Excitation of Plasmon Resonances on Nanowire and Nanoshell by a Complex Source Point

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***Abstract*—Transient pulsed excitation of the localized surface plasmons on nanowire and nanoshell is visualized and analyzed. The complex source point concept is used to simulate an incident transient beam. Rigorous mathematical method based on the Laplace transformation is applied. Time domain field representation is obtained through the evaluation of the residues at singular points associated with the eigenvalues of the structure and integrals along the branch-cuts on the complex plane.**

***Keywords*—localised surface plasmons; plasmon resonances; complex source point.**

I. Introduction

Recently, localized surface plasmons (SPs) have attracted great amount of attention due to their potential use for the subwavelength field enhancement and localization that are explored in single molecule detection, transmission through a subwavelength aperture, subwavelength imaging, and improvement in the performance of conventional photonics components such as modulators and switches and others [1-4]. It is known that SPs can exist on a metal wire that can be considered as a plasma cylinder in the optical region.

In this paper we consider the excitation of SPs on a nanowire and a nanoshell by a transient pulsed beam. The external beam is modelled by a pulsed source point [5-8] with complex coordinates. To find the excited fields we use a rigorous mathematical tool that allows analysing problems both in the frequency and in the time domains. By applying the Laplace transformation directly to a wave equation we derive an analytical solution in the frequency domain; the time dynamics of the electromagnetic field is recovered by the inverse Laplace transformation. In this way we evaluate the residues at singular points associated with the eigenvalues of the structure and the integral along the branch-cuts in the complex plane. This approach guarantees the calculation with controllable accuracy and allows us to extract and to interpret physical phenomena. This method, introduced by C. Baum (singularity expansion method) in the 1970-s, has been successfully used in variety of ultra-wide-band antenna and target identification problems [9,10], and has been successfully applied by the authors to a variety of 1D, 2D and 3D time domain problems with nondispersive media [11-13].

II. Mathematical background: formulation and solution

In this paper, we consider 2D problem of the excitation of localized SPs on a nanowire and a nanoshell by a transient external beam. The complex source point concept is used to simulate an incident transient beam. The model is based on the idea of analytic continuation of the functions of real point source into the complex space [12]. H-polarized fields will be considered.

At first, assume that the source is at a point with the real coordinates represented by a radius vector $\vec{\rho}_s$. Using the expression for the 2D Green's function in the time domain,

$$g(t,t',\vec{\rho},\vec{\rho}_s)=\frac{1}{2\pi}\frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}_s|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}_s|^2/c^2}}, \qquad (1)$$

where $\theta(..)$ is the unit Heaviside function, we can write the expression for the magnetic field in free space,

$$h_0(t,\vec{\rho})=\frac{1}{2\pi}\int_0^\infty dt'\int_0^\infty \rho' d\rho'\int_0^{2\pi} d\varphi' \frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}'|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}'|^2/c^2}}\varepsilon_0\frac{\partial}{\partial t'}\hat{j}(t',\vec{\rho}') \qquad (2)$$

If the source is located in the real point of the space, $\hat{j}(t,\vec{\rho})=j(t)\delta(\vec{\rho}-\vec{\rho}_s)/|\vec{\rho}-\vec{\rho}_s|$, where $\delta(..)$ is the Dirac delta function, then

$$h_0(t,\vec{\rho})=\frac{\varepsilon_0}{2\pi}\int_0^\infty dt'\frac{\theta(t-t'-|\vec{\rho}-\vec{\rho}_s|/c)}{\sqrt{(t-t')^2-|\vec{\rho}-\vec{\rho}_s|^2/c^2}}\frac{\partial}{\partial t'}j(t'). \qquad (3)$$

Laplace transform of the equation (3) has the form

$$H_0(p,\vec{\rho})=\frac{\varepsilon_0}{2\pi}K_0\left(\frac{p}{c}|\vec{\rho}-\vec{\rho}_s|\right)pJ(p), \qquad (4)$$

where $J(p)$ is image of the function $j(t)$, $H_0(p,\vec{\rho})$ is the image of the function $h_0(t,\vec{\rho})$, and $K_0(..)$ is the modified Bessel function of the second kind. Let us further assume that the vector $\vec{\rho}_s$ is complex $\vec{\rho}_s=\vec{\rho}_{cs}$, where $\vec{\rho}_{cs}$ is given by

$$x_{cs}=x_0+ib\cos\beta,\ y_{cs}=y_0+ib\sin\beta, \qquad (5)$$

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and x_0, y_0, b, β are real numbers.

In this case, the distance between the point source and the observation point is complex as well, $\left|\vec{\rho}-\vec{\rho}_{cs}\right|=\sqrt{(x-x_{cs})^2+(y-y_{cs})^2}$.

Let us consider the transient dynamics of localized SPs excited by the pulsed complex source point on the nanowire (Fig. 1 (a)).

Dispersive medium is described by the Drude model,

$$\varepsilon=1-\frac{\omega_{pe}^2}{\omega(\omega-i\gamma_e)}, \qquad (6)$$

where ω_{pe} is the plasma frequency and γ_e is the material absorption. The susceptibility of plasma in the frequency domain is of the form

$$\chi(\omega)=-\frac{\omega_{pe}^2}{\omega(\omega-i\gamma_e)} \qquad (7)$$

The susceptibility of the medium in the time domain can be found using the inverse Fourier transform

$$\chi(t)=-\frac{\omega_{pe}^2}{\gamma_e}(1-e^{-\gamma_e t})\Theta(t) \qquad (8)$$

In Fourier-transform domain, we look for the solution to the problem associated with a nanowire in the following form:

$$H=\sum_{m=-\infty}^{\infty}A_m I_m\left(\bar{n}_p q\rho\right)e^{im(\varphi-\varphi_{cs})},\ \rho<a, \qquad (9)$$

$$H=\sum_{m=-\infty}^{\infty}B_m K_m(q\rho)e^{im(\varphi-\varphi_{cs})},\ \rho>a,$$

where $I_k(..)$, $K_k(..)$ are the modified Bessel functions of the first and second kind, respectively. Representation of the external field in the form of functions $K_k(..)$ guarantees the agreement with the radiation conditions at infinity.

Applying the boundary conditions, which represent continuity of the tangential field components at the circular boundary $\rho=a$, we come up to the system (provided that the source is located outside, $\rho_0>a$) to determine the unknown coefficients A_m and B_m,

$$A_m I_m(\bar{n}_p qa)-B_m K_m(qa)=I_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \qquad (10)$$

$$A_m I'_m(\bar{n}_p qa)-\bar{n}_p B_m K'_m(qa)=\bar{n}_p I'_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \qquad (11)$$

For the nanoshell (Fig. 1 (b)) the solution can be found in the similar form,

- if $\rho<b$, $H^{(1)}=\sum_{m=-\infty}^{\infty}A_m^{(1)}I_m\left(q\rho\right)e^{im(\varphi-\varphi_{cs})}$, (12)
- if $b<\rho<a$,

$$H^{(2)}=\sum_{m=-\infty}^{\infty}\left(A_m^{(2)}I_m(n_p q\rho)+B_m^{(2)}K_m(n_p q\rho)\right)e^{im(\varphi-\varphi_{cs})}, \qquad (13)$$

- if $\rho>a$, $H^{(3)}=\sum_{m=-\infty}^{\infty}B_m^{(3)}K_m(q\rho)e^{im(\varphi-\varphi_{cs})}$. (14)

Similarly, after the application of the boundary conditions, we determine the unknown coefficients from the solution to the system

$$A_m^{(1)}I_k(qb)-A_m^{(2)}I_m(n_p qb)-B_m^{(2)}K_m(n_p qb)=0 \qquad (15)$$

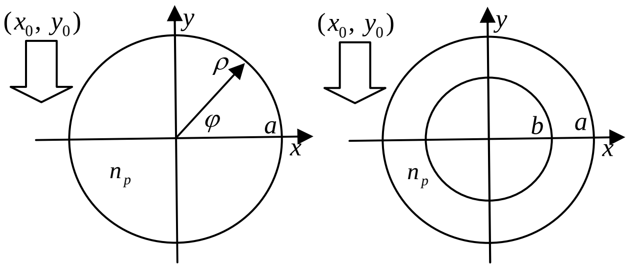
$$n_p A_m^{(1)}I'_k(qb)-A_m^{(2)}I'_m(n_p qb)-B_m^{(2)}K'_m(n_p qb)=0 \qquad (16)$$

$$A_m^{(2)}I_m(n_p qa)+B_m^{(2)}K_m(n_p qa)-B_m^{(3)}K_m(qa)= \\ =I_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \qquad (17)$$

$$A_m^{(2)}I'_m(n_p qa)+B_m^{(2)}K'_m(n_p qa)-n_p B_m^{(3)}K'_m(qa)= \\ =n_p I'_m(qa)K_m(q\rho_{cs})pJ(p)\frac{\varepsilon_0}{2\pi} \qquad (18)$$

The coefficients of expansions have the poles corresponding to the plasmon modes and also the branch point. The eigenfrequencies of the plasmon modes are complex, $\omega=\omega'+i\omega''$. The dependence on the time of the transient pulse source is

$$j(t)=e^{i\omega_0 t}\left[\Theta(t)-\Theta(t-\tau)\right] \qquad (19)$$



(a) (b)

Figure 1. Schematic diagram of the investigated phenomenon.

III. Numerical results and disscusion

In this paper, we consider normalized plasma frequency $w_p=\omega_p a/c$, $w_p=1$, $\gamma=10^{-3}\cdot w_p$, $b/a=0.5$. Figure 2 represents the dependences of the scattering cross sections (SCS) of nanowire (dashed line) and nanoshell (solid line) on the normalized frequency *ka*. In the spectrum of the nanowire SCS, one can see the resonances on the dipole ($\mathrm{Re}(ka)=0.63$) and the quadrupole ($\mathrm{Re}(ka)=0.675$) plasmons. In the spectrum of the nanoshell SCS, two types of plasmon resonances are present: even and odd. The odd plasmon resonances are shifted to lower frequencies ($\mathrm{Re}(ka)=0.48$ and $\mathrm{Re}(ka)=0.6$) and the even plasmon resonances are shifted to the region of higher frequencies ($\mathrm{Re}(ka)=0.77$ and $\mathrm{Re}(ka)=0.83$). Figure 3 shows the spectral density of the field in the nanoshell at its excitation by a complex source point beam. We use the normalized frequency of the source $w_0=\omega_0 a/c$. The solid line corresponds to the case when the frequency of the source coincides with the real part of the frequency of even dipolar plasmon and the dashed line corresponds to an odd dipole plasmon. The duration of the pulse is $\tau = 2\pi a/c$. In this case, unlike the case of the incident harmonic wave (see Fig. 2), in the spectrum multiple peaks that are associated with higher plasmons are present.

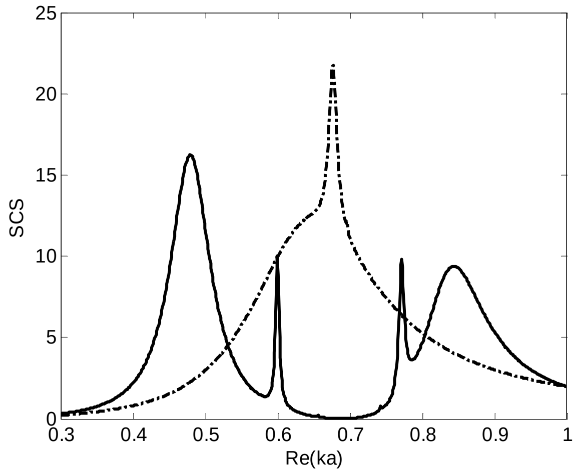


Figure 2. SCS of the nanowire (dashed line) and nanoshell (solid line) as a function of the normalized frequency.

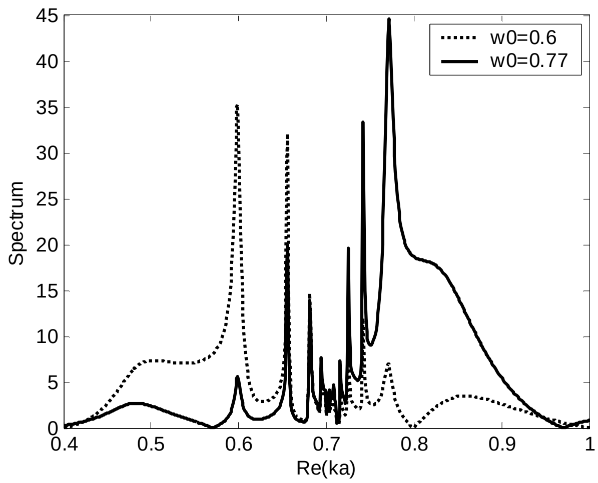


Figure 3. Spectral density of the field in nanoshell ($w_p = 1$, $\tau = 2\pi a/c$) as a function of the normalized frequency.

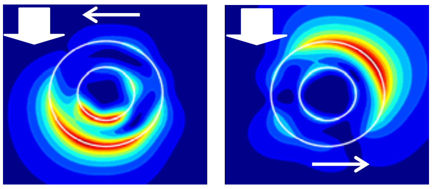


Figure 4. Snapshots of the dynamic SP propagation along the nanoshell. From left to right: $T = 100\pi$, $T = 140\pi$ ($T = tc/a$ is the normalized time, where t is real time, a is radius of wire, c is velocity of light in vacuum).

Figure 4 represents snapshots of the absolute value of the magnetic field of the transient SPP travelling around the metal shell excited by the external transient beam with the eigenfrequency $\omega_0 = 0.83$.

We see that the beating of the simultaneously excited SPs gives rise to asymmetric running field pattern. The field on the surface of the nanoshell is more complicated because in this case excited are both even and odd plasmons with different quality factors and characteristic fields.

Conclusions

In this paper we built an analytic solution, in the form of the Laplace transform, of the problem of transient plasmon excitation by a localized directive pulse source. For the simulation of transient external beam we used the concept of complex source point beam. The model is based on the idea of analytic continuation of the functions of real source point into the complex space. This kind of source is a very useful model for describing the excitation of non-stationary waves. It was shown that the beating of the simultaneously excited SPs gives rise to asymmetric running field pattern.

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