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PHYSICS

Diffraction of Electromagnetic Waves on an Imperfectly Conducting Conical Structure of a Particular Shape

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When studying problems of diffraction of electromagnetic waves on imperfectly conducting structures, Shchukin–Leontovich–Rytov boundary conditions are used [1, 2]. However, they can hardly be used in investigation of the scattering properties of imperfectly conducting surfaces with a variable curvature. Therefore, equivalent boundary conditions taking into account the specific geometry of such structures are needed. In this work, we propose equivalent impedance-type boundary conditions on the surface of a thin conical surface taking into account its curvature; they are obtained from the exact solution for a thick conical grating [3]. These conditions are used for investigation of the problem of diffraction of electromagnetic waves on an unclosed imperfectly conducting conical structure.

STATEMENT OF THE PROBLEM

Let us consider the problem of diffraction of electromagnetic waves on an unbounded conical impedance structure Σ consisting of two coaxial cones Σ_1, Σ_2 ($\Sigma = \Sigma_1 \cup \Sigma_2$) with a common apex, each cone having N slots periodically notched along the generatrix (Fig. 1). Denote the aperture of cone Σ_j ($j = 1, 2$) by $2\gamma_j$, the width of its slots by d_j, and the period of the conical structure by $l = \dfrac{2\pi}{N}$. The slot period and width determine the values of the dihedral angles formed by the intersection of the planes passing through the axis of the structure and the edges of the conical stripes. In the thus introduced spherical coordinate system r, θ, φ with the origin at the apex of the conical structure, each cone Σ_j is determined by the equation $\theta = \gamma_j$. Let the structure be exposed to the field of a point source (either an electric, $\chi = 1$, or magnetic, $\chi = 2$, dipole) located at a point $B_0(\mathbf{r}_0)$, and let this field vary in time by the law $e^{ia\omega t}$ ($a = \pm 1$). Electromagnetic fields $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ in the isotropic homogeneous medium with parameters ε and μ, where the conical structure Σ and the source are located, satisfy the Maxwell equations; the two-sided impedance-type boundary condition on the conical stripes

$$\mathbf{n}_j \times \{\mathbf{n}_j \times (\mathbf{E}^+ + \mathbf{E}^-)\} = -\tilde{\zeta}_j^{(\chi)} \mathbf{n}_j \times (\mathbf{H}^+ - \mathbf{H}^-), \quad (1)$$

$$\mathbf{n}_j \times \mathbf{E}^+ - \mathbf{n}_j \times \mathbf{E}^- = 0, \quad \Sigma_j: \theta = \gamma_j, \quad (2)$$

$$\tilde{\zeta}_j^{(\chi)} = 2w\tilde{R}_j^{(\chi)}(\sin\gamma_j)^{\tilde{\rho}(\chi)}, \quad \tilde{\rho}(\chi) = (-1)^{\chi - 1},$$

$$w = \sqrt{\frac{\mu}{\varepsilon}},$$

$$\tilde{R}_j^{(\chi)} = \tilde{R}_{j,1}^{(\chi)} + ia\tilde{R}_{j,2}^{(\chi)}, \quad \tilde{R}_{j,1}^{(\chi)} \geq 0, \quad \mathbf{E}^{\pm} = \mathbf{E}|_{\theta = \gamma_j \pm 0},$$

and the conditions of radiation at infinity and limited energy value. Boundary conditions (1), (2) simulate the conditions for the electromagnetic field on the surface of thin resistive or thin dielectric (with the relative dielectric permittivity of a large magnitude) structures. The boundary value problem formulated in this way has a unique solution.

Let us represent the sought fields $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ in the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \mathbf{E}_1(\mathbf{r}), \quad (3)$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_0(\mathbf{r}) + \mathbf{H}_1(\mathbf{r}), \quad (4)$$

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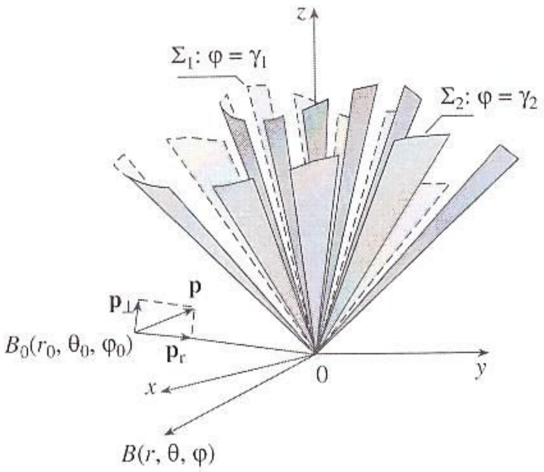

Fig. 1. Geometry of the structure.

where $\mathbf{E}_0$, $\mathbf{H}_0$ is the source field (primary field) and $\mathbf{E}_1$, $\mathbf{H}_1$ is the field due to the presence of the conical structure (secondary field). In solving the electrodynamic boundary value problem, we use the electric $\upsilon^{(1)}(\mathbf{r})$ and magnetic $\upsilon^{(2)}(\mathbf{r})$ Debye potentials and express the field components in terms of these potentials. The potential $\upsilon^{(\chi)}(\mathbf{r})$ satisfies the Helmholtz equation everywhere outside the source and the conical structure; it satisfies the boundary conditions corresponding to (1), (2); the limiting absorption principle; and the condition of finiteness of energy. In the general statement, the dipole moment is arbitrarily oriented. The basic problem for the original one is the problem of scattering of the field of a radial dipole with the moment

$$\mathbf{P}_r^{(\chi)}(\mathbf{r}) = M_r^{(\chi)}\mathbf{e}_r\delta(\mathbf{r}-\mathbf{r}_0)$$

by the conical structure Σ.

According to (3), (4), the potential $\upsilon^{(\chi)}(\mathbf{r})$ can be written as

$$\upsilon^{(\chi)}(\mathbf{r}) = \upsilon_0^{(\chi)}(\mathbf{r}) + \upsilon_1^{(\chi)}(\mathbf{r}),$$

where $\upsilon_0^{(\chi)}(\mathbf{r}) = \dfrac{\hat{p}_\chi}{r_0}\dfrac{e^{-qR}}{4\pi R}$ corresponds to the primary field, $\hat{p}_\chi = \dfrac{M_r^{(\chi)}}{\varepsilon^{2-\chi}\mu^{\chi-1}}$, $q = iak$, k is the wavenumber, and $R = |\mathbf{r}-\mathbf{r}_0|$. It is convenient to seek the potential $\upsilon_1^{(\chi)}$ for the secondary field using the Kantorovich–Lebedev integral transformation [3]

$$\upsilon_1^{(\chi)} = \frac{2\hat{p}_\chi}{\pi^2 r_0}\int_0^\infty \tau\sinh\pi\tau\frac{K_{i\tau}(qr)}{\sqrt{r}}\sum_{m=-\infty}^{\infty} a_{m\tau}^{(\chi)}\hat{U}_{m,i\tau}^{(\chi)}(\theta,\varphi)d\tau, \quad (5)$$

$$\hat{U}_{m,i\tau}^{(\chi)} = \begin{cases} \sum\limits_{n=-\infty}^{+\infty}\hat{\alpha}_{mn}^{(\chi)}P_{-1/2+i\tau}^{m+nN}(\cos\theta)e^{i(m+nN)\varphi}, \\ 0<\theta<\gamma_1, \\ \sum\limits_{n=-\infty}^{+\infty}[\hat{\beta}_{mn}^{(\chi)}P_{-1/2+i\tau}^{m+nN}(\cos\theta) \\ +\hat{\xi}_{mn}^{(\chi)}P_{-1/2+i\tau}^{m+nN}(-\cos\theta)]e^{i(m+nN)\varphi}, \\ \gamma_1<\theta<\gamma_2, \\ \sum\limits_{n=-\infty}^{+\infty}\hat{\eta}_{mn}^{(\chi)}P_{-1/2+i\tau}^{m+nN}(-\cos\theta)e^{i(m+nN)\varphi}, \\ \gamma_2<\theta<\pi, \end{cases} \quad (6)$$

where $K_{i\tau}(qr)$ is the Macdonald function; $a_{m\tau}^{(\chi)}$ are the known coefficients depending on the location of the source; $P_{-1/2+i\tau}^{m+nN}(\cos\theta)$ is the Legendre function of the first kind; and $\hat{\alpha}_{mn}^{(\chi)}$, $\hat{\beta}_{mn}^{(\chi)}$, $\hat{\xi}_{mn}^{(\chi)}$, and $\hat{\eta}_{mn}^{(\chi)}$ are the unknown coefficients, which are independent of parameter q. For obtaining functional equations in order to determine the unknown coefficients, boundary conditions (1), (2) and the field conjugation conditions in the slots should be used.

SOLITARY IMPEDANCE CONE WITH LONGITUDINAL SLOTS

In the case of diffraction of electromagnetic waves on a solitary cone Σ_1, function $\hat{U}_{m,i\tau}^{(\chi)}$ (6) has the form

$$\hat{U}_{m,i\tau}^{(\chi)} = \sum_{n=-\infty}^{+\infty} x_{m,n+m_0}^{(\chi)}\frac{P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)}{\dfrac{d^{\chi-1}}{d\gamma_2^{\chi-1}}P_{-1/2+i\tau}^{m+nN}(\pm\cos\gamma_1)}e^{i(m+nN)\varphi}, \quad (7)$$

where the upper signs in (7) correspond to the region $0<\theta<\gamma_1$ and the lower signs correspond to the region $\gamma_1<\theta<\pi$, $\dfrac{m}{N} = m_0+\nu$, m_0 is the closest integer to $\dfrac{m}{N}$, and $-\dfrac{1}{2}\le\tilde{\nu}<\dfrac{1}{2}$. We assume that the impedance parameter $\tilde{R}_j^{(\chi)}$ depends on the radial coordinate

$$\tilde{R}_1^{(\chi)} = \frac{\zeta^{(\chi)}}{(qr)^{\tilde{\rho}(\chi)}}. \quad (8)$$

The unknown coefficients $x_{m,n}^{(\chi)}$ are the solutions to the functional equations of the form

$$\sum_{m=-\infty}^{+\infty}\left\{1+2\hat{\zeta}^{(\chi)}\frac{[N(n+\nu)]^{\bar{\rho}(\chi)}\frac{|n|}{n}(1-\varepsilon_n^{(\chi)})}{(\tau^2+1/4)^{\bar{\rho}(\chi)}\sin^{\bar{\rho}(\chi)}\gamma_1}\right\}$$

$$\times x_{m,n}^{(\chi)}e^{inN\varphi} = e^{im_0N\varphi}, \quad \frac{\pi d_1}{l}<|N\varphi|<\pi, \tag{9}$$

$$\sum_{m=-\infty}^{+\infty}[N(n+\nu)]^{\bar{\rho}(\chi)}\frac{|n|}{n}(1-\varepsilon_n^{(\chi)})x_{m,n}^{(\chi)}e^{inN\varphi} = 0,$$

$$|N\varphi|<\frac{\pi d}{l}, \tag{10}$$

$$[N(n+\nu)]^{\bar{\rho}(\chi)}\frac{|n|}{n}(1-\varepsilon_n^{(\chi)})$$

$$= \frac{(-1)^{(n+\nu)N+\chi-1}\cosh(\pi\tau)}{\pi(\sin\gamma_1)^{1-\bar{\rho}(\chi)}}\frac{\Gamma(1/2+i\tau+(n+\nu)N)}{\Gamma(1/2+i\tau-(n+\nu)N)}$$

$$\times\frac{1}{\frac{d^{\chi-1}}{d\gamma_1^{\chi-1}}P_{-1/2+i\tau}^{(n+\nu)N}(-\cos\gamma_1)\frac{d^{\chi-1}}{d\gamma_1^{\chi-1}}P_{-1/2+i\tau}^{(n+\nu)N}(\cos\gamma_1)}.$$

If the source of the primary field is a radial magnetic dipole ($\chi = 2$), then Eqs. (9) and (10) can be solved by the semi-inversion method [3, 4]. Then, the original electrodynamic problem is reduced to the solution of the following system of linear algebraic Fredholm equations of the second kind:

$$\frac{1}{\nu}\frac{2P_{\nu-1}(-u)}{P_\nu(-u)+P_{\nu-1}(-u)}y_0^{(2)} = -\tilde{h}_{i\tau}^{(m_0+\nu)N}V^{m_0}(u)$$

$$+\sum_{p=-\infty}^{+\infty}y_0^{(2),m_0}\tilde{f}_{i\tau}^{(p+\nu)N}V^p(u),$$

$$\tilde{y}_n^{(2)} = -\tilde{h}_{i\tau}^{(m_0+\nu)N}V_{n-1}^{m_0-1}(u)+\sum_{p=-\infty}^{+\infty}\tilde{f}_{i\tau}^{(p+\nu)N}y_0^{(2)}V_{n-1}^{p-1}(u)$$

$$+y_0^{(2)}P_n(u), \quad n = \pm1, \pm2, \pm3, \ldots,$$

$$\tilde{h}_{i\tau}^{(n+\nu)N} = \frac{1}{1+\hat{\zeta}^{(2)}2(\tau^2+1/4)\sin\gamma_1\tilde{D}_{i\tau}^{(n+\nu)N}}\frac{|n|}{n}(1-\varepsilon_n^{(2)}),$$

$$\tilde{f}_{i\tau}^{(n+\nu)N} = \frac{|n|}{n}\varepsilon_n^{(2)}$$

$$+N(n+\nu)\frac{\hat{\zeta}^{(2)}2\left(\tau^2+\frac{1}{4}\right)\sin\gamma_1(\tilde{D}_{i\tau}^{(n+\nu)N})^2}{1+\hat{\zeta}^{(2)}2\left(\tau^2+\frac{1}{4}\right)\sin\gamma_1\tilde{D}_{i\tau}^{(n+\nu)N}},$$

$$u = \cos\frac{\pi d_1}{l}, \quad \tilde{D}_{i\tau}^{(n+\nu)N} = \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_n^{(2)}),$$

$$\delta_n^{m_0} = \begin{cases}1, & n = m_0,\\ 0, & n\neq m_0.\end{cases}$$

The functions $V_{n-1}^{p-1}(u)$ and $V^p(u)$ are presented in [4]. In the case of a large number of sufficiently narrow (as compared to the period of the conical structure) slots, under the condition of existence of the limit

$$W_2 = \lim_{\substack{N\to+\infty\\ d_1/l\to0}}\left(-\frac{1}{N}\ln\sin\frac{\pi d_1}{2l}\right)$$

the potential $\upsilon_1^{(2)}$ can be represented in the form

$$\upsilon_1^{(2)} = -\frac{4W_2}{\pi^2}\hat{p}_2\int_0^{+\infty}\tau\sinh\pi\tau\frac{K_{i\tau}(qr)}{\sqrt{r}}$$

$$\times\sum_{m=-\infty}^{+\infty}\frac{a_{m\tau}^{(2)}\Phi_{i\tau}^{(m)}}{\tilde{D}_{i\tau}^{(m)}(2W_2+\Phi_{i\tau}^{(m)})}\frac{P_{-1/2+i\tau}^m(\pm\cos\theta)}{\frac{d}{d\gamma}P_{-1/2+i\tau}^m(\pm\cos\gamma_1)}e^{im\varphi}d\tau,$$

$$\Phi_{i\tau}^{(m)} = \frac{\tilde{D}_{i\tau}^m}{1+\hat{\zeta}^{(2)}2\left(\tau^2+\frac{1}{4}\right)\sin\gamma\tilde{D}_{i\tau}^m}.$$

In a nonabsorbing medium, in the presence of a cone with reactive impedance ($0<(-\hat{\zeta}^{(2)})\ll1$), one of the components of magnetic field (4) has the form ($\theta_0 = \pi$, $\gamma_1<\theta<\pi$):

$$H_\theta = \frac{-\hat{p}_2}{2rr_0\sqrt{r_0}}\Omega_{i\tau}(\gamma_1)\frac{\partial}{\partial r}[\sqrt{r}T_{i\tau}(r,r_0)]$$

$$\times\frac{P_{-1/2+i\tau}^{-1}(\cos\gamma_1)}{P_{-1/2+i\tau}^{-1}(-\cos\gamma_1)}P_{-1/2+i\tau}^{-1}(-\cos\theta)\Big|_{\tau=\tau^*}$$

$$-\frac{\hat{p}_2}{rr_0\sqrt{r_0}}\Omega_{\hat{\mu}}(\gamma_1)\frac{\partial}{\partial r}[\sqrt{r}T_{\hat{\mu}}(r, r_0)]$$

$$\times\frac{P^{-1}_{-1/2+\hat{\mu}}(\cos\gamma_1)}{P^{-1}_{-1/2+\hat{\mu}}(-\cos\gamma_1)}P^{-1}_{-1/2+\hat{\mu}}(-\cos\theta)\Big|_{\hat{\mu}=\breve{\mu}_0}$$

$$-\frac{\hat{p}_2}{rr_0\sqrt{r_0}}\sum_{n=1}^{+\infty}\Omega_{\hat{\mu}}(\gamma_1)\frac{\partial}{\partial r}[\sqrt{r}T_{\hat{\mu}}(r, r_0)]$$

$$\times\frac{P^{-1}_{-1/2+\hat{\mu}}(\cos\gamma_1)}{P^{-1}_{-1/2+\hat{\mu}}(-\cos\gamma_1)}P^{-1}_{-1/2+\hat{\mu}}(-\cos\theta)\Big|_{\hat{\mu}=\breve{\mu}^*_n}, \quad (11)$$

$$\Omega_{\hat{\mu}}(\gamma) = \frac{\hat{\mu}\left(\hat{\mu}^2-\frac{1}{4}\right)^2 S_{\hat{\mu}}(\gamma)}{\cos\pi\hat{\mu}\frac{d}{d\hat{\mu}}\hat{F}_{\hat{\mu}}(\gamma)},$$

$$\hat{F}_{\hat{\mu}}(\gamma) = 2\left(\hat{\mu}^2-\frac{1}{4}\right)\hat{D}^{(2),0}_{\hat{\mu}}(\gamma)+\frac{1}{2W_2},$$

$$S_{\hat{\mu}}(\gamma) = \frac{\pi}{2}\frac{\hat{\mu}^2-\frac{1}{4}}{\cos\pi\hat{\mu}}$$

$$\times\sin^2\gamma P^{-1}_{-1/2+\hat{\mu}}(\cos\gamma)P^{-1}_{-1/2+\hat{\mu}}(-\cos\gamma),$$

$$\hat{D}^{(2),0}_{\hat{\mu}}(\gamma) = S_{\hat{\mu}}(\gamma)-\hat{\zeta}^{(2)},$$

$$T_{\hat{\mu}}(r, r_0) = \begin{cases} I_{\hat{\mu}}(qr)K_{\hat{\mu}}(qr_0), & r<r_0, \\ K_{\hat{\mu}}(qr)I_{\hat{\mu}}(qr_0), & r>r_0, \end{cases}$$

$$P^{-1}_{-1/2+\breve{\mu}^{1\pm}_n}(\pm\cos\gamma_1) = 0,$$

$$W_2\sin\gamma_1 \gg 1,$$

$$\breve{\mu}^*_n = \breve{\mu}^{\pm}_n+\frac{1}{2W_2}\frac{1}{\left(\hat{\mu}^2-\frac{1}{4}\right)\frac{d}{d\hat{\mu}}\hat{D}^{(2),0}_{\hat{\mu}}\Big|_{\hat{\mu}=\breve{\mu}^{\pm}_n}}+O(W_2^{-2}),$$

$$\breve{\tau}^* \approx -\frac{\sin\gamma_1}{2\hat{\zeta}^{(2)}}+\frac{2}{\sin\gamma_1 W_2} \gg 1, \quad (12)$$

$$\breve{\mu}_0 \approx \xi-\frac{\hat{\zeta}^{(2)}}{W_2(\sin^2\gamma+2\hat{\zeta}^{(2)})}, \quad (13)$$

$$\xi = \frac{1}{2}+\frac{1}{2W_2\sin^2\gamma_2}. \quad (14)$$

The first term in (11) is the mode due to the reactive impedance; it corresponds to the spectral value $\breve{\mu}^* = i\breve{\tau}^*$. In representation (12), the term $-\frac{\sin\gamma_1}{2\hat{\zeta}^{(2)}}$ corresponds to the spectral value of the mode in the case of excitation by a radial magnetic dipole of a solid cone with two-sided boundary conditions of the type (1), (2) satisfied on its surface and the term $\frac{2}{W_2\sin\gamma}$ characterizes the influence of inhomogeneities in the form of longitudinal slots. The second term in (11) is the mode corresponding to the spectral value $\breve{\mu}_0$; representation (13) of the latter involves quantity ξ, whose value (14) corresponds to the TEM wave in the structure of the field scattered by a semitransparent cone, which is the limiting case of an ideally conducting cone with periodical longitudinal slots [5].

SEMITRANSPARENT CONE ON THE IMPEDANCE PLANE

A semitransparent cone Σ_1 (the limiting case of an ideally conducting cone Σ_1 with longitudinal slots) with the transparency parameter

$$W_1 = \lim_{\substack{N\to+\infty \\ \frac{d_1}{l}\to 1}}\left(-\frac{1}{N}\ln\cos\frac{\pi d_1}{2l}\right)$$

is located on the impedance plane Σ_2: $\theta = \frac{\pi}{2}$ and represents a model of an imperfectly conducting cone with the components of the electromagnetic field ($\chi = 1$) satisfying the following averaged boundary conditions on its surface:

$$\tilde{E}_r = 0,$$

$$\frac{-q}{wW_1\sin\gamma_1}E_r\Big|_{\Sigma_1} = \left(\frac{\partial^2}{\partial r^2}-q^2\right)(r\tilde{H}_\varphi), \quad \tilde{A} = A^+-A^-.$$

The plane is a model of an opaque flat solid substrate surface with a thin absorbing layer, on which the Shchukin–Leontovich–Rytov impedance conditions are set:

$$\mathbf{n}\times\mathbf{E}|_{\Sigma_2} = -wR^{(\chi)}\mathbf{n}\times(\mathbf{n}\times\mathbf{H})|_{\Sigma_2}.$$

Under the assumption of the varying impedance parameter

$$R^{(\chi)} = \frac{\zeta^{(\xi)}}{(qr)^{\bar{p}(\chi)}}$$

and the radial electric dipole being located on the axis of the semitransparent cone ($\chi = 1$, $\theta_0 = 0$), using the Kantorovich–Lebedev integral transformations, we obtain potential $\upsilon_1^{(1)}$ and write it in the form

$$\upsilon_1^{(1)} = \upsilon_{1,\,\mathrm{imp.pl.}}^{(1)} + \bar{\upsilon}_{1,\,\mathrm{con.pl.}}^{(1)}, \quad \gamma < \theta < \frac{\pi}{2},$$

where $\upsilon_{1,\,\mathrm{imp.pl.}}^{(1)}$ is the Debye potential for the impedance plane without the cone, which can be written as

$$\upsilon_{1,\,\mathrm{imp.pl.}}^{(1)} = -\frac{2}{\pi^2}\hat{p}_1 \int_0^{+\infty} \tau \sinh\pi\tau a_{0\tau}^{(1)} \Lambda_{i\tau} \frac{K_{i\tau}(qr)}{\sqrt{r}} d\tau,$$

$$\Lambda_{i\tau} = \frac{P_{-1/2+i\tau}(0) + \zeta^{(1)} \frac{d}{d\theta} P_{-1/2+i\tau}(\cos\theta)|_{\theta=\pi/2}}{P_{-1/2+i\tau}(0) - \zeta^{(1)} \frac{d}{d\theta} P_{-1/2+i\tau}(\cos\theta)|_{\theta=\pi/2}},$$

and the term $\bar{\upsilon}_{1,\,\mathrm{con.pl.}}^{(1)}$ is due to the presence of the semitransparent cone,

$$\bar{\upsilon}_{1,\,\mathrm{con.pl.}}^{(1)} = \frac{2}{\pi^2} \int_0^{+\infty} \tau \sinh(\pi\tau) a_{0\tau}^{(1)} \frac{N_{i\tau}^{(1)} P_{-1/2+i\tau}(-\cos\gamma_1)}{N_{i\tau}^{(1)} + 2W_1}$$

$$\times \left[B_{i\tau} \frac{P_{-1/2+i\tau}(\cos\theta)}{P_{-1/2+i\tau}(\cos\gamma_1)} - \frac{P_{-1/2+i\tau}(-\cos\theta)}{P_{-1/2+i\tau}(-\cos\gamma_1)} \right] \frac{K_{i\tau}(qr)}{\sqrt{r}} d\tau, \quad (15)$$

$$N_{i\tau}^{(1)} = \frac{\pi}{\cosh\pi\tau} P_{-1/2+i\tau}(\cos\gamma_1) P_{-1/2+i\tau}(-\cos\gamma_1)(1 - B_{i\tau}),$$

$$B_{i\tau} = \Lambda_{i\tau} \frac{P_{-1/2+i\tau}(\cos\gamma_1)}{P_{-1/2+i\tau}(-\cos\gamma_1)}.$$

The difference of fractions in the integrand in (15) containing Legendre functions of arguments with opposite signs determines the influence of the plane on the radiation from the cone.

CONCLUSIONS

A numerically analytical method for solving three-dimensional boundary value problems of electrodynamics for imperfectly conducting unclosed conical structures with two-sided impedance-type boundary conditions taking into account the surface curvature has been proposed and rigorously substantiated. The advantage of the method is that it provides the possibility to obtain an analytical solution to the problem of diffraction of electromagnetic waves on impedance cones and bicones with longitudinal slots. The proposed method can be used efficiently in investigating problems of pulsed excitation of superconducting conical structures.

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