, m} \) cutting off a plurality of set \( S'_j \supseteq S^{E} \). Equations planes passing through \( m \) set points determined by the expression (15).

In the next step to determine a subset of viable options \( S^{E} \subseteq S \) satisfying the correctness of the condition (7) on the resulting EARS \( S'_j \supseteq s \) implemented method of paired comparisons.

The proposed modification of the "segment" is being implemented more difficult from a computational point of view of the algorithm, but it allows to determine the approximate subset \( S'_j \) much smaller than the original version of the method of "segment" (\( S'_2 \)) and the method of "sectors" (\( S'_1 \)) (Fig. 2). For two local criteria \( m = 2 \) and using one reference embodiment \( r = 1 \) EARS reduction \( S'_j \); relative \( S \) it amounts to 40.2 times; about \( S'_j \) - 9.7 times; about \( S'_2 \) - 3.5 times. For \( r = 2 \) EARS reduction \( S'_j \) increases and is as follows: with respect to \( S \) is 88.73 times; about \( S'_j \) - 22.4 times; about \( S'_2 \) - 8.3 times.

With increasing number of local criteria \( m \) and the number of reference for this \( r \) observed acceleration of the reduction of rough sets effective options \( S'_j \). However, this method increases time complexity due to the need \( r \)-fold solution of problem (18) - (19) and the deployment of the determinants (15) of larger size.

**Evaluation methods temporal complexity**

Formation of a subset \( S^{E} \) method combinatorial directly from the set of feasible variants \( S \) in the worst case it requires pairwise comparison of options \( s \in S \) all local criteria \( k_{j}(s) \), \( j=1,m \). To do this, on the set \( S \), consisting of \( N = Card(S) \) elements required to execute the order comparisons

\[
f_{0}(m, N) = o[m \cdot C_{N}^{2}], \quad N = Card(S)
\]

Determination EARS \( S'_j \) by "sector" involves choosing the best options for each of the criteria (of the order \( m \cdot N \) comparison operations), the formation of boundaries (the order \( m^2 \) operations), hit testing of each option on all of the criteria in the selected border (about \( 2 \cdot m \cdot N \) operations). Thus, the time complexity of the method of forming \( S'_j \) is

\[
f_{j}(m, N) = o[m^2 + 3m \cdot N].
\]

The first two stages of the definition of \( S'_j \) by “segments” coincide with the stages of the method of “sector” and includes a number of order \( m \cdot N + m^2 \) operations. Preparation hyperplane equation (15) requires the deployment size of the determinant of the matrix \( m \times m \) (about \( m!m \) operations). The decision whether the point \( S'_2 \) it involves calculating the values of \( F \{ \bar{k}(s'_i) \} \) \((2 \cdot m \cdot N \) operations for all the points of the set of permissible variants \( S \)). In view of this, the time complexity of the method “segment” of the order

\[
f_{2}(m, N) = o[m!m + m^2 + 3m \cdot N].
\]

In view of the fact that in practice \( N \gg m \) it can be assumed that the considered methods are substantially the same temporal complexity \( f(m, N) = o[3m \cdot N] \).

Modification of the method “segment” implies a further construction with two criteria problems \( r+1 \) plane, which requires deployment \( r+1 \) determinant dimension \( 2 \times 2 \). Given the fact that the \( N \gg m \) it has virtually no effect on the time complexity of the method.
Fig. 2. The boundaries of subsets $S'_1$, $S'_2$ and $S'_3$ on the convex set of alternatives for $r = 2$, $k_1 \rightarrow \max$ and $k_2 \rightarrow \max$

The problems with the amount of local criteria $m \geq 3$ with one embodiment, the support $r = 1$ and large scale set of feasible solutions $N = \text{Card}(S)$ the amount of computation is not significantly increased at $m \geq 3$ and $r \geq 2$ task of building $S'_i$ substantially more complicated, as it requires solutions auxiliary tasks determining a plurality of support options and system construction planes forming convex surface. This makes inefficient use of this modification of the method “segment” with the number of support options $r \geq 2$.

Given the complexity of forming temporary estimates obtained above procedures subset effective design solutions $S^E$ are as follows:

- according to the scheme $S \rightarrow S'_1 \rightarrow S^E$
  $$f_1(m, N, N_1) = o \left[ m^2 + 3m \cdot N + m \cdot C_{N_1}^2 \right];$$
- according to the scheme $S \rightarrow S'_2 \rightarrow S^E$
  $$f_2(m, N, N_2) = o \left[ m!m + m^2 + 3m \cdot N + m \cdot C_{N_2}^2 \right];$$
- according to the scheme $S \rightarrow S'_3 \rightarrow S^E$
  $$f_3(m, N, N_3) = o \left[ m!m + m^2 + 3m \cdot N + m \cdot C_{N_3}^2 \right],$$

where: $N_1, N_2, N_3$ – size of subsets $S'_1, S'_2, S'_3$, $N_i = \text{Card}(S'_i)$, $i = 1, 3$.

It should be borne in mind that, in practice, a large scale set of feasible solutions $N = \text{Card}(S)$ and a large number of local criteria $m \geq 3$: $N_1 \gg N_2 \gg N_3$. The difference in size sets $N_1$, $N_2$ and $N_3$ sharply increases with increasing number of alternatives in the original set of options $N = \text{Card}(S)$ and the number of partial criteria $m$.

The analysis time reducing the degree of difficulty for methods based on preliminary allocation EARSM showed that the most efficient scheme is to use $S \rightarrow S'_1 \rightarrow S^E$.

Conclusions. As a result of analysis of the current state of the problem of support of acceptance of design decisions revealed that due to the combinatorial nature of most tasks synthesis number of alternative solutions dramatically increases with the dimension of design problems. The vast majority of options is ineffective. They can be improved at the same time on all the quality parameters. This leads to the need to develop methods for the selection of subsets of adaptive technology of
effective design solutions tailored to the features of the original sets, as the complexity of the requirements and the accuracy of the solution. To meet the challenges of various dimensions on convex and non-convex set of feasible options to choose the exact and approximate methods based on pair-wise analysis of the options, theorems Karlin and Germeyer.

According to the analysis of the computational complexity estimates methods as a function of the dimension of the original set of alternatives and the amount of local optimization criteria established that the selection of sets of approximate effective solutions at high power initial set of alternatives is almost always it is appropriate. This can significantly reduce the complexity of solving the decision-making tasks without loss of effective alternatives. The analysis time complexity methods revealed that the most efficient for large-scale problems is to use a scheme based on a modified method “segment”. Application isolation technology subsets effective solutions possible to significantly reduce the time of solving practical design problems [19].

The results can be used in the procedures for the adoption of multi-factor solutions in the design and management systems. Their use will improve the degree of automation of processes to reduce decision-making time by reducing the time complexity of procedures and ensure the quality of the decisions made by the choice of only a subset of them effective.

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ВИДІЛЕННЯ ПІДМНОЖИН ЕФЕКТИВНИХ ВАРІАНТІВ В ТЕХНОЛОГІЯХ ПРИЙНЯТТЯ ПРОЕКТНИХ РІШЕНЬ

Анотація: У статті розглядаються теоретичні аспекти виділення підмножин ефективних з множин допустимих варіантів в технологіях прийняття проектних рішень. За результатами аналізу сучасного стану проблеми виявлено, що з огляду на комбінаторний характер більшості задач інтерпретація кількість альтернативних варіантів рішень звісно збільшується з зростанням розмірності задач прийняття. При цьому переважна більшість варіантів є неефективними. Вони можуть бути позиціоновані одночасно за всіма показниками якості. Ці призводять до необхідності розробки методів для процедури виділення підмножин ефективних проектних рішень з урахуванням особливостей вихідних множин, вимог по трудомісткості та точності розв'язання задачі. Для розв'язання задач різної розмірності на опуклих і неопуклих множинах допустимих варіантів обрані точні і наближені методи, засновані на настільному аналізі варіантів, теоремах Карліна, Гермейера. Для зниження часових складностей методів розв'язання задач запропоновано використовувати наближені модифікації ефективних рішень методами «сектора» і «сегмента». За результатами аналізу обчислюваної складності методів як функцій від розмірності вихідних множин альтернатив і кількості локальних критеріїв оптимізації встановлено, що виділення наближених множин ефективних рішень при великій потужності вихідних множин альтернатив завжди є доцільним. Це дозволяє істотно зменшити трудомісткість розв'язання задач прийняття рішення без втрати ефективних альтернатив. Проведений аналіз часових складностей методів дозволив встановити, що найбільш раціональним для задач великої розмірності є використання схеми, що базується на модифікованому методі «сегмента». Отримані результати рекомендуються до використання в процедурних прийняття багатофакторних рішень у системах прийняття та управління. Їх застосування дозволить підвищити ступінь автоматизації процесів, скоротити час прийняття рішення за рахунок зменшення часових складностей процедур і гарантувати якість прийнятих рішень за раціональну вибірку їх із підмножин ефективних.

Ключові слова: технології прийняття рішень; інформаційна технологія; прийняття рішення; множина допустимих рішень; критерії оптимізації; множина компромісів

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ОПРЕДЕЛЕНИЕ ПОДМНОЖЕСТВ ЭФФЕКТИВНЫХ ВАРИАНТОВ В ТЕХНОЛОГИЯХ ПРИНЯТИЯ ПРОЕКТНЫХ РЕШЕНИЙ

Аннотация: В статье рассматриваются теоретические аспекты выделения подмножеств эффективных из множеств допустимых вариантов в технологиях принятия проектных решений. В результате анализа современного состояния проблемы выявлено, что ввиду комбинаторного характера большинства задач синтеза количество альтернативных вариантов решений резко увеличивается с ростом размерности задачи проектирования. При этом подавляющее большинство вариантов является неэффективными. Они могут быть улучшены одновременно по всем показателям качества. Это приводит к необходимости разработки методов для процедур выделения подмножеств эффективных проектных решений с учетом особенностей исходных множеств, требований по трудоемкости и точности решения задачи. Для решения задач различной размерности на выпуклых и невыпуклых множествах допустимых вариантов выбраны точные и приближенные методы, основанные на попарном анализе вариантов, теоремах Карлсона, Гермейера. Для снижения временной сложности методов решения задачи предложено предварительно выделять приближенные множества эффективных решений с учетом особенностей исходного множества. По результатам анализа оценок временной сложности методов как функций от размерности исходного множества альтернатив и количества локальных критериев оптимизации установлено, что выделение приближенных множеств эффективных решений при большой мощности исходного множества альтернатив всегда является целесообразным. Это позволяет существенно снижать трудоемкость решения задачи принятия решений без потери эффективных альтернатив. Проведенный анализ временной сложности методов позволил установить, что наиболее рациональным для задач большой размерности является использование схемы, базирующейся на модифицированном методе «сегмента». Полученные результаты рекомендуются для использования в процедурах принятия многофакторных решений в системах проектирования и управления. Их применение позволит повысить степень автоматизации процессов, сократить время принятия решений благодаря снижению временной сложности процедур и гарантировать качество принимаемых решений за счет выбора их только из подмножеств эффективных.

Ключевые слова: технология проектирования; информационная технология; принятие решений; множество допустимых решений; критерий оптимизации; множество компромиссов

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