## **QOS GUARANTEES FOR THE MULTIFLOW ROUTING TENSOR MODEL**

Lemeshko O.V., Yeremenko O.S.

Department of Telecommunication Systems, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine, E-mail: <u>oleksandr.lemeshko@nure.ua</u>, <u>oleksandra.yeremenko.ua@ieee.org</u>

**Abstract** – The routing tensor model with the QoS guarantees by the packet rate and end-to-end delay for the multiflow case in the coordinate system of interpolar paths and internal node pairs is presented. The use of selected coordinate system of tensor representation as network model allowed to obtain the solution of routing problems with reduced computational complexity, where the end-to-end delay in all paths converged within the update timer to the values of QoS-requirements.

Keywords: QoS; tensor; model; routing; delay.

#### I. Introduction

The presence of requirements for providing guaranteed Quality of Service (QoS) over the multiple parameters, on the one hand, and taking into account the dynamics of network state, on the other, determined the need of dynamic tensor communications network models. In turn, it is important to choose an appropriate coordinate system in tensor representation with the network geometrization. At the same time in solving the routing problems the approach of the transition to the coordinate system of interpolar paths and internal node pairs is of interest.

## II. QoS Guarantees For The Multiflow Routing Tensor Model

In the multiflow routing model the network is presented by the graph S = (U, V) with the sets  $U = \{u_i, i = \overline{1, m}\}$  of network routers and  $V = \{v_z = (i, j); z = \overline{1, n}; i, j = \overline{1, m}; i \neq j\}$  of edges. Here the *z*th link, which connects the *i*th and *j*th routers, is modeled by the edge  $v_z = (i, j) \in V$ . Link capacity  $\Phi(i, j)$ assumed to be known and has dimension of the number of packets per second (1/s). Within the model the routing variables  $x_{(i,j)}^k$  have to be calculated for determining the fraction of intensity of each *k*th flow ( $k \in K$ , where *K* is the set of flows in the network) from the *i*th node to the *j*th node.

Flow conservation condition with the view of network nodes overload prevention can be written for the source, transit, and destination nodes as follows [1]:

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$$\begin{cases} \sum_{\substack{j:(i,j)\in V \\ \sum j:(i,j)\in V \\ j:(j,i)\in V \\ \sum j:(j,i)\in V \\ j:(j,i)\in V \\ x_{(j,i)}^{k} = -1, k \in K, i = d_{k}, \end{cases} x_{(j,i)}^{k} = 0, k \in K, i \neq s_{k}, d_{k}; (1)$$

where  $s_k$  is the source node for the *k*th flow, and  $d_k$  is the corresponding destination node.

Implementation of the multipath routing strategy in this model depends on the fulfillment of condition  $0 \le x_{(i,j)}^k \le 1$ , related to the control variables. At the same time, the capacity constraints on the network links utilization take place:

$$\sum_{k \in K} \lambda_{req}^k x_{(i,j)}^k \le \varphi_{(i,j)}, \quad (i,j) \in E , \qquad (2)$$

where  $\lambda_{req}^k$  is the average intensity of the *k*th flow.

Finally, the objective function should be minimized:

$$J = \sum_{(i,j)\in E} \sum_{k\in K} h_{(i,j)}^{x} \cdot \lambda_{req}^{k} \cdot x_{(i,j)}^{k}, \qquad (3)$$

where  $h_{(i,j)}^x$  is metric of routing for the link between the corresponding *i*th and the *j*th network nodes.

Within the routing tensor model the network is introduced by the anisotropic space-structure constructed by the sets of the interpolar paths and internal node pairs, where the poles represented by the source and destination nodes, and the internal node pairs include all node pairs except of the source and destination. If S is a connected network, then the number of interpolar paths is determined as  $\kappa(S) = \mu + 1 = n - m + 2$ , where n is the total number of edges in the network, m is the total number of nodes and  $\mu = n - m + 1$  is the cyclomatic number  $\mu(S)$ . The number of internal node pairs can be found as  $\vartheta(S) = \rho - 1 = m - 2$ , where  $\rho = m - 1$  is the total number of node pairs.

Finally,  $n = \Theta(S) + \kappa(S)$  in the *n*-dimensional space and the mixed divalent tensor can be introduced as  $Q = T \otimes \Lambda$ , where  $\otimes$  is the operator of direct tensor multiplication; *T* is the univalent covariant tensor of the average packet delay with coordinates  $\tau_j$  of the packet delay along the *j*th coordinate path

(s);  $\Lambda$  is the univalent contravariant tensor of the traffic intensity with coordinates  $\lambda^i$  of the packet flow intensity along the *i*th coordinate path (1/s).

There are two coordinate systems (CS) considering in the presented model: the CS of edges (type v), and the CS of interpolar paths and internal node pairs (type  $\gamma \varepsilon$ ). In addition,  $\kappa(S)$  of the linearly independent paths  $\{\gamma_i, i = \overline{1, \kappa}\}$  from all the possible interpolar paths (end-to-end paths from the source node to destination) selected, and the set of internal node pairs  $\{\varepsilon_j, j = \overline{1, 9}\}$  is defined. These two sets in the CS of  $\gamma \varepsilon$  -type form the basis of the *n*-dimensional space-structure.

According to the Kron's second generalization postulate [2] the tensor Q in the CS of edges defined in the form of vector equation as:

$$\Lambda_{\nu}(t) = G_{\nu}(t)T_{\nu}(t), \qquad (4)$$

where  $\Lambda_{\nu}(t)$  is the projection of tensor  $\Lambda(t)$ ,  $T_{\nu}(t)$  is the projections of the tensor T(t), and  $G_{\nu}(t) = \left\|g_{\nu}^{ij}(t)\right\|$  is the

 $n \times n$  diagonal matrix. Vectors  $\Lambda_v(t)$  and  $T_v(t)$  have dimension n and correspond to the flow intensities and packet delays in the edges. Using the approach proposed in [1] and the application of PSFFA M/G/1 [3, 4] model for the description of the dynamics of the interface state, for the network edges  $\{v_i, i = \overline{1, n}\}$  diagonal elements of  $G_v(t)$  can be calculated as:

$$g_{\nu}^{ll}(t) = \lambda_{\nu}^{l}(\varphi - \lambda) / [(\varphi \cdot W(0, -(\lambda \cdot \exp(-(\lambda + (t - (\lambda + \varphi \cdot \ln(\exp(-(\lambda \cdot (\tau_{0}\lambda - \tau_{0}\varphi + 1))/\varphi) \times (\tau_{0}\lambda - \tau_{0}\varphi + 1)))/(\varphi - \lambda)^{2}) \cdot (\varphi - \lambda)^{2})/(\varphi))/(\varphi))/(\lambda + 1], (5)$$

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where  $\lambda_{v}^{i}$  is the *k*th flow intensity in the link modeled by the *i*th edge;  $W(\cdot)$  is the Lambert W function;  $\exp(\cdot)$  is the exponential function;  $\tau_{0}$  is the average delay at the interface at the initial moment of time;  $i = \overline{1, n}$  shows the number of link in the network; the index v shows the type of CS.

As before [1] the coordinate transformation rules are linear:  

$$\Lambda_{\nu}(t) = C \Lambda_{\gamma \varepsilon}(t) , \qquad (6)$$

where  $\Lambda_{\gamma \varepsilon}(t)$  are *n*-dimensional vectors (projection of tensor  $\Lambda(t)$  in CS of type  $\gamma \varepsilon$ ) with components:

$$\Lambda_{\gamma\varepsilon}(t) = \begin{bmatrix} \Lambda_{\gamma}(t) \\ -- \\ \Lambda_{\varepsilon}(t) \end{bmatrix}, \ \Lambda_{\gamma}(t) = \begin{bmatrix} \lambda_{\gamma}^{1}(t) \\ \vdots \\ \lambda_{\gamma}^{\kappa}(t) \end{bmatrix}, \ \Lambda_{\varepsilon}(t) = \begin{bmatrix} \lambda_{\varepsilon}^{1}(t) \\ \vdots \\ \lambda_{\varepsilon}^{9}(t) \end{bmatrix}, (7)$$

where  $\Lambda_{\gamma}(t)$  is the  $\kappa$  -dimensional subvector of flow intensities in the end-to-end (interpolar) paths;  $\Lambda_{\varepsilon}(t)$  is the 9dimensional subvector of flow intensities in the internal node pairs;  $\lambda_{\gamma}^{i}(t)$  is the flow intensity in the end-to-end path  $\gamma_{i}$ ;

# $\lambda_{\varepsilon}^{j}(t)$ is the flow intensity in the internal node pair $\varepsilon_{j}$ .

The tensor of average delays T(t) projection in the CS of type  $\gamma \varepsilon$  is represented with the structure:

$$T_{\gamma\varepsilon}(t) = \begin{bmatrix} T_{\gamma}(t) \\ -- \\ T_{\varepsilon}(t) \end{bmatrix}, \ T_{\gamma}(t) = \begin{bmatrix} \tau_{1}^{\gamma}(t) \\ \vdots \\ \tau_{\kappa}^{\gamma}(t) \end{bmatrix}, \ T_{\varepsilon}(t) = \begin{bmatrix} \tau_{1}^{\varepsilon}(t) \\ \vdots \\ \tau_{\vartheta}^{\varepsilon}(t) \end{bmatrix}, \ (8)$$

where  $\tau_i^{\gamma}(t)$  is the average packet delay in the path  $\gamma_i$ ;  $\tau_j^{\varepsilon}(t)$  is the average packet delay in the internal node pair  $\varepsilon_i$ .

The covariant character of delay tensor T(t) induces the law of coordinate transformation:

$$T_{\nu}(t) = A \ T_{\nu \varepsilon}(t) , \qquad (9)$$

where A is the  $n \times n$  matrix of covariant coordinate transformation in transition from CS of interpolar paths and internal node pairs to CS of edges. The matrix A is connected to the matrix of contravariant coordinate transformation C by orthogonality condition  $CA^t = I$ , where I is the  $n \times n$  unit matrix, and  $[\cdot]^t$  is the transposition operator. The expression (4) in the CS of type  $\gamma \varepsilon$  is determined:

$$\Lambda_{\gamma\varepsilon}(t) = G_{\gamma\varepsilon}(t) T_{\gamma\varepsilon}(t) , \qquad (10)$$

where  $G_{\gamma\varepsilon}(t)$  is the projection of the tensor G(t) in the CS of interpolar paths and internal node pairs. Then G(t) is the diva-

lent contravariant metric tensor:

$$G_{\gamma\varepsilon}(t) = A^t G_v(t) A . \tag{11}$$

In vector-matrix form the expression (10) is rewritten:

$$\begin{bmatrix} \Lambda_{\gamma}(t) \\ -- \\ \Lambda_{\varepsilon}(t) \end{bmatrix} = \begin{vmatrix} G_{\gamma\varepsilon}^{\langle 1 \rangle}(t) & | & G_{\gamma\varepsilon}^{\langle 2 \rangle}(t) \\ --- & + & --- \\ G_{\gamma\varepsilon}^{\langle 3 \rangle}(t) & | & G_{\gamma\varepsilon}^{\langle 4 \rangle}(t) \end{vmatrix} \begin{bmatrix} T_{\gamma}(t) \\ -- \\ T_{\varepsilon}(t) \end{bmatrix}, \quad (12)$$

where

$$\begin{vmatrix} G_{\gamma\varepsilon}^{\langle 1 \rangle}(t) & | & G_{\gamma\varepsilon}^{\langle 2 \rangle}(t) \\ --- & + & --- \\ G_{\gamma\varepsilon}^{\langle 3 \rangle}(t) & | & G_{\gamma\varepsilon}^{\langle 4 \rangle}(t) \end{vmatrix} = G_{\gamma\varepsilon}(t) ,$$

 $G_{\gamma\varepsilon}^{\langle 1 \rangle}(t)$  is the square  $\kappa \times \kappa$  submatrix;  $G_{\gamma\varepsilon}^{\langle 4 \rangle}(t)$  is the square  $\vartheta \times \vartheta$  submatrix;  $G_{\gamma\varepsilon}^{\langle 2 \rangle}(t)$  is the  $\kappa \times \vartheta$  submatrix;  $G_{\gamma\varepsilon}^{\langle 3 \rangle}(t)$  is the  $\vartheta \times \kappa$  submatrix; supposed that  $\Lambda_{\varepsilon}(t) = 0$ .

Let us formulate the conditions for the QoS guarantees in communications network according to the routing tensor model in coordinate system of interpolar paths and internal node pairs (4)-(12). Suppose, that the following numerical requirements are known: the average packet end-to-end delay  $\tau_{req}^k$  and average packet rate  $\lambda_{req}^k$  for the *k*th flow. Then the conditions for the QoS guarantees can be stated as follows:

$$\Lambda_{\gamma}(t) \leq \left(G_{\gamma\varepsilon}^{\langle 1 \rangle}(t) - G_{\gamma\varepsilon}^{\langle 2 \rangle}(t) \left[G_{\gamma\varepsilon}^{\langle 4 \rangle}(t)\right]^{-1} G_{\gamma\varepsilon}^{\langle 3 \rangle}(t)\right] T_{\gamma}(t), (13)$$

where  $\sum_{i=1}^{\kappa} \lambda_{\gamma}^{i}(t) = \lambda_{req}^{k}$ ,  $\tau_{i}^{\gamma} = \tau_{req}^{k}$   $(i = \overline{1, \kappa})$ ,  $G_{\gamma \varepsilon}^{\langle 1 \rangle}(t)$  and

 $G_{\gamma\varepsilon}^{\langle 4 \rangle}(t)$  are the square  $\kappa \times \kappa$  and  $\vartheta \times \vartheta$  submatrices,  $G_{\gamma\varepsilon}^{\langle 2 \rangle}(t)$  is the  $\kappa \times \vartheta$  submatrix,  $G_{\gamma\varepsilon}^{\langle 3 \rangle}(t)$  is the  $\vartheta \times \kappa$  submatrix. Besides, all components of the presented model are the functions of time. Thus, the inequality (16) is a condition for the QoS guarantees over the set of metrics: the average packet rate  $\lambda_{req}^k$  and average packet end-to-end delay  $\tau_{req}^k$  concurrently.

### **III.** Conclusion

Thus, within the presented routing tensor model in the basis of interpolar paths and internal node pairs the QoS guarantees are provided for all paths, and the average delays along different paths will not exceed the acceptable value according to the use of defined coordinate system in compliance with the requirements for the packet end-to-end delay.

During the research it was found, that the use of selected coordinate system of tensor representation as network model allowed to obtain the solution of routing problems, where the end-to-end delay in all paths converged within the update timer to the values corresponding to QoSrequirements. Moreover, in contrast to the use of CS of circuits and node pairs [1], the end-to-end delay of the paths may not be equal, reducing the computational complexity in obtaining the desired solutions.

#### **IV. References**

- O.V. Lemeshko, O.S. Yeremenko, and Ahmad M. Hailan, "Investigation of Multipath QoS-Routing Dynamic Tensor Model," International Conference on Electronics and Information Technology (EIT'2016), Conference Proceedings, Odesa, Ukraine, pp. 59-62, May 23–27, 2016.
- [2] G. Kron, "Tensor analysis of networks," Wiley, New York, 1939, 635 p.
- [3] K. Xu, D. Tipper, Y. Qian, P. Krishnamurthy, and S. Tipmongkonsilp, "Time-Varying Performance Analysis of Multihop Wireless Networks With CBR Traffic," IEEE TRANSACTIONS ON VE-HICULAR TECHNOLOGY, VOL. 63, NO. 7, pp. 3397–3409, September 2014.
- [4] O.S. Yeremenko, T.M. Lebedenko, T.V. Vavenko, and M.V. Semenyaka, "Investigation of Queue Utilization on Network Routers by the Use of Dynamic Models," Second International IEEE Conference Problems of Infocommunications. Science and Technology, pp. 46-49, October 13-15, 2015.

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