A Self-Consistent Potential Formalism in the Electrodynamics

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Abstract: *An attempt is made to complete logically the potential formalism in the electromagnetic theory basing on treatment of the Minkowski space-time as an electromagnetic oscillating system with distributed parameters. The Lagrange function and the energy-momentum spatial densities for the system are written using no the electromagnetic field tensor. Some physical consequences of the offered mathematical tool implementation are considered.*

Keywords: electromagnetic oscillating system; electromagnetic potential; Lagrange function; electromagnetic energy and momentum densities.

Introduction

Advance in the electromagnetic theory may be caused by development of new computational methods in electrodynamics of UWB electromagnetic pulses in passive and active microwave devices. One of lines of the development is so-called matrix electrodynamics based on decomposition of the electromagnetic potentials in new spatially localized basis functions – partial functions (oscillets) [1]. The spatial localization of the oscillets makes those very suitable for simulation of short electromagnetic pulses.

However, a matrix theory of electromagnetic oscillating systems [2] is not completed yet, because one does not describe energetic characteristics of electromagnetic phenomena. A cause of the problem is incompleteness of contemporary electromagnetic theory stating electromagnetic energy and momentum densities only in the terms of the field formalism (electric field strength $\vec{E}$ and magnetic induction $\vec{B}$). On the contrary, founded on the second-order (D'Alembert) operator the matrix electrodynamics is formulated in the terms of the potential formalism (scalar-vector $\Phi - \vec{A}$ or four-vector $\vec{A}^{\mathrm{f}}$, in the Lorentz gauge).

Thus, the completion of the matrix electrodynamics depends on the creation of a self-consistent potential formalism, which does not use the field terms. One of ways is treatment of the electromagnetic energy and the energy flux (momentum) as energy and momentum of a distributed electromagnetic oscillating system, not as properties of the electric and the magnetic fields. The Minkowski 4D space-time [3] might be such system. However, the Lagrange function and the energy-momentum densities for this oscillating system must be formulated in some different manner than in the field formalism [3]. Namely, these items cannot contain the electromagnetic field tensor.

Lagrange Function of an Electromagnetic System

The Lagrange function $\Lambda(t)$ for a distributed electromagnetic oscillating system can be constructed by analogy with a mechanical oscillating system. For N charged particles with rest masses m_{0n} and charges q_n moving in the four-vector potential $\vec{A}^{\mathrm{f}}(t,x,y,z) = \{A_t, A_x, A_y, A_z\}$ this function consists from three components: "mechanical" part $\Lambda^P(t)$ for the particles; "interaction" part $\Lambda^I(t)$ between the particles and the potential; "own" Lagrange function $\Lambda^S(t)$ for the oscillating system:

$$\Lambda(t) \equiv \Lambda^P(t) + \Lambda^I(t) + \Lambda^S(t) = -\frac{1}{\varepsilon_0\mu_0}\sum_{n=1}^{N} m_{0n} \times$$
$$\times\sqrt{1-(dx_n/dt)^2-(dy_n/dt)^2-(dz_n/dt)^2} -$$
$$-\frac{1}{\sqrt{\varepsilon_0\mu_0}}\sum_{n=1}^{N} q_n \times$$
$$\times\left[A_{nt}-(dx_n/dt)A_{nx}-(dy_n/dt)A_{ny}-(dz_n/dt)A_{nz}\right]-$$
$$-\frac{1}{2\mu_0}\int_V dxdydz \times$$
$$\times\left[(\partial\vec{A}^{\mathrm{f}}/\partial t)^2-(\partial\vec{A}^{\mathrm{f}}/\partial x)^2-(\partial\vec{A}^{\mathrm{f}}/\partial y)^2-(\partial\vec{A}^{\mathrm{f}}/\partial z)^2\right],$$

where $\vec{A}_n^{\mathrm{f}}(t) \equiv \vec{A}^{\mathrm{f}}[t, x_n(t), y_n(t), z_n(t)]$ is the potential for n-th particle location $\{x_n, y_n, z_n\}$; V is the system volume; ε_0 is the electric constant; μ_0 is the magnetic constant; t is the time coordinate measured in the length units, which are defined as product of the time by the light velocity in the vacuum. Note that the scalar product of four-vectors $\vec{a}^{\mathrm{f}}$ and $\vec{b}^{\mathrm{f}}$ is defined as $\vec{a}^{\mathrm{f}} \cdot \vec{b}^{\mathrm{f}} = a_t b_t - a_x b_x - a_y b_y - a_z b_z$.

Determining a four-vector of the current density as

$$\vec{j}^{\mathrm{f}}(t,x,y,z) = \frac{1}{\sqrt{\varepsilon_0\mu_0}}\sum_{n=1}^{N}\rho_n\{1, dx_n/dt, dy_n/dt, dz_n/dt\},$$

where $\rho_n(t,x,y,z)$ is n-th particle charge density in the fixed system of coordinates, the spatial density of the Lagrange function "electromagnetic" part can be written as

$$\lambda(t,x,y,z) \equiv \lambda^I + \lambda^S = -\vec{j}^{\mathrm{f}} \cdot \vec{A}^{\mathrm{f}} - \frac{1}{2\mu_0}\times$$
$$\times\left[(\partial\vec{A}^{\mathrm{f}}/\partial t)^2-(\partial\vec{A}^{\mathrm{f}}/\partial x)^2-(\partial\vec{A}^{\mathrm{f}}/\partial y)^2-(\partial\vec{A}^{\mathrm{f}}/\partial z)^2\right].$$

In spite of the negative sign in $(\partial\vec{A}^{\mathrm{f}}/\partial t)^2$ term, the Lagrange function does not increase infinitely while the frequency grows (due to the Lorentz gauge of the potential four-vector).

Electromagnetic Energy and Momentum Densities

Expressions for the electromagnetic energy and momentum spatial densities in the self-consistent potential formalism can be derived from the density of the "electromagnetic" part of the Lagrange function in the usual manner [3]. The energy density is

$$w(t,x,y,z)=\frac{1}{\sqrt{\varepsilon_0\mu_0}}\rho A_t-\frac{1}{2\mu_0}\times$$
$$\times\left[(\partial\vec{A}^{\mathrm{f}}/\partial t)^2+(\partial\vec{A}^{\mathrm{f}}/\partial x)^2+(\partial\vec{A}^{\mathrm{f}}/\partial y)^2+(\partial\vec{A}^{\mathrm{f}}/\partial z)^2\right],$$

where $\rho(t,x,y,z)$ is the temporal component of $\vec{j}^{\mathrm{f}}$. A ξ-th component of the momentum density is

$$p_\xi(t,x,y,z)=\rho A_\xi+\sqrt{\frac{\varepsilon_0}{\mu_0}}(\partial\vec{A}^{\mathrm{f}}/\partial t)\cdot(\partial\vec{A}^{\mathrm{f}}/\partial\xi),$$

where ξ means any spatial coordinate (*x*, *y* or *z*).

The expressions for p_ξ (known as the Umov vector components) are equivalents of the Poynting vector components in the self-consistent potential formalism. The electromagnetic momentum is the energy flux of a distributed oscillating system. Possible carriers of this may be both free vibrations of the system (i.e., electromagnetic waves) and moving spatial gradient of the potential around relocating charged particles together with the flux of the interaction energy between the particles and the system.

Some Physical Consequences

The above definition of the Lagrange function for an electromagnetic oscillating system results in slightly different treatment of some physical objects and phenomena comparing with the field formalism. Those are:

1). In the field formalism, energy of a closed system formed by charged particles and electromagnetic fields consists from positive "mechanical" energy of the particles (including their rest energy) and positive energies of the electric and the magnetic fields. In the potential formalism, energy of the similar system consists from the same positive "mechanical" energy of the particles, positive or negative interaction energy between the charged particles and A_t component of the potential, negative energy of varying in the space-time A_t component and positive energy of varying in the space-time A_x, A_y, A_z components.

2). In the field formalism, oscillating solutions of the wave equation for A_t and divergent parts of A_x, A_y, A_z components do not exist in the free space. In the potential formalism, such solutions exist (so-called zero magnetic *ZM* or potential *P* waves [4]). Wave of A_t component carries negative energy and momentum; waves of A_x, A_y, A_z components carry positive ones. However, due to the current continuity law, these solutions do not exert influence on charged particles at far distance from their sources (at least, in the classical electrodynamics).

3). In the field formalism, the electromagnetic field quanta (photons) are considered as ordinary particles. In the potential formalism, the energy of a distributed oscillating system is quantized instead. Thus, photons are quasi-particles here, like the phonons in a crystal. The full analogy is achieved, if the oscillating system is treated as a lattice of the partial oscillators [2].

Negative densities of the energy and the momentum produced by gradient of A_t component in the space-time may be explained from the viewpoint of the quantum electrodynamics [5], as reducing of the zero-point energies of the electromagnetic quantum oscillators around a charged particle. This may be considered as appearance of "negative" energy of the physical vacuum.

Conclusion

The offered variant of a self-consistent potential formalism does not conflict with the classical electrodynamics, as it seems. However, the ultimate conclusion can be made only after examination of various electromagnetic systems behavior basing on the above assumptions. A practical value of the self-consistent potential formalism consists in simplification and clarification of the electromagnetic theory and prospective completion of the matrix electrodynamics.

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