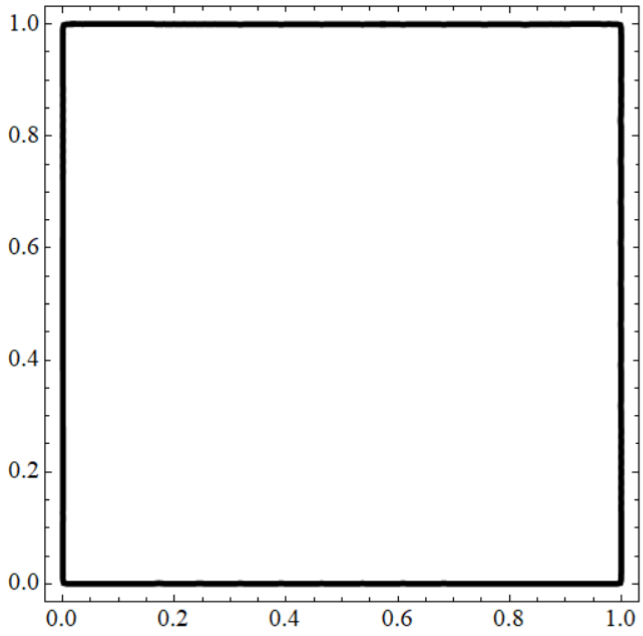
ДОДАТОК А

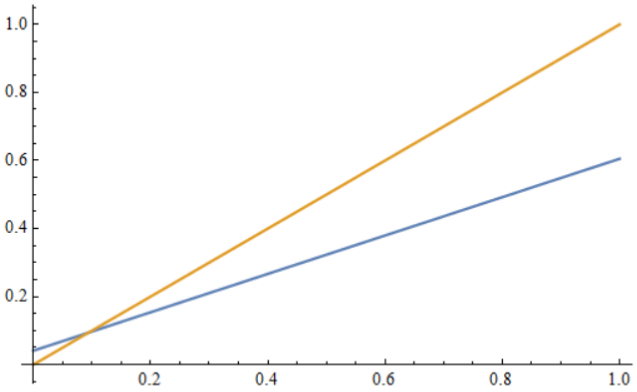
Лістінг програми

```
t1 = AbsoluteTime[];

ϵ = 10^-4;

con[x1_, x2_] = x1 + x2 - Sqrt[x1^2 + x2^2];

a = 1; b = 1;

s1[x1_, x2_] = x1 (a - x1) / a;

s2[x1_, x2_] = x2 (b - x2) / b;

Om[x1_, x2_] = con[s1[x1, x2], s2[x1, x2]];

Om[x1, x2]

(1 - x1) x1 + (1 - x2) x2 - Sqrt[(1 - x1)^2 x1^2 + (1 - x2)^2 x2^2]

f[x1_, x2_, u_] = λ (1 + u)^p;

f̂[x1_, x2_, v_, w_] = λ (1 + v)^p;

Gr0 = ContourPlot[Om[x1, x2] == 0, {x1, 0 - 0.01, a + 0.01}, {x2, 0 - 0.01, b + 0.01}, AspectRatio → Automatic,
  ContourStyle → {Black, Thickness[0.01]}, BaseStyle → {14, FontFamily → "Times"}]
```



```
(*zadanie kvazafunkcyi Grina-Rvachova*)

G2[x1_, x2_, ξ1_, ξ2_] = 1/(2 Pi) (Log[1/Sqrt[(x1 - ξ1)^2 + (x2 - ξ2)^2]] - Log[1/Sqrt[(x1 - ξ1)^2 + (x2 - ξ2)^2 + 4 Om[x1, x2] Om[ξ1, ξ2]]]);

K[x1_, x2_, ξ1_, ξ2_] =
  -1/(2 Pi) (D[Log[1/Sqrt[(x1 - ξ1)^2 + (x2 - ξ2)^2 + 4 Om[x1, x2] Om[ξ1, ξ2]]], {ξ1, 2}] + D[Log[1/Sqrt[(x1 - ξ1)^2 + (x2 - ξ2)^2 + 4 Om[x1, x2] Om[ξ1, ξ2]]], {ξ2, 2}]);

K1[x1_, x2_, ξ1_, ξ2_] = Max[0, K[x1, x2, ξ1, ξ2]];

K2[x1_, x2_, ξ1_, ξ2_] = Max[0, -K[x1, x2, ξ1, ξ2]];

(*formula Gaussa*)
NN = 16;

Uzly = {-0.989400934991650, -0.944575023073233, -0.865631202387832, -0.755404408355003, -0.617876244402644, -0.458016777657227,
   -0.281603550779259, -0.095012509837637, 0.095012509837637, 0.281603550779259, 0.458016777657227, 0.617876244402644,
   0.755404408355003, 0.865631202387832, 0.944575023073233, 0.989400934991650};

Vesa = {0.027152459411754, 0.062253523938648, 0.095158511682493, 0.124628971255534, 0.149595988816577, 0.169156519395003,
   0.182603415044924, 0.189450610455069, 0.189450610455069, 0.182603415044924, 0.169156519395003, 0.149595988816577,
   0.124628971255534, 0.095158511682493, 0.062253523938648, 0.027152459411754};


nGx1 = 1;

nGx2 = 1;

aG = a;

bG = b;

ax1 = Table[iG aG/nGx1, {iG, 0, nGx1}];

ax2 = Table[jG bG/nGx2, {jG, 0, nGx2}];

Nx1 = 20; (*тут можно взять 20 или 30*)

Nx2 = 20;(*тут можно взять 20 или 30*)

Δx1 = N[a/Nx1];

Δx2 = N[b/Nx2];

X1 = Table[i Δx1, {i, 0, Nx1}];

X2 = Table[j Δx2, {j, 0, Nx2}];

B1[t_] = Piecewise[{{t + 1, -1 ≤ t < 0}, {1 - t, 0 ≤ t ≤ 1}, {0, t < -1}, {0, t > 1}}];

u0[x1_, x2_] := NIntegrate[G2[x1, x2, ξ1, ξ2], {ξ1, 0, a}, {ξ2, 0, b}]


 M = u0[a / 2, b / 2]
 0.0409306

 KK[x1_, x2_] := NIntegrate[Abs[K[x1, x2, ξ1, ξ2]], {ξ1, 0, a}, {ξ2, 0, b}]

 M1 = KK[a / 2, b / 2]
 0.708195

 Beep[]
```

```
(*podbor parametrov*)

λ = 1; p = 0.1;

L = λ p;

γ = M1 + L M

0.712288

γ < 1

True

KK1[x1_, x2_] := NIntegrate[K1[x1, x2, ξ1, ξ2], {ξ1, 0, a}, {ξ2, 0, b}]

MM1 = KK1[a / 2, b / 2]

0.561201

Beep[]

KK2[x1_, x2_] := NIntegrate[K2[x1, x2, ξ1, ξ2], {ξ1, 0, a}, {ξ2, 0, b}]


sol = Reduce[β MM1 + M f̂[x1, x2, β, 0] ≤ β, β]

sol = Reduce[β MM1 + M f̂[x1, x2, β, 0] ≤ β, β]

β ≥ 0.0941217

Plot[{β MM1 + M f̂[x1, x2, β, 0], β}, {β, 0, 1}]
```




```
β = sol[[2]]

0.0941217


v0[x1_, x2_] = 0;

w0[x1_, x2_] = β;

FLow[x1_, x2_, ξ1_, ξ2_] = K1[x1, x2, ξ1, ξ2] v0[ξ1, ξ2] - K2[x1, x2, ξ1, ξ2] w0[ξ1, ξ2] + G2[x1, x2, ξ1, ξ2] f̂[ξ1, ξ2, v0[ξ1, ξ2], w0[ξ1, ξ2]];

v1[x1_, x2_] = (a b / 4) Sum_{i=1}^{NN} Sum_{j=1}^{NN} (Vesa[[i]] Vesa[[j]] FLow[x1, x2, a/2 (1 + Uzly[[i]]), b/2 (1 + Uzly[[j]])]);

v1table = ParallelTable[v1[X1[[i]], X2[[j]]], {i, 1, Nx1 + 1}, {j, 1, Nx2 + 1}];
```

```
Do[{v1table[[Nx1 + 1, j]] = 0, v1table[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{v1table[[i, Nx2 + 1]] = 0, v1table[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
MatrixForm[v1table]
```

0	0	0	0	0	0	0	0	0	0	0	0
0	-0.00725862	-0.00528302	-0.0037926	-0.0022067	-0.00132844	-0.000655028	5.49738×10^{-6}	0.000372463	0.000601524	0.000619146	0.00060
0	-0.00528302	-0.00258623	-0.000430099	0.00175832	0.0030793	0.00395974	0.00537848	0.00525223	0.00633423	0.00566389	0.00633
0	-0.0037926	-0.000430099	0.00246431	0.00528883	0.00708395	0.00856934	0.00991688	0.0103568	0.0112487	0.0109005	0.0112
0	-0.0022067	0.00175832	0.00528883	0.00874326	0.0108089	0.0124763	0.0146628	0.0146745	0.0165841	0.0153802	0.0165
0	-0.00132844	0.0030793	0.00708395	0.0108089	0.0134987	0.0154757	0.0175377	0.0181368	0.0194338	0.0189965	0.0194
0	-0.000655028	0.00395974	0.00856934	0.0124763	0.0154757	0.0177989	0.0199292	0.0208291	0.0219701	0.0218173	0.0219
0	5.49738×10^{-6}	0.00537848	0.00991688	0.0146628	0.0175377	0.0199292	0.0228078	0.0230454	0.025499	0.024124	0.0254
0	0.000372463	0.00525223	0.0103568	0.0146745	0.0181368	0.0208291	0.0230454	0.0243351	0.0254639	0.0254066	0.0254
0	0.000601524	0.00633423	0.0112487	0.0165841	0.0194338	0.0219701	0.025499	0.0254639	0.0294088	0.0265861	0.0294
0	0.000619146	0.00566389	0.0109005	0.0153802	0.0189965	0.0218173	0.024124	0.0254066	0.0265861	0.0267088	0.0265
0	0.000601524	0.00633423	0.0112487	0.0165841	0.0194338	0.0219701	0.025499	0.0254639	0.0294088	0.0265861	0.0294
0	0.000372463	0.00525223	0.0103568	0.0146745	0.0181368	0.0208291	0.0230454	0.0243351	0.0254639	0.0254066	0.0254
0	5.49738×10^{-6}	0.00537848	0.00991688	0.0146628	0.0175377	0.0199292	0.0228078	0.0230454	0.025499	0.024124	0.0254
0	-0.000655028	0.00395974	0.00856934	0.0124763	0.0154757	0.0177989	0.0199292	0.0208291	0.0219701	0.0218173	0.0219
0	-0.00132844	0.0030793	0.00708395	0.0108089	0.0134987	0.0154757	0.0175377	0.0181368	0.0194338	0.0189965	0.0194
0	-0.0022067	0.00175832	0.00528883	0.00874326	0.0108089	0.0124763	0.0146628	0.0146745	0.0165841	0.0153802	0.0165
0	-0.0037926	-0.000430099	0.00246431	0.00528883	0.00708395	0.00856934	0.00991688	0.0103568	0.0112487	0.0109005	0.0112
0	-0.00528302	-0.00258623	-0.000430099	0.00175832	0.0030793	0.00395974	0.00537848	0.00525223	0.00633423	0.00566389	0.00633
0	-0.00725862	-0.00528302	-0.0037926	-0.0022067	-0.00132844	-0.000655028	5.49738×10^{-6}	0.000372463	0.000601524	0.000619146	0.00060
0	0	0	0	0	0	0	0	0	0	0	0

$$v1Spl[x1\_, x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} v1table[[i+1, j+1]]\, B1\left[\frac{Nx1\, x1}{a} - i\right] B1\left[\frac{Nx2\, x2}{b} - j\right];$$

```
v1[x1_, x2_] = v1Spl[x1, x2];
```

```
FHigh[x1_, x2_, ξ1_, ξ2_] = K1[x1, x2, ξ1, ξ2] w0[ξ1, ξ2] - K2[x1, x2, ξ1, ξ2] v0[ξ1, ξ2] + G2[x1, x2, ξ1, ξ2] f̂[ξ1, ξ2, w0[ξ1, ξ2], v0[ξ1, ξ2]];
```

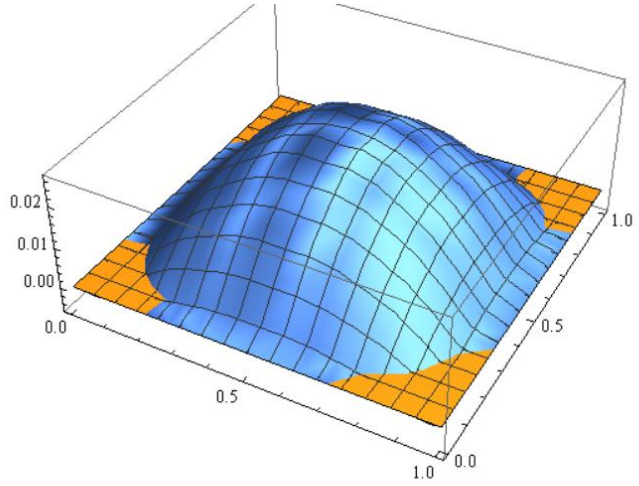
$$w1[x1\_, x2\_] = \frac{a\,b}{4}\sum_{i=1}^{NN}\sum_{j=1}^{NN}\left(Vesa[[i]]\, Vesa[[j]]\, FHigh\left[x1, x2, \frac{a}{2}(1 + Uzly[[i]]), \frac{b}{2}(1 + Uzly[[j]])\right]\right);$$

```
w1table = ParallelTable[w1[X1[[i]], X2[[j]]], {i, 1, Nx1 + 1}, {j, 1, Nx2 + 1}];

Do[{w1table[[Nx1 + 1, j]] = 0, w1table[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{w1table[[i, Nx2 + 1]] = 0, w1table[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
MatrixForm[w1table]
```

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0.0317614	0.0324342	0.0307848	0.0323543	0.0307892	0.0290208	0.0324675	0.0274421	0.0330906	0.0271531	0.0330906	0.0274421	0.0324675	0.
0	0.0324342	0.0420554	0.0454047	0.0469987	0.0476897	0.0479376	0.0486358	0.0482005	0.049119	0.0482908	0.049119	0.0482005	0.0486358	0.
0	0.0307848	0.0454047	0.0525324	0.056377	0.0587832	0.0600661	0.0613985	0.0615912	0.0624104	0.0620641	0.0624104	0.0615912	0.0613985	0.
0	0.0323543	0.0469987	0.056377	0.0630355	0.0665054	0.0688167	0.0713878	0.0714859	0.0735188	0.0722853	0.0735188	0.0714859	0.0713878	0.
0	0.0307892	0.0476897	0.0587832	0.0665054	0.0714595	0.0748581	0.0776801	0.0786653	0.0802484	0.0797933	0.0802484	0.0786653	0.0776801	0.
0	0.0290208	0.0479376	0.0600661	0.0688167	0.0748581	0.0790148	0.0821877	0.0837944	0.0853265	0.0852101	0.0853265	0.0837944	0.0821877	0.
0	0.0324675	0.0486358	0.0613985	0.0713878	0.0776801	0.0821877	0.0866083	0.087666	0.0905414	0.0892757	0.0905414	0.087666	0.0866083	0.
0	0.0274421	0.0482005	0.0615912	0.0714859	0.0786653	0.0837944	0.087666	0.0898282	0.0914775	0.0917171	0.0914775	0.0898282	0.087666	0.
0	0.0330906	0.049119	0.0624104	0.0735188	0.0802484	0.0853265	0.0905414	0.0914775	0.0960103	0.0933992	0.0960103	0.0914775	0.0905414	0.
0	0.0271531	0.0482908	0.0620641	0.0722853	0.0797933	0.0852101	0.0892757	0.0917171	0.0933992	0.0935591	0.0933992	0.0917171	0.0892757	0.
0	0.0330906	0.049119	0.0624104	0.0735188	0.0802484	0.0853265	0.0905414	0.0914775	0.0960103	0.0933992	0.0960103	0.0914775	0.0905414	0.
0	0.0274421	0.0482005	0.0615912	0.0714859	0.0786653	0.0837944	0.087666	0.0898282	0.0914775	0.0917171	0.0914775	0.0898282	0.087666	0.
0	0.0324675	0.0486358	0.0613985	0.0713878	0.0776801	0.0821877	0.0866083	0.087666	0.0905414	0.0892757	0.0905414	0.087666	0.0866083	0.
0	0.0290208	0.0479376	0.0600661	0.0688167	0.0748581	0.0790148	0.0821877	0.0837944	0.0853265	0.0852101	0.0853265	0.0837944	0.0821877	0.
0	0.0307892	0.0476897	0.0587832	0.0665054	0.0714595	0.0748581	0.0776801	0.0786653	0.0802484	0.0797933	0.0802484	0.0786653	0.0776801	0.
0	0.0323543	0.0469987	0.056377	0.0630355	0.0665054	0.0688167	0.0713878	0.0714859	0.0735188	0.0722853	0.0735188	0.0714859	0.0713878	0.
0	0.0307848	0.0454047	0.0525324	0.056377	0.0587832	0.0600661	0.0613985	0.0615912	0.0624104	0.0620641	0.0624104	0.0615912	0.0613985	0.
0	0.0324342	0.0420554	0.0454047	0.0469987	0.0476897	0.0479376	0.0486358	0.0482005	0.049119	0.0482908	0.049119	0.0482005	0.0486358	0.
0	0.0317614	0.0324342	0.0307848	0.0323543	0.0307892	0.0290208	0.0324675	0.0274421	0.0330906	0.0271531	0.0330906	0.0274421	0.0324675	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	

$$w1Spl[x1\_, x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} w1table[[i+1, j+1]]\, B1\left[\frac{Nx1\, x1}{a} - i\right] B1\left[\frac{Nx2\, x2}{b} - j\right];$$

```
w1[x1_, x2_] = w1Spl[x1, x2];

Plot3D[{0, v1[x1, x2] - v0[x1, x2]}, {x1, 0, a}, {x2, 0, b}]
```



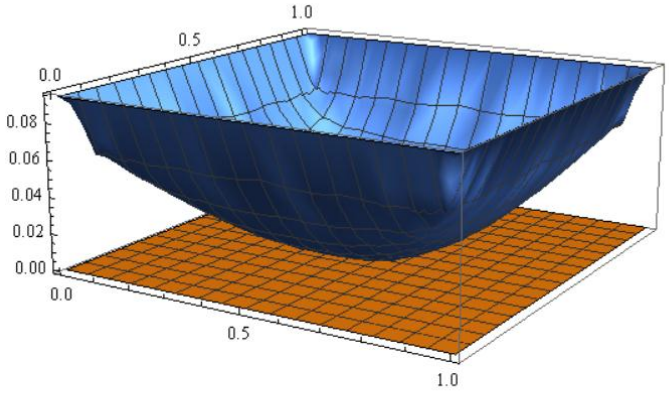

```
v1[0.01, 0.01] - v0[0.01, 0.01]
```

```
-0.000290345
```

```
w0[a / 2, b / 2] - w1[a / 2, b / 2]
```

```
0.000562673
```

```
Plot3D[{0, w0[x1, x2] - w1[x1, x2]}, {x1, 0, a}, {x2, 0, b}]
```




```
Beep[]
```

```
(*initial iteration*)
```

```
eps = {};
```

```
v0[x1_, x2_] = 0;
```

```
w0[x1_, x2_] = β;
```

```
mm = Maximize[{w0[x1, x2] - v0[x1, x2], 0 ≤ x1 ≤ a, 0 ≤ x1 ≤ b}, {x1, x2}]
```

```
{0.0941217, {x1 → 0, x2 → 0}}
```

```
eps = Append[eps, 1 / 2. mm[[1]]]
```

```
{0.0470609}
```

```
Last[eps] < ϵ
```

```
False
```

```
vtab = {};

wtab = {};

v0tab = ParallelTable[Re[v0[i Δx1, j Δx2]], {i, 0, Nx1}, {j, 0, Nx2}];
Do[{v0tab[[Nx1 + 1, j]] = 0, v0tab[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{v0tab[[i, Nx2 + 1]] = 0, v0tab[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
MatrixForm[v0tab]
```

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
w0tab = ParallelTable[Re[w0[i Δx1, j Δx2]], {i, 0, Nx1}, {j, 0, Nx2}];
Do[{w0tab[[Nx1 + 1, j]] = 0, w0tab[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{w0tab[[i, Nx2 + 1]] = 0, w0tab[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
MatrixForm[w0tab]
```

```
0 0         0         0         0         0         0         0         0         0         0         0         0         0
0 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.
0 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.
0 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.
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0 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.0941217 0.
0 0         0         0         0         0         0         0         0         0         0         0         0         0
```

```
vtab = Append[vtab, v0tab];

wtab = Append[wtab, w0tab];

k = 0;
```

$$\text{vprev}[x1\_,\ x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} \text{vtab}[[k+1,\ i+1,\ j+1]]\ \text{B1}\left[\frac{Nx1\ x1}{a} - i\right]\ \text{B1}\left[\frac{Nx2\ x2}{b} - j\right];$$

$$\text{wprev}[x1\_,\ x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} \text{wtab}[[k+1,\ i+1,\ j+1]]\ \text{B1}\left[\frac{Nx1\ x1}{a} - i\right]\ \text{B1}\left[\frac{Nx2\ x2}{b} - j\right];$$

```
FLow[x1_, x2_, s1_, s2_] = K1[x1, x2, s1, s2] vprev[s1, s2] - K2[x1, x2, s1, s2] wprev[s1, s2] +
    G2[x1, x2, s1, s2] f̂[s1, s2, vprev[s1, s2], wprev[s1, s2]];
```

```
vtmp[x1_, x2_] :=
  Sum[Sum[((ax1[[i1 + 1]] - ax1[[i1]]) (ax2[[i2 + 1]] - ax2[[i2]]))/4
      Sum[Sum[Vesa[[j1]] Vesa[[j2]] FLow[x1, x2, 1/2 (ax1[[i1 + 1]] + ax1[[i1]] + (ax1[[i1 + 1]] - ax1[[i1]]) Uzly[[j1]]),
          1/2 (ax2[[i2 + 1]] + ax2[[i2]] + (ax2[[i2 + 1]] - ax2[[i2]]) Uzly[[j2]])], {j2, 1, NN}], {j1, 1, NN}], {i2, 1, nGx2}], {i1, 1, nGx1}];

t3 = AbsoluteTime[];

vtmp[a / 2, b / 2]

0.0331196

t4 = AbsoluteTime[];

tt = t4 - t3

0.7100010


tt * 200 / 60

2.366670

tt * 800 / 60 / 4 / 60 * 12

0.4733340


Beep[]

While[Last[eps] > ϵ, {t3 = AbsoluteTime[]; vprev[x1_, x2_] = Sum[Sum[vtab[[k + 1, i + 1, j + 1]] B1[Nx1 x1/a - i] B1[Nx2 x2/b - j], {j, 0, Nx2}], {i, 0, Nx1}];

  wprev[x1_, x2_] = Sum[Sum[wtab[[k + 1, i + 1, j + 1]] B1[Nx1 x1/a - i] B1[Nx2 x2/b - j], {j, 0, Nx2}], {i, 0, Nx1}];

  FLow[x1_, x2_, s1_, s2_] = K1[x1, x2, s1, s2] vprev[s1, s2] - K2[x1, x2, s1, s2] wprev[s1, s2] +
    G2[x1, x2, s1, s2] f̂[s1, s2, vprev[s1, s2], wprev[s1, s2]];
  vtmp[x1_, x2_] :=
  Sum[Sum[((ax1[[i1 + 1]] - ax1[[i1]]) (ax2[[i2 + 1]] - ax2[[i2]]))/4
      Sum[Sum[Vesa[[j1]] Vesa[[j2]] FLow[x1, x2, 1/2 (ax1[[i1 + 1]] + ax1[[i1]] + (ax1[[i1 + 1]] - ax1[[i1]]) Uzly[[j1]]),
          1/2 (ax2[[i2 + 1]] + ax2[[i2]] + (ax2[[i2 + 1]] - ax2[[i2]]) Uzly[[j2]])], {j2, 1, NN}], {j1, 1, NN}], {i2, 1, nGx2}], {i1, 1, nGx1}];
FHigh[x1_, x2_, s1_, s2_] = K1[x1, x2, s1, s2] wprev[s1, s2] - K2[x1, x2, s1, s2] vprev[s1, s2] +
  G2[x1, x2, s1, s2] f̂[s1, s2, wprev[s1, s2], vprev[s1, s2]];
wtmp[x1_, x2_] :=
  Sum[Sum[((ax1[[i1 + 1]] - ax1[[i1]]) (ax2[[i2 + 1]] - ax2[[i2]]))/4
      Sum[Sum[Vesa[[j1]] Vesa[[j2]] FHigh[x1, x2, 1/2 (ax1[[i1 + 1]] + ax1[[i1]] + (ax1[[i1 + 1]] - ax1[[i1]]) Uzly[[j1]]),
          1/2 (ax2[[i2 + 1]] + ax2[[i2]] + (ax2[[i2 + 1]] - ax2[[i2]]) Uzly[[j2]])], {j2, 1, NN}], {j1, 1, NN}], {i2, 1, nGx2}], {i1, 1, nGx1}];
```

```
vktab = ParallelTable[Re[vtmp[i Δx1, j Δx2]], {i, 0, Nx1}, {j, 0, Nx2}];
Do[{vktab[[Nx1 + 1, j]] = 0, vktab[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{vktab[[i, Nx2 + 1]] = 0, vktab[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
wktab = ParallelTable[Re[wtmp[i Δx1, j Δx2]], {i, 0, Nx1}, {j, 0, Nx2}];
Do[{wktab[[Nx1 + 1, j]] = 0, wktab[[1, j]] = 0}, {j, 1, Nx2 + 1}];
Do[{wktab[[i, Nx2 + 1]] = 0, wktab[[i, 1]] = 0}, {i, 1, Nx1 + 1}];
vtab = Append[vtab, vktab];
wtab = Append[wtab, wktab];
```

$$vnew[x1\_, x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} vtab[[k+2, i+1, j+1]]\, B1\left[\frac{Nx1\, x1}{a} - i\right] B1\left[\frac{Nx2\, x2}{b} - j\right];$$

$$wnew[x1\_, x2\_] = \sum_{i=0}^{Nx1}\sum_{j=0}^{Nx2} wtab[[k+2, i+1, j+1]]\, B1\left[\frac{Nx1\, x1}{a} - i\right] B1\left[\frac{Nx2\, x2}{b} - j\right];$$

```
eps = Append[eps, 1 / 2 (wnew[a / 2, b / 2] - vnew[a / 2, b / 2])];
k = k + 1; t4 = AbsoluteTime[]; Print["Iteration ", k, " is complete, ", "eps=", Last[eps], "    t=", t4 - t3]; Beep[]}]
```

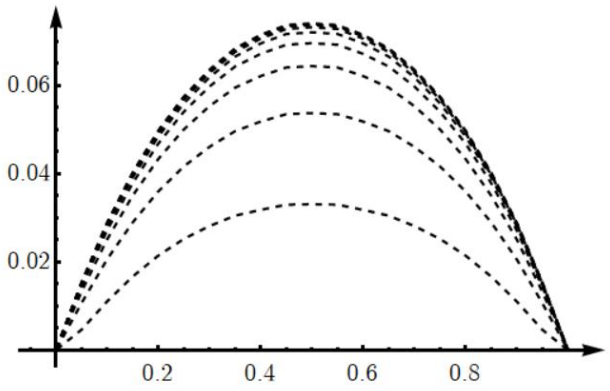
```
Iteration 1 is complete, eps=0.0302186   t=250.9304936
Iteration 2 is complete, eps=0.0164904   t=515.5105382
Iteration 3 is complete, eps=0.00822583   t=518.6117491
Iteration 4 is complete, eps=0.00393248   t=531.5753713
Iteration 5 is complete, eps=0.00184239   t=567.0498325
Iteration 6 is complete, eps=0.000854643   t=522.0705540
Iteration 7 is complete, eps=0.000394467   t=520.1981796
Iteration 8 is complete, eps=0.000181601   t=522.3265518
Iteration 9 is complete, eps=0.0000834922   t=527.5973643
```

```
Print["Всего выполнено итераций ", k]
```

```
Всего выполнено итераций 9
```

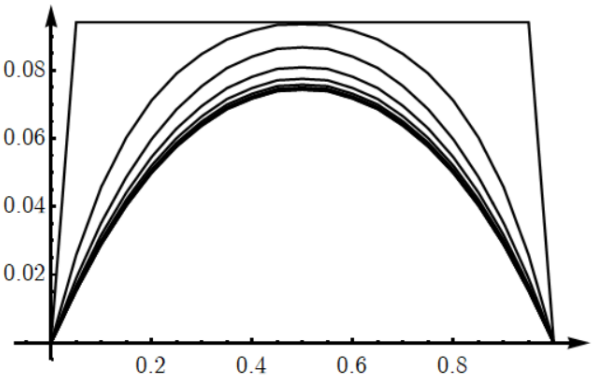
$$v[x1\_, x2\_] = Table\left[\sum_{i1=0}^{Nx1}\sum_{j1=0}^{Nx2} vtab[[j, i1+1, j1+1]]\, B1\left[\frac{Nx1\, x1}{a} - i1\right] B1\left[\frac{Nx2\, x2}{b} - j1\right], \{j, 1, k+1\}\right];$$

```
gr1 = Plot[v[x1, b / 2], {x1, -.05, a + .05}, PlotStyle → {Black, Dashed}, BaseStyle → {14, FontFamily → "Euclid"},
  AxesStyle → Directive[Arrowheads[0.04], Thick], PlotPoints → 200]
```



$$w[x1\_, x2\_] = Table\left[\sum_{i1=0}^{Nx1}\sum_{j1=0}^{Nx2} wtab[[j, i1+1, j1+1]]\, B1\left[\frac{Nx1\, x1}{a} - i1\right] B1\left[\frac{Nx2\, x2}{b} - j1\right], \{j, 1, k+1\}\right];$$

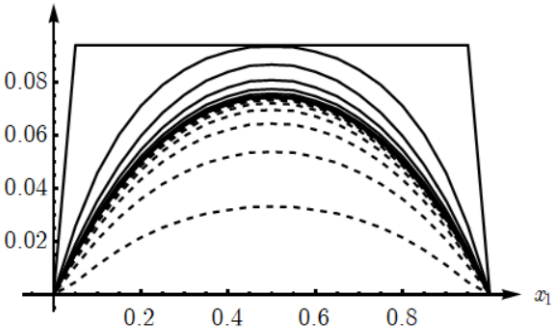
```
gr2 = Plot[w[x1, b / 2], {x1, -.05, a + .05}, PlotStyle → {Black}, BaseStyle → {14, FontFamily → "Euclid"},
  AxesStyle → Directive[Arrowheads[0.04], Thick], PlotPoints → 200]
```




```
Show[gr1, gr2, AxesLabel → {"x1", "w^(k)(x1, 1/2), v^(k)(x1, 1/2)"}, PlotRange -> {-0.001, w[a / 2, b / 2][[1]] + 0.01}]
```


```
Show[gr1, gr2, AxesLabel → {"x1", "w^(k)(x1, 1/2), v^(k)(x1, 1/2)"}, PlotRange -> {-0.001, w[a / 2, b / 2][[1]] + 0.01}]
```


$w^{(k)}(x_1,\frac{1}{2}),\ v^{(k)}(x_1,\frac{1}{2})$



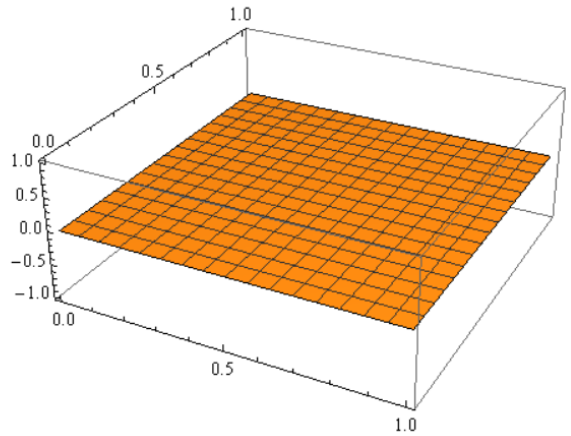

```
ulast[x1_, x2_] = (v[x1, x2][[k + 1]] + w[x1, x2][[k + 1]]) / 2;

hx1 = a / 10;

hx2 = b / 10;

nnx2 = Quotient[b, hx2]
10

Plot3D[v[x1, x2][[1]], {x1, 0, a}, {x2, 0, b}]
```




```
VTab = Transpose[Table[v[i hx1, j hx2], {i, 0, nnx1}, {j, 0, nnx2}]];
```

```
utab = Table[ulast[i hx1, j hx2], {i, 0, nnx1}, {j, 0, nnx2}];
Do[{utab[[nnx1 + 1, j]] = 0, utab[[1, j]] = 0}, {j, 1, nnx2 + 1}];
Do[{utab[[i, nnx2 + 1]] = 0, utab[[i, 1]] = 0}, {i, 1, nnx1 + 1}];
```

```
utab
```

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0.0128684, 0.0208956, 0.0254811, 0.0280669, 0.0289144, 0.0280669, 0.0254811, 0.0208956, 0.0128684, 0},
 {0, 0.0208956, 0.0352198, 0.0434676, 0.0482791, 0.0498547, 0.0482791, 0.0434676, 0.0352198, 0.0208956, 0},
 {0, 0.0254811, 0.0434676, 0.0550088, 0.0615614, 0.0636959, 0.0615614, 0.0550088, 0.0434676, 0.0254811, 0},
 {0, 0.0280669, 0.0482791, 0.0615614, 0.0691495, 0.0716259, 0.0691495, 0.0615614, 0.0482791, 0.0280669, 0},
 {0, 0.0289144, 0.0498547, 0.0636959, 0.0716259, 0.0742172, 0.0716259, 0.0636959, 0.0498547, 0.0289144, 0},
 {0, 0.0280669, 0.0482791, 0.0615614, 0.0691495, 0.0716259, 0.0691495, 0.0615614, 0.0482791, 0.0280669, 0},
 {0, 0.0254811, 0.0434676, 0.0550088, 0.0615614, 0.0636959, 0.0615614, 0.0550088, 0.0434676, 0.0254811, 0},
 {0, 0.0208956, 0.0352198, 0.0434676, 0.0482791, 0.0498547, 0.0482791, 0.0434676, 0.0352198, 0.0208956, 0},
 {0, 0.0128684, 0.0208956, 0.0254811, 0.0280669, 0.0289144, 0.0280669, 0.0254811, 0.0208956, 0.0128684, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

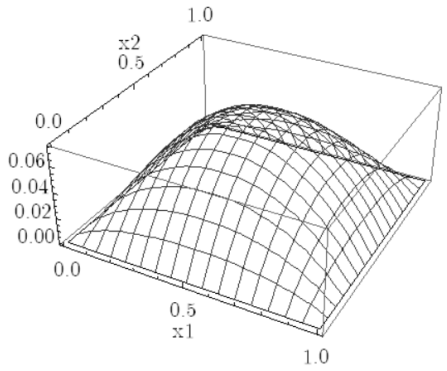
```
MatrixForm[utab]
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0128684 & 0.0208956 & 0.0254811 & 0.0280669 & 0.0289144 & 0.0280669 & 0.0254811 & 0.0208956 & 0.0128684 & 0 \\
0 & 0.0208956 & 0.0352198 & 0.0434676 & 0.0482791 & 0.0498547 & 0.0482791 & 0.0434676 & 0.0352198 & 0.0208956 & 0 \\
0 & 0.0254811 & 0.0434676 & 0.0550088 & 0.0615614 & 0.0636959 & 0.0615614 & 0.0550088 & 0.0434676 & 0.0254811 & 0 \\
0 & 0.0280669 & 0.0482791 & 0.0615614 & 0.0691495 & 0.0716259 & 0.0691495 & 0.0615614 & 0.0482791 & 0.0280669 & 0 \\
0 & 0.0289144 & 0.0498547 & 0.0636959 & 0.0716259 & 0.0742172 & 0.0716259 & 0.0636959 & 0.0498547 & 0.0289144 & 0 \\
0 & 0.0280669 & 0.0482791 & 0.0615614 & 0.0691495 & 0.0716259 & 0.0691495 & 0.0615614 & 0.0482791 & 0.0280669 & 0 \\
0 & 0.0254811 & 0.0434676 & 0.0550088 & 0.0615614 & 0.0636959 & 0.0615614 & 0.0550088 & 0.0434676 & 0.0254811 & 0 \\
0 & 0.0208956 & 0.0352198 & 0.0434676 & 0.0482791 & 0.0498547 & 0.0482791 & 0.0434676 & 0.0352198 & 0.0208956 & 0 \\
0 & 0.0128684 & 0.0208956 & 0.0254811 & 0.0280669 & 0.0289144 & 0.0280669 & 0.0254811 & 0.0208956 & 0.0128684 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

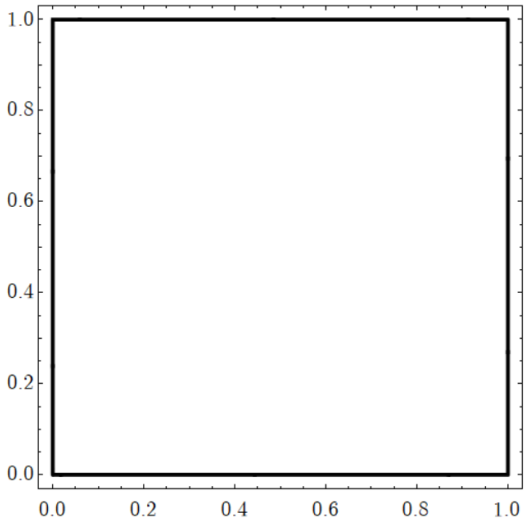
```
ulast[a / 2, b / 2]
```

```
0.0742172
```

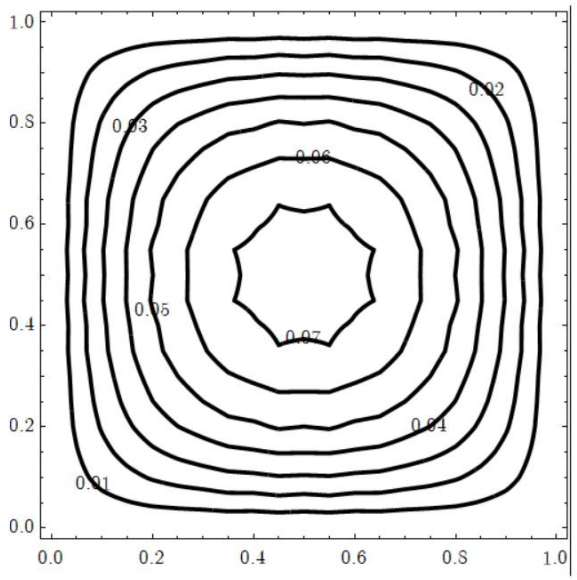
```
Plot3D[ulast[x1, x2], {x1, 0, a}, {x2, 0, b}, BaseStyle → {14, FontFamily → "Euclid"}, PlotStyle → None, AxesLabel → {"x1", "x2"}]
```



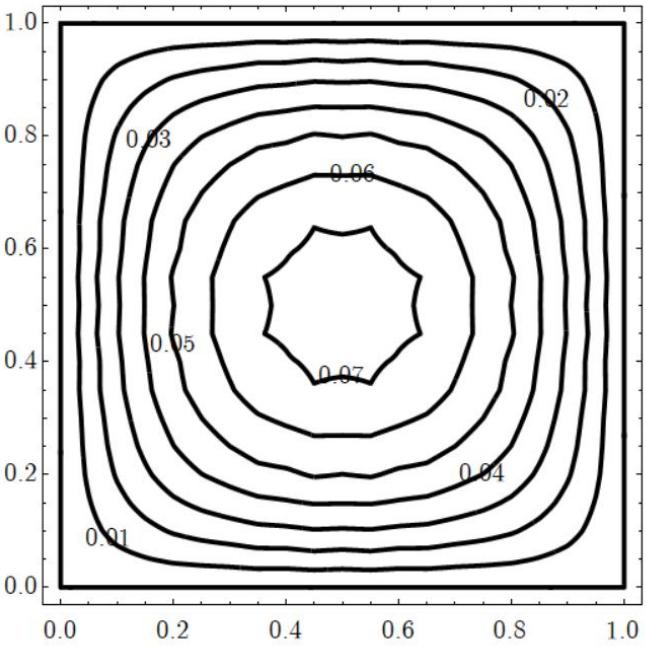

```
Gr0 = ContourPlot[Om[x1, x2] == 0, {x1, 0 - 0.01, a + 0.01}, {x2, 0 - 0.01, b + 0.01}, AspectRatio → Automatic,
  ContourStyle → {Black, Thickness[0.008]}, ContourLabels → Function[{x1, x2, z}, Text[Framed[z], {x1, x2}, Background → White]],
  BaseStyle → {14, FontFamily → "Euclid"}, ContourShading → None, PlotPoints → 200]
```



```
Gru5 = ContourPlot[ulast[x1, x2], {x1, 0, a}, {x2, 0, b}, AspectRatio → Automatic, ContourStyle → Thickness[0.008],
  ContourLabels → All, Contours → {0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07}, BaseStyle → {12, FontFamily → "Euclid"},
  ContourShading → None, PlotPoints → 200]
```




```
Show[Gr0, Gru5]
```



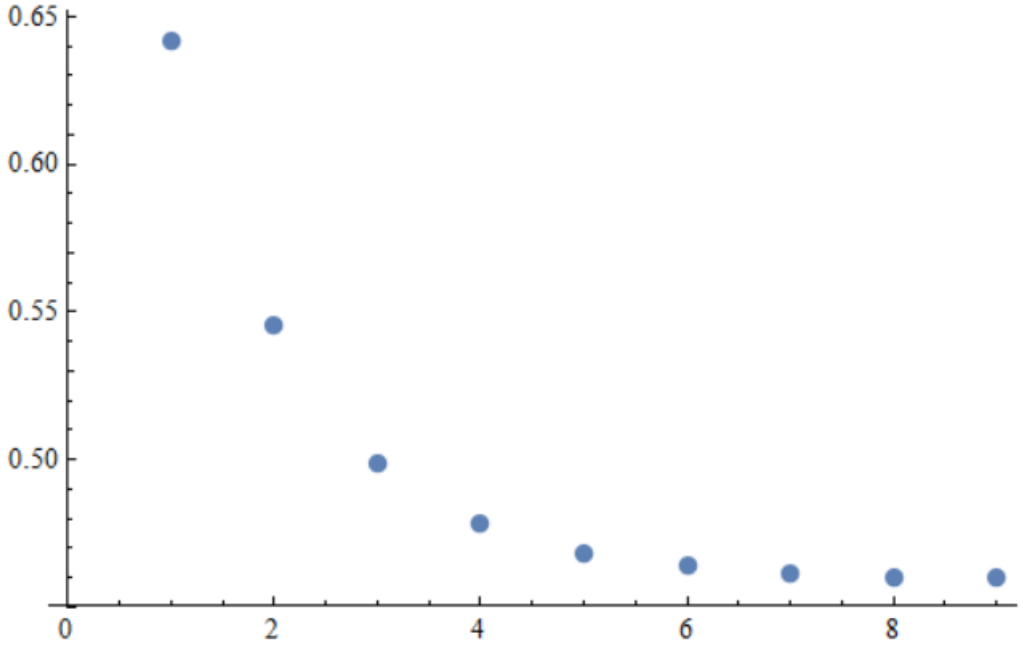

```
eps
```

```
{0.0470609, 0.0302186, 0.0164904, 0.00822583, 0.00393248, 0.00184239, 0.000854643, 0.000394467, 0.000181601, 0.0000834922}
```

```
qqq = Table[eps[[j + 1]] / eps[[j]], {j, 1, k}]
```

```
{0.642118, 0.545702, 0.498827, 0.478065, 0.468505, 0.463878, 0.461558, 0.460371, 0.459756}
```

```
ListPlot[qqq, PlotRange → All]
```




```
Median[qqq]
```

```
0.468505
```

```
Mean[qqq]
```

```
0.497642
```

```
t2 = AbsoluteTime[];
```

```
(t2 - t1) / 60
```

```
82.402771183
```

```
Beep[]
```

```
a1 = ulast[a / 2, b / 2]
```

```
0.0742172
```

```
b1 = 0.07584
```

```
0.07584
```

$$\frac{\text{Abs}[a1 - b1]}{b1} * 100$$

```
2.13971
```

ВІДОМІСТЬ АТЕСТАЦІЙНОЇ РОБОТИ

Позначення	Найменування	Дод. відомості
	Текстові документи	
1	Пояснювальна записка	64 с.
2	Презентаційний матеріал	25 с.
	Інші документи	
3	Роздруківки програм	12 с.
4	Рецензія	2 с.
5	Відгук керівника	1 с.

					Застосування методу двобічних наближень на основі використання квазіфункції Гріна-Рвачова до розв'язання еліптичних крайових задач з ізотонними нелінійностями			
Змін	**Арк.**	**Номер докум.**	**Підп.**	**Дата**				
Розроб.		**Юхименко В.Є.**			**(Тема роботи) Відомість атестаційної роботи**		**Аркуш**	**Аркушів**
Перевір.		**Тевяшев А.Д.**						
Н. контр.		**Сидоров М.В.**				**ХНУРЕ Кафедра ПМ**		
Затв.		**Тевяшев А.Д.**						