Investigation of Time Series of Original Values of Currency Rates Measured on Small Time Frames on FOREX Using Methods of Chaos Theory

Olga Dabi-Prashad (MPhil), Ludmila Kirichenko (PhD)
Department of Applied Mathematics
Kharkiv National University of Radio Electronics
Ukraine

***Abstract*—Lately, the linear paradigm with its idea of normal distribution of profits has been replaced with the non-linear approach and Chaos Theory which gives the explanation of the complex behavior of financial markets. It has been discovered that time series of profits measured on long time frames on currency and stock markets (time series of monthly prices etc.) are chaotic. This paper is concentrated on investigation of time series of original values of currency rates measured on short time frames on FOREX (hourly, 4-hourly, daily prices) using methods of Chaos Theory (Time-delay reconstruction method, Grassberger-Procaccia method, estimation of the Lyapunov exponent) in order to define if such time series are chaotic as well.**

***Index Terms*—FOREX, Chaos Theory, memory in financial time series, predictability of financial time series, small (short) time frames.**

I. INTRODUCTION

FOREX is the market where currency is sold or bought freely for another currency according to a currency rate. FOREX is also the most liquid market. Many companies and private persons conduct conversion operations on the currency market with various purposes. It is known, that the currency rates on the FOREX market are affected by many factors which makes the currency price movement very complicated [1][2]. It is obvious that to conduct the conversion operations with success it is very important to have a model which would provide a deep understanding of the complicated behavior of the FOREX market. Lately, the linear approach to modeling of the financial markets and the idea of normal distribution of profits (e.g. the portfolio theory, CAPM) [3-5] had been replaced with the non-linear approach and Chaos Theory [6-8]. Within the frame of the non-linear approach, the time series on stock and currency markets have been investigated using the methods of Chaos Theory and it has been shown that the investigated time series adhere to a law of deterministic chaos and are not stochastic as it was stated by the linear paradigm [6,7]. *Chaos* has been defined as a behavior of a deterministic dynamical system which has sensitive dependence on the initial conditions [9,10]. Since the chaotic data are produced with a deterministic system there are non-linear correlations between stages of the system. Thus, if it is discovered that some time series is chaotic, it means there is a memory of the time series about its values in the past and we can predict its values in the future. But still, in the investigations of time series on the FOREX market with the methods of Chaos Theory the time series were used which were constructed on long time frames, such as one day, one month and more. Also, original values of currency rates were transformed to profits which were used in all the calculations instead of original values [6].

Most of conversion operations on FOREX are of speculative character and conducted with the aim to gain profit on the currency rates fluctuations. Such operations are normally conducted using original values of currency rates and short time frames, such as one hour (H1), four hours (H4), one day (D1), one week (1WEEK). Thus, it is useful to investigate time series of original values of currency rates on the FOREX market, which are measured on such small time frames, using methods of Chaos Theory and define if they are chaotic as well.

The main methods which allow to test whether time series is chaotic are [11,12]: reconstruction of attractors with the delay-time reconstruction method, estimation of the correlation dimension in m-dimension space, estimation

Manuscript received December 19, 2009.

Olga Dabi-Prashad (MPhil), has graduated from Wessex Institute of Technology (University of Wales) (e-mail: radharani_ua@yahoo.com).

Dr. Ludmila Kirichenko, Associate professor of Department of Applied Math, Kharkiv National University of Radio Electronics (e-mail: ludmila@kture.kharkov.ua).

of the Lyapunov exponent. In the following sections of the paper these methods will be used to test the time series of original values of currency rates constructed on short time frames on FOREX. Section 2 describes the reconstruction of attractors of dynamical system with the delay-time reconstruction method using one single component. This method allows to estimate visually if the reconstructed attractor is an attractor of a chaotic system. Section 3 describes the test of the considered time series with the Grassberger-Procaccia method. It provides the quantitative estimation of the fractal dimension which saturates at some finite value for a deterministic process. In section 4 it is defined either the considered time series have a sensitive dependence on the initial conditions. The results of the investigation are provided and the paper is concluded in section 5. All estimations in the paper have been conducted using Mathematica 5.0.

II. Reconstruction of attractors with the delay-time reconstruction method

Strange attractor [13] is a set of all the trajectories of a chaotic system, and whatever the initial conditions a trajectory of the chaotic system runs from, it falls on one of the trajectories of the strange attractor. Thus, if a dynamical system adheres to a law of a deterministic chaos, its trajectories run within a strict space, whereas trajectories of a stochastic system look like a cloud of points which tends to fill the entire phase space.

The *delay-time reconstruction method* is based on the idea of the reconstruction of an attractor of a dynamical system using a one-dimensional time series, which are generated by this dynamical system [14]. According to this method, the reconstructed attractor and the original one are topologically equivalent. Vectors of a reconstructed attractor can be formed using the formula:

$$y(t) = (x(t), x(t-\tau), x(t-2\tau), ..., x(t-(m-1)\tau)) \qquad (1)$$

where $x = x(t)$ is one-dimensional time series; τ is time delay; m is the dimension of a reconstructed phase space (lag space).

Let us first reconstruct the attractor for a Lorenz system which is described by equations [15]:

$$\begin{cases} \dfrac{dx}{dt} = \partial(y-x), \\ \dfrac{dy}{dt} = x(\rho - z) - y, \\ \dfrac{dz}{dt} = xy - \beta z. \end{cases} \qquad (2)$$

where parameters $\partial, \rho, \beta > 0$ and are constant. Let the parameters be defined as follows: $\sigma = 10$, $\rho = 28$, $\beta = 8/3$.

Let us use the measurement of a single variable *y* to reconstruct the Lorenz attractor. Then the reconstructed Lorenz attractor looks as follows:

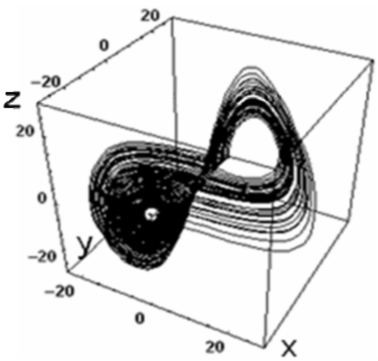

Fig. 1. Reconstructed Lorenz attractor in 3-dimensional lag space and $\tau = 10$

It is known, that the original Lorenz attractor looks as follows:

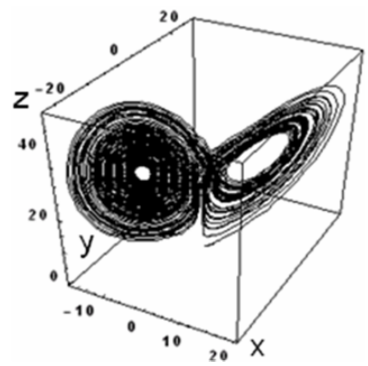

Fig. 2. Original Lorenz attractor.

As we can see from Fig.1 and Fig.2, the reconstructed attractor of the chaotic Lorenz system and the original one look very similar. For chaotic system they are topologically equivalent. Here the value of time delay τ has been chosen experimentally in order to achieve the best result (the reconstructed attractor should not look too stretched or too spread in the lag space).

Let us now reconstruct an attractor for a random system which is described by the autoregression equation $X(t) = \phi \cdot X(t-1) + \varepsilon(t)$, where ϕ is a constant and $\varepsilon(t)$ is a random component which follows Gaussian distribution.

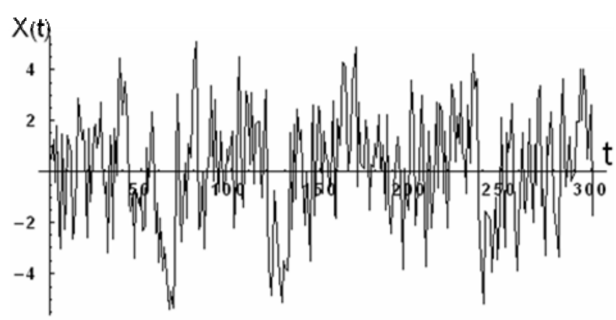

Fig. 3. Realisation of autoregression process for $\phi = 0.5$

The reconstructed attractor of a random system looks like a cloud of point which tends to fill the entire lag space. This happens for any value of time delay. We can see that in Fig. 4.

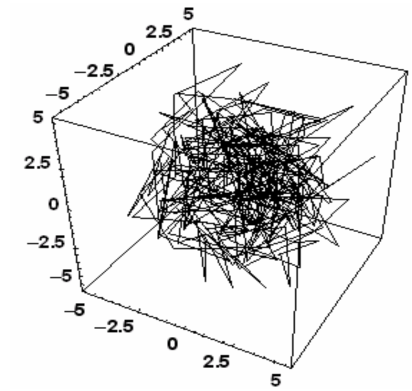

Fig. 4. Reconstructed attractor for autoregression process in a 3-dimensional space, $\tau = 10$

Let us now reconstruct attractors for time series of original values of currency rates on the FOREX market, which are measured using small time frames, which are the most common used for conducting of speculative trade operations, i.e. H1, H4, D1, 1WEEK.

Reconstruction of the strange attractor for EUR/USD, time frame H1

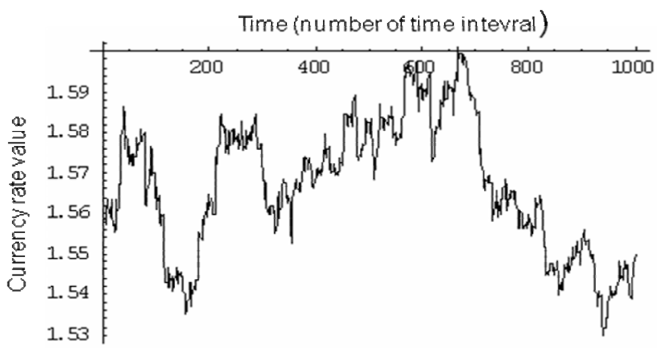

Fig. 5. Realisation of currency rate time series of currency pair EUR/USD, time frame H1

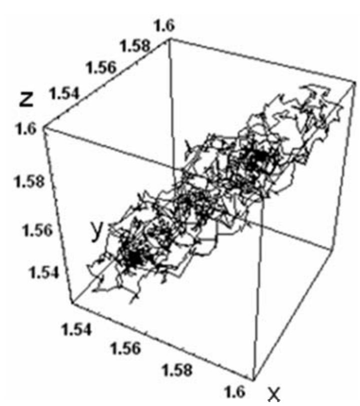

Fig. 6. Reconstructed attractor for EUR/USD in a 3-dimensional lag space, H1, optimal $\tau = 10$

Reconstruction of the strange attractor for NZD/CAD, time frame H4

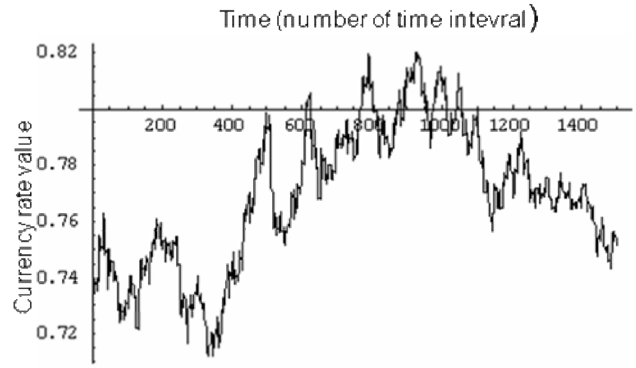

Fig. 7. Realisation of currency rate time series of currency pair NZD/CAD, time frame H4

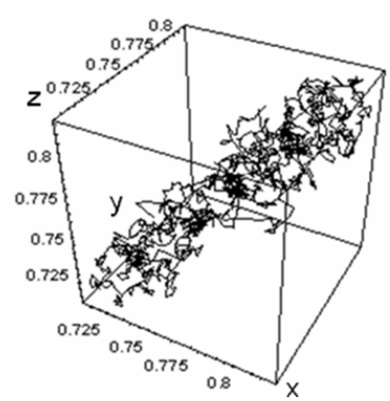

Fig. 8. Reconstructed attractor for NZD/CAD in a 3-dimensional lag space, H4, optimal $\tau = 15$

Reconstruction of the strange attractor for GBP/JPY, time frame D1

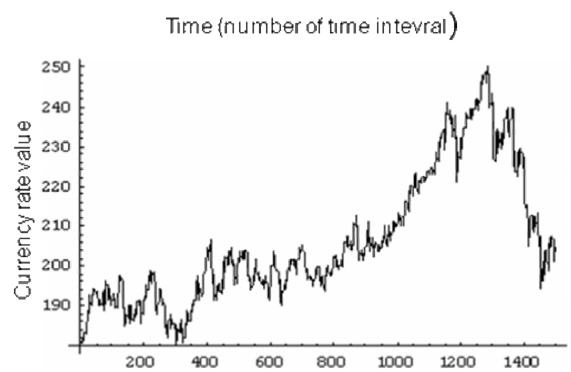

Fig. 9. Realisation of currency rate time series of currency pair GBP/JPY, time frame D1

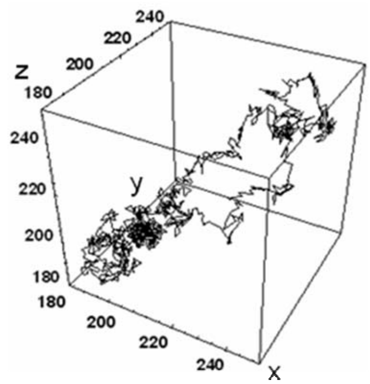

Fig. 10. Reconstructed attractor for GBP/JPY in a 3-dimensional lag space, D1, optimal $\tau = 30$

Reconstruction of the strange attractor for CAD/JPY, time frame 1WEEK

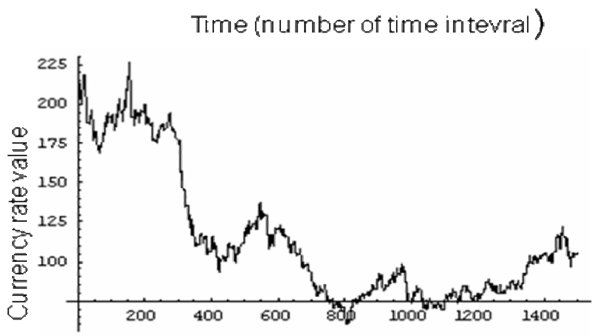

Fig. 11. Realisation of currency rate time series of currency pair CAD/JPY, time frame 1WEEK

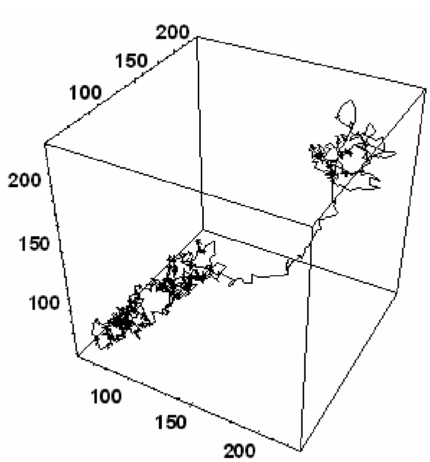

Fig. 12. Reconstructed attractor for CAD/JPY in a 3-dimensional lag space, 1WEEK, optimal $\tau = 20$

As we can see from Fig. 5 – 12, the trajectories of the reconstructed attractors run within strict space and do not look like a cloud of point as it is shown for the random system (Fig. 4). Thus, we can make a preliminary conclusion that the reconstructed attractors are strange attractors and are produced by chaotic systems.

Here the optimal value of time delay τ has been chosen experimentally in order to achieve the best result (the reconstructed attractor should not look too stretched or too spread in the lag space)

III. Estimation of the Correlation dimension in m-dimension space

A strange attractor of a chaotic system is a fractal and has fractal dimension [13][16]. For a true chaotic system its reconstructed attractor saves its dimensionality even if it is embedded into a lag space with a higher dimensionality.

According to the Grassberger-Procaccia method, a good approximation of the fractal dimension of a strange attractor is the *correlation dimension* [17]. Thus, if we reconstruct an attractor using the time-delay reconstruction method for various values of m (here m is called embedding dimension) and estimate the correlation dimension of the reconstructed attractor for various values of embedding dimension, then for a true chaotic system the value of correlation dimension will saturate at its true value which is a finite number. And the correlation dimension for the reconstructed attractor of a random system never stops growing.

If there is time series $x_1, x_2, \dots, x_M$ and an attractor is reconstructed in m-dimensional lag space, then the *correlation sum* is the probability of that a pair of points on the reconstructed attractor lie within a distance ε. The correlation sum can be estimated using the formula

$$C(\varepsilon) = \frac{1}{M^2} \cdot \sum_{ij=1}^{M} \theta(\varepsilon - \| x_i - x_j \|), \tag{3}$$

where $\theta(x)$ is the Heaviside step function:

$$\theta(x) = \begin{cases} 1, & \text{if } \varepsilon - \| x_i - x_j \| > 0 \\ 0, & \text{if } \varepsilon - \| x_i - x_j \| \le 0 \end{cases}$$

If $C(\varepsilon)$ is estimated for various ε, then $C(\varepsilon) \sim \varepsilon^{D(m)}$, where D(m) is the correlation dimension. D(m*)* can be estimated as a slope of a line fitted to a small ε-tail of the curve on a log-log plot of $C(\varepsilon)$ against ε.

Let us first estimate the correlation dimension for the Lorenz system:

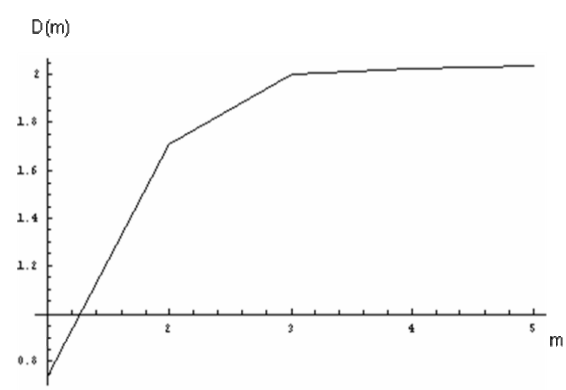

Fig. 13. Lorenz system. Estimation of the correlation dimension

We can see that for a true chaotic system the correlation dimension saturates at a finite value (D(m) = 2.03 for the Lorenz system). The true value of D(m) has been reached at the value of embedding dimension m=3.

Let us now estimated the correlation dimension for the random time series generated by the autoregression equation:

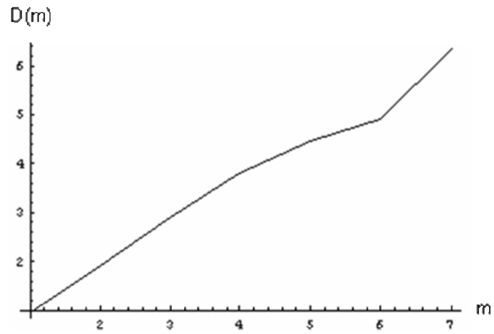

Fig. 14. Autoregression process. Estimation of the correlation dimension

As we can see in Fig. 14, the correlation dimension for a random system never stops growing.

Let us now estimate the correlation dimension for the currency rates time series for which the attractors have already been reconstructed.

Estimation of the correlation dimension for EUR/USD, time frame H1

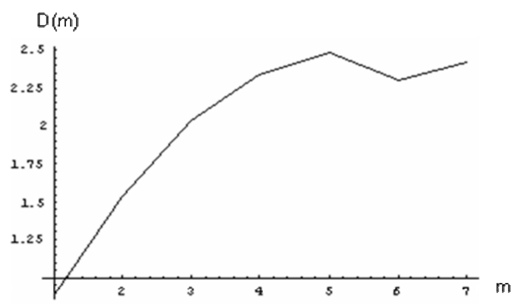

Fig. 15. Currency rates time series for currency pair EUR/USD, time frame H1. Estimation of the correlation dimension

For the considered time series the correlation dimension saturates at the value D(m)=2.5 starting from the embedding dimension m=5.

Estimation of the correlation dimension for NZD/CAD, time frame H4

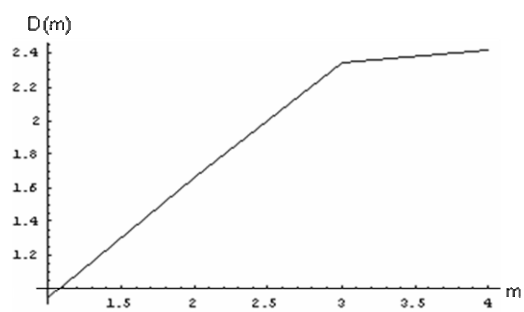

Fig. 16. Currency rates time series for currency pair NZD/CAD, time frame H4. Estimation of the correlation dimension

For the considered time series the correlation dimension saturates at the value D(m)=2.4 starting from the embedding dimension m=3.

Estimation of the correlation dimension for GBP/JPY, time frame D1

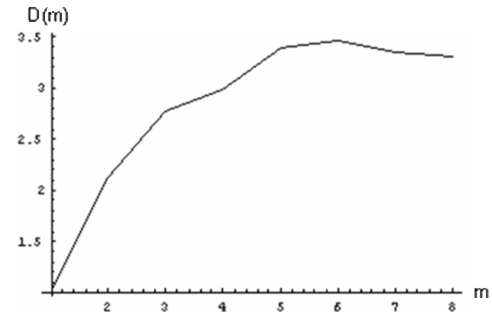

Fig. 17. Currency rates time series for currency pair GBP/JPY, time frame D1.Estimation of the correlation dimension

For the considered time series the correlation dimension saturates at the value D(m)=3.4 starting from the embedding dimension m=5.

Estimation of the correlation dimension for CAD/JPY, time frame 1WEEK

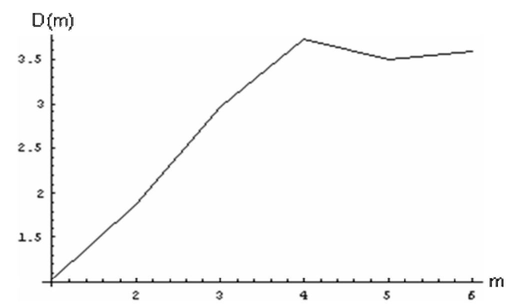

Fig. 17. Currency rates time series for currency pair CAD/JPY, time frame 1WEEK. Estimation of the correlation dimension

For the considered time series the correlation dimension saturates at the value D(m)=3.7 starting from the embedding dimension m=4.

Summary of the results of the estimation of the correlation dimension of the considered data can be found in Table I.

TABLE I
SUMMARY OF THE ESTIMATION OF THE CORRELATION DIMENSION

	D(m)					
m	Lorenz system	Random system	EUR/USD H1	NZD/CAD H4	GBP/JPY D1	CAD/JPY W1
1	0.7352	0.9812	0.8913	0.9427	1.0266	1.0174
2	1.7111	1.9102	1.5353	1.6627	2.1257	1.8776
3	2.0212	2.8898	2.0287	2.3553	2.7628	2.9508
4	2.0246	3.8084	2.3383	2.4235	2.9858	3.7092
5	2.038	4.4817	2.4801		3.3863	3.4996
6		4.8971	2.2998		3.4693	3.5864
7		6.3489	2.4148		3.3494	
8					3.3198	

For the estimation of the correlation dimension for all the considered samples of time series we have used the optimal values of time delay τ which have been obtained during the procedure of the attractor reconstruction. Also, the values of ε should not bee too small in order to obtain enough point for the statistical estimation of the correlation sum [17]. On other hand, ε should not be too big, otherwise it becomes comparable to the size of the attractor. For the estimation of the correlation dimension for EUR/USD we have used M=1000 points. And for all other currency pairs M=1500 points have been used. According to the Eckmann and Ruelle estimation [18] the maximum value of correlation dimension which is allowed to estimate using number of point M is defined by the formula:

$$D_{max} \cong 2\lg M \quad (4)$$

Thus, for M=1000 it is allowed to estimate the correlation dimension $D(m) \leq 6$. For all the considered samples of time series the estimated correlation dimension does not exceed D(m) = 3.5. Thus, the estimation of the correlation dimension is reliable for the given samples.

We have estimated the correlation dimension for currency rates time series of various currency pairs considered on small time frames, such as H1, H4, D1, 1WEEK. For all them the correlation dimension saturates at some finite value. It shows that on small time frames the currency rates time series are produced with deterministic system.

IV. Estimation of the Lyapunov Exponent

Deterministic chaos can appear in a dynamical system only if there is a sensitive dependence on its initial conditions. Such sensitivity is measured by the largest Lyapunov exponent (it is often called just the Lyapunov exponent). The Lyapunov exponent measures the divergence of initially close trajectories. For chaotic systems the Lyapunov exponent is always $\Lambda > 0$ [19].

The Lyapunov exponent has been estimated for the currency rates times series for which the attractors have been reconstructed and the correlation dimension has been estimated above in this paper. To estimate the Lyapunov exponent the Wolf algorithm has been used [18], which allows to estimate the exponent just using a one-dimensional time series. Also, to prove the efficiency of the estimation method the Lyapunov exponent has been estimated for the Lorenz system. The results of the estimation can be found in Table II:

TABLE II
The results of estimation of the Lyapunov exponent for the currency rates time series

	EUR/USD H1	NZD/CAD H4	GBP/JPY D1	CAD/JPY W1	Lorenz system
Λ	0.37	0.39	0.42	0.4	1.37

As it can be seen in Table II, all the investigated time series have the Lyapunov exponent greater than zero and thus, they are chaotic.

Since the Lyapunov exponent measures the divergence of initially close trajectories, then the less is the value of Λ, the less the initially trajectories diverge. Thus, the rate $1/\Lambda$ defines the predictability of a system.

V. Conclusion

In this paper we investigate the time series of original values of currency rates on the FOREX market which have been measured on small times frames such as 1 hour (H1), 4 hours (H4), 1 day (D1), 1 week (1WEEK). Such time frames are the most common used by traders during speculative trade operations. As a result of the investigation of the considered time series with the methods of Chaos Theory, it has been found that the time series of original values of currency rates on the FOREX market are chaotic on small time frames. Thus, such time series have a memory about its values in the past and we can predict its values in the future. This can be used in development of more efficient technical indicators for predicting price movement on FOREX which will help to traders to conduct the conversion operations with better success. Also, this allows to consider use of time series measured on small time frames in other prediction methods where only long time frames have been used before.

The fact, that the time series of original values of currency rates are chaotic, gives the opportunity to develop new prediction methods based on using original values of currency rates without pre-processing of initial data (transformation it to profits etc.). This can save time and resources while using such prediction methods. This is even more essential when the forecast needs to be done in the real-time mode (e.g. while conducting trade operations on FOREX)

References

[1] *Eric Naiman*. Short Trader's Encyclopedia. Alpina Business Books. (2007)

[2] FOREX Online User Manual for Successful Trading. (www.instantforexincome.com)

[3] *Markowitz H.M.* Portfolio Selection, Journal of Finance 7 (1953)

[4] *Litner, J.* The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. Rev. Econ. Statist. (Feb 1965)

[5] *Mossin J*. Equilibrium in a Capital Asser Market. Econometrica 34 (1966)

[6] *Edgar E. Peters*. Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility. Wiley; 2 edition (August 1996)

[7] *Edgar E. Peters*. Fractal Market Analysis: Applying Chaos Theory to Investment and Economics. Wiley; 1 edition (January 12, 1994)

[8] *A.S. Soofi & L. Cao*. Modelling and forecasting financial data: techniques of nonlinear dynamics. Norwell, Massachusetts: Kluwer academic publishers (2002

[9] *F. Mun*. Chaotic Vibrations: Wiley, New York (1987)

[10] *Shuster, H. G*. Deterministic Chaos: An Introduction, second revised edition. VCH Publishers, New York (1988)

[11] *M. Corazza & A.G. Malliaris*. Multi-Fractality in Foreign Currency Markets, Multinational Finance Journal, 6(2), 65-98 (2002)

[12] *Gian Italo Bischi, Carl Chiarella, Laura Gardini*. Nonlinear Dynamics in Economics, Finance and the Social Sciences: Springer; 1st Edition. edition (January 12, 2010)

[13] *D. Ruelle*. Small random perturbations of dynamical systems and the definition of attractors. Communications of Mathematical Physics 82: 137–151. (1981)

[14] *F. Takens*, *D. A. Rand* and *L.-S. Young*. Detecting strange attractors in turbulence. Dynamical Systems and Turbulence, Lecture Notes in Mathematics, vol. 898: 366–381, Springer-Verlag (1981)
[15] *Edward Lorenz*. The Essence of Chaos. University of Washington Press. University of Washington Press (April 1996)
[16] *Benoît B. Mandelbrot*. The Fractal Geometry of Nature. New York: W. H. Freeman and Co., (1982)
[17] *P. Grassberger, T. Schreiber, and C. Schaffrath, Int. J.* Bifurcation and Chaos 1, 521 (1991)
[18] *Eckmann J.-P., Kamphorst S., Ruelle D., Ciliberto S.* Lyapunov exponents from a time series //Phys.Rev.A. -1986. -V.34. -P.4971-4980
[19] *V. I. Oseledets*. Multiplicative ergodic theorem: Characteristic Lyapunov exponents of dynamical systems, Trudy MMO 19, 179–210 (1968)

Olga Dabi-Prashad was born in 1983, Ukraine.

In 2004 graduated from Kharkiv National University of Radio Electronics, Ukraine, with Bachelor's degree in Applied Mathematics.

In 2009 defended her thesis entitled "Using Chaos Theory and Oscillators for Predicting Price Movement in FOREX" at the Wessex Institute of Technology (University of Wales), the United Kingdom, and has been awarded the Degree of Master of Philosophy in Engineering.

Ms. Dabi-Prashad's scientific interests focus on quantitative analysis in financial markets, Chaos Theory, fractal and multifractal analysis.

Ludmila Kirichenko was born in 1964.

In 1986 graduated from Kharkiv Institute of Radio Electronics, Ukraine, with Master's Degree in Applied Mathematics.

In 2000 defended the thesis for a Candidate Degree at Kharkov State Technical University of Radioelectronics, Ukraine, and has been awarded the Candidated Degree in Technical Science (specialization: mathematical modelling and computing methods).

At present time, Dr. Kirichenko holds the position of Associate professor in Department of Applied Math, Kharkiv National University of Radio Electronics.

Dr. Kirichenko's scientific interests focus on deterministic and chaotic systems, stochastic self-similar processes, time series modelling and forecasting, fractal and multifractal analysis. She is the author of over 50 publications on the listed subjects.