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Resonant Degenerate Crystal Made of Spheres Located in Magnetodielectric Medium

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Abstract The paper addresses the computational method we proposed to analyze the phenomena which occur when electromagnetic waves are scattered by a finite resonant crystal with a cubic lattice having small resonant magnetodielectric spheres in its nodes. The method is based on the second kind Fredholm integral equations. Properties of the degenerate resonant crystal are investigated when it is located in magnetodielecrtic media with either positive or negative values of permittivity ε_0 and permeability μ_0. It is shown that media with positive and negative values of permittivity and permeability differently influence on scattering properties of the degenerate resonant crystal. When the permittivity and permeability jump, the effect of its degeneration cancellation is found such that the lattice resonance and the internal crystal sphere resonance are no more coincident, and the resonance curves split. Graphic dependences given in the paper numerically define a magnitude of the splitting. The influence of the presence of a cavity in the resonant crystal on a structure of the scattered field in the Fresnel zone is graphically estimated.

Keywords Magnetodielectric, Integral Equation, Crystal

1. Introduction

Our attention was attracted by works[1-6] wherein unusual properties of substances with simultaneously negative values of permittivity ε and permeability μ were considered. It is of interest to study scattering properties of a finite discrete resonant crystal made of magnetodielectric spheres, if the crystal is located in external media with simultaneously either positive or negative permittivity ε and permeability μ. Investigated in this paper are the crystal properties when the degenerate resonance (st+m+e) is excited in it, which occurs when the structural (lattice) crystal resonance (st) coincides with the internal magnetic (m) and electric (e) resonances of spheres (also coincided). We study the case being equivalent to that of roentgen crystals optics, with $a/\lambda << 1$; a/λ_g ~1; $d,h,l/\lambda$ ~1 where a is the sphere radius; λ, λ_g are the lengths of the waves scattered outside and inside the spheres; d,h,l are the orthogonal lattice constants. We have investigated the multimode structure (d, h, l~ $2\lambda_r^{m+e}$) of the resonant (r) field inside and outside the cubic lattice in the Fresnel and Fraunhofer zones for external media with the permittivity and permeability $\varepsilon_0 = \mu_0 = \pm 1$.

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An effect of the resonant crystal degeneracy cancellation is considered. The solution is obtained on the basis of the second type Fredholm integral electromagnetic equations[7, 8,9].

2. Theoretical Details

Let us find the field scattered over the known internal field of scatterers via the electric $\vec{\Pi}^e(\vec{r},t)$ and magnetic $\vec{\Pi}^m(\vec{r},t)$ Hertz potentials of the spatial lattice

$$\begin{aligned}\vec{E}_{scatt} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^e(\vec{r},t) - ik\mu_0\left[\nabla,\vec{\Pi}^m(\vec{r},t)\right],\\ \vec{H}_{scatt} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^m(\vec{r},t) + ik\varepsilon_0\left[\nabla,\vec{\Pi}^e(\vec{r},t)\right].\end{aligned} \quad (1)$$

The lattice Hertz potentials are presented as a superposition of those for separate lattice spheres. The electric Hertz potential of the lattice looks like

$$\begin{aligned}\vec{\Pi}^e(\vec{r},t) &= \sum_{c=1}^{N}\frac{1}{k_1^3}\left(\sin k_1a_c - k_1a_c\cos k_1a_c\right)\mathrm{x}\\ &\mathrm{x}\left(\frac{\varepsilon_{cef}}{\varepsilon_0}-1\right)\vec{E}^0_{c(p,s,t)}(\vec{r}\,',t)\frac{e^{-ik_1r_c}}{r_c}.\end{aligned} \quad (2)$$

The expression (2) describes the scattered field at an arbitrary distance r_c from the sphere centres to observation points outside the spheres. Here $\vec{E}^0_c(\vec{r}\,',t)$ is the induced internal field of the interacting spheres. The field can be found via the algebraic system of inhomogeneous equations[3], N is the number of spheres. The field scattered by the system of spheres $\vec{E}_{scatt}(\vec{r},t)$ is found via (1), (2) as

$$\vec{E}_{scatt}(\vec{r},t)=\sum_{c=1}^{N}\frac{1}{k_1^3}(\sin k_1 a-k_1 a\cos k_1 a)\text{x}$$
$$\text{x}\left\{\left(\frac{\varepsilon_{cef}}{\varepsilon_0}-1\right)\widehat{L}_c\vec{E}_c^0(\vec{r}\,')-ik\mu_0\left(\frac{\mu_{cef}}{\mu_0}-1\right)\text{x}\right. \quad (3)$$
$$\left.\text{x}\,\widehat{P}_c\vec{H}_c^0(\vec{r}\,')\right\}e^{i(\omega t-k_1 r_c)},$$

where $\widehat{L}_c$ and $\widehat{P}_c$ are the functional matrices

$$\hat{L}_c=\begin{bmatrix}\Psi_{xxc} & \Psi_{xyc} & \Psi_{xzc}\\ \Psi_{yxc} & \Psi_{yyc} & \Psi_{yzc}\\ \Psi_{zxc} & \Psi_{zyc} & \Psi_{zzc}\end{bmatrix};\ \hat{P}_c=\begin{bmatrix}0 & \Psi_{zc} & \Psi_{yc}^0\\ \Psi_{zc}^0 & 0 & \Psi_{xc}\\ \Psi_{yc} & \Psi_{xc}^0 & 0\end{bmatrix}$$

The element Ψ_{xxc} of the matrix $\widehat{L}_c$ is

$$\Psi_{xxc}=\frac{1}{r_c}k_1^2+\frac{3(x-x_{c0})^2-r_c^2}{r_c^5}-k_1^2\frac{(x-x_{c0})^2}{r_c^3}+$$
$$+ik_1\frac{3(x-x_{c0})^2-r_c^2}{r_c^4},$$

where (x,y,z) are the coordinates of the observation point; (x_{c0},y_{c0},z_{c0}) are the coordinates of the spheres' centres.

The expression (3) describes the scattered field that consists of propagating and damped spatial harmonics inside and outside the lattice in the Fresnel and Fraunhofer zones.

The entire field at an arbitrary medium point outside the spheres is defined as

$$\vec{E}(\vec{r},t)=\vec{E}_0(\vec{r},t)+\vec{E}_{scatt}(\vec{r},t), \quad (4)$$

where $\vec{E}_0(\vec{r},t)$ is the undisturbed field of the scattered wave.

The expressions (3) and (4) are numerically analysed for the degenerate resonant cubic crystal. The analysis results are given in Figs. 1, 2, 3. The number of spheres here is N = 64000; their radii are a = 0.5 cm; their permittivity and permeability are $\varepsilon=\mu=$ 9.75; the lattice constants are $d=h=l=2\lambda_r^{m+e}=$ 14.82 cm (Fig. 1 *a*) and

$d=h=l=2\lambda_r^{m+e}=$ 12.52 cm (Fig. 1 *b*).

Figs. 1 *a* and 1 *b* show the fields modules, (4) and (3), versus scattered wavelength λ for the degenerate resonant crystals in the medium with $\varepsilon_0=\mu_0=+1$ (curve 1, Fig. 1 *a*) and in the medium with $\varepsilon_0=\mu_0=-1$ (curve 1, Fig. 1 *b*), respectively. Considered for the same crystals are also the same dependences for the cases when the medium parameters vary and become $\varepsilon_0=\mu_0=-1$ (curve 2 in Fig. 1 *a*) and $\varepsilon_0=\mu_0=+1$ (curve 2 in Fig. 1 *b*). Here resonances of lattices and spheres do not coincide.

Figs. 2 *a,b* and 3 *a,b* show field modules, (4) and (3) , versus changes in coordinates x, y, respectively, in the direction of scattered wave propagation (like in Fig. 1 *b*), inside (when x and y are in the vicinity of zero) and outside the degenerate crystal in media with $\varepsilon_0=\mu_0=+1$ (Fig. 2 *a,b*) and $\varepsilon_0=\mu_0=-1$ (Fig. 3 *a,b*).

Figs. 2 *c* and 3 *c* show the damped component of the multimode scattered field in the Fresnel zone in media with $\varepsilon_0=\mu_0=+1$ (Fig. 2 *c*), and $\varepsilon_0=\mu_0=-1$ (Fig. 3 *c*). Curve 1 is in both figures relates to crystals with internal cavities[10,11].

Figs. 2 *d* and 3 *d* show field modules, (4) and (3), versus *z*-coordinate for the crystal side area along the y-axis (like in Fig. 1 *b*) in the Fraunhofer zone in media with $\varepsilon_0=\mu_0=$ +1 (Fig. 2 *d*) and $\varepsilon_0=\mu_0=-1$ (Fig. 3 *d*).

It follows from the numerical analysis that the resonant degenerate cubic crystal in media with simultaneously either positive or negative permittivity and permeability $\varepsilon_0=\mu_0=\pm1$ has different electromagnetic properties. In a medium with $\varepsilon_0=\mu_0=-1$ the crystal has the pronounced reflecting properties. It can play a role of reflector and resonator with a strong internal field. By forming nodes and antinodes on crystal faces, one can control the scattering properties of the crystal. In a medium with $\varepsilon_0=\mu_0=+1$, a phenomenon of resonant propagation of the scattered wave is inherent in the crystal. This can be used when creating non-reflecting devices.

By varying the medium parameters ε_0 and μ_0 , the effect of degeneracy cancellation in the resonance degenerate crystal can be obtained, which entails violation of the crystal resonances (st+ m +e) superposition. As an example, this effect is shown in Fig.4 where one can see resonant curves splitting as a consequence of the effect shown in Fig.1 *b* , and occurrence of noncoincident resonances (st), (m), (e) (Fig. 4 *a*) and (e), (m) (Fig. 4 *b*). In the case of resonance curve 1 (st+ m +e) splitting in Fig. 4 *a* , parameters of the medium have the values: $\varepsilon_0=-1$; $\mu_0=-1.71$. After the degeneracy is cancelled, the structural resonance (st) of curve 1 (st+ m +e) (Fig. 4 *a*) shifts to the domain of the wavelength $\lambda_r^{st}=8.2$, and therefore is not presented in Fig. 4*a*, as it is beyond the figure limit.

In the case of curve 1 (m + e) splitting in Fig. 4 *b* , the medium parameters have the values: $\varepsilon_0=+1$; $\mu_0=+1.71$.

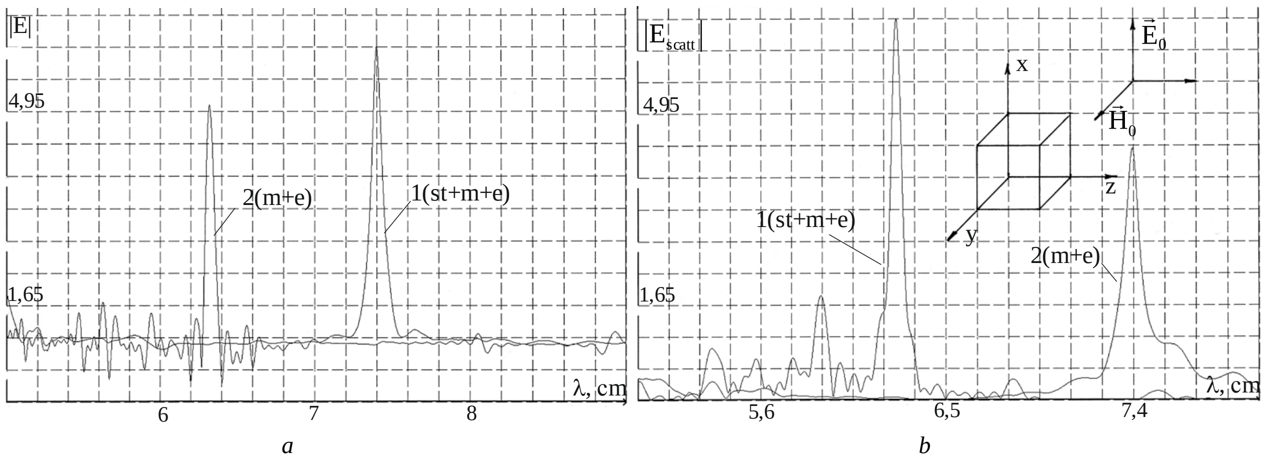

Figure 1. Dispersion dependences of entire (a) and scattered (b) fields in media with $\varepsilon_0 = \mu_0 = \pm 1$

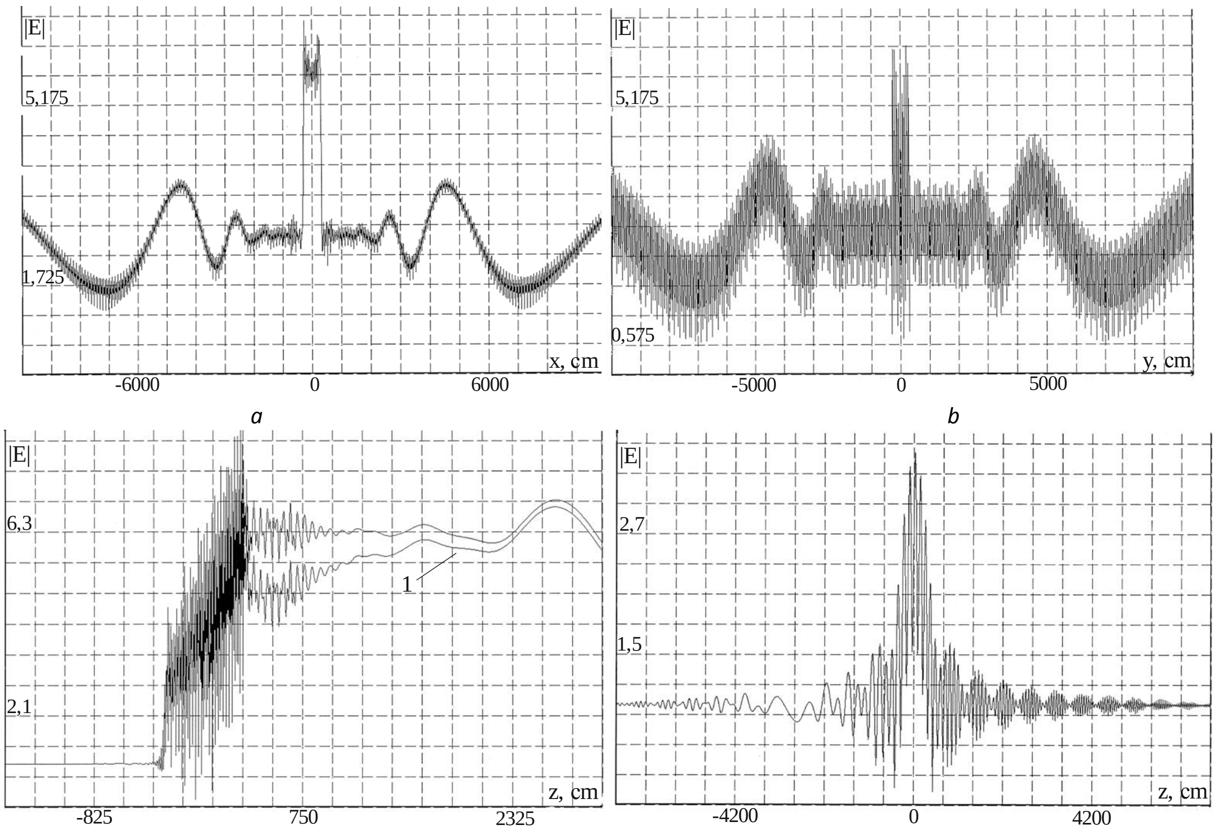

Figure 2. The entire field in the medium with $\varepsilon_0 = \mu_0 = +1$

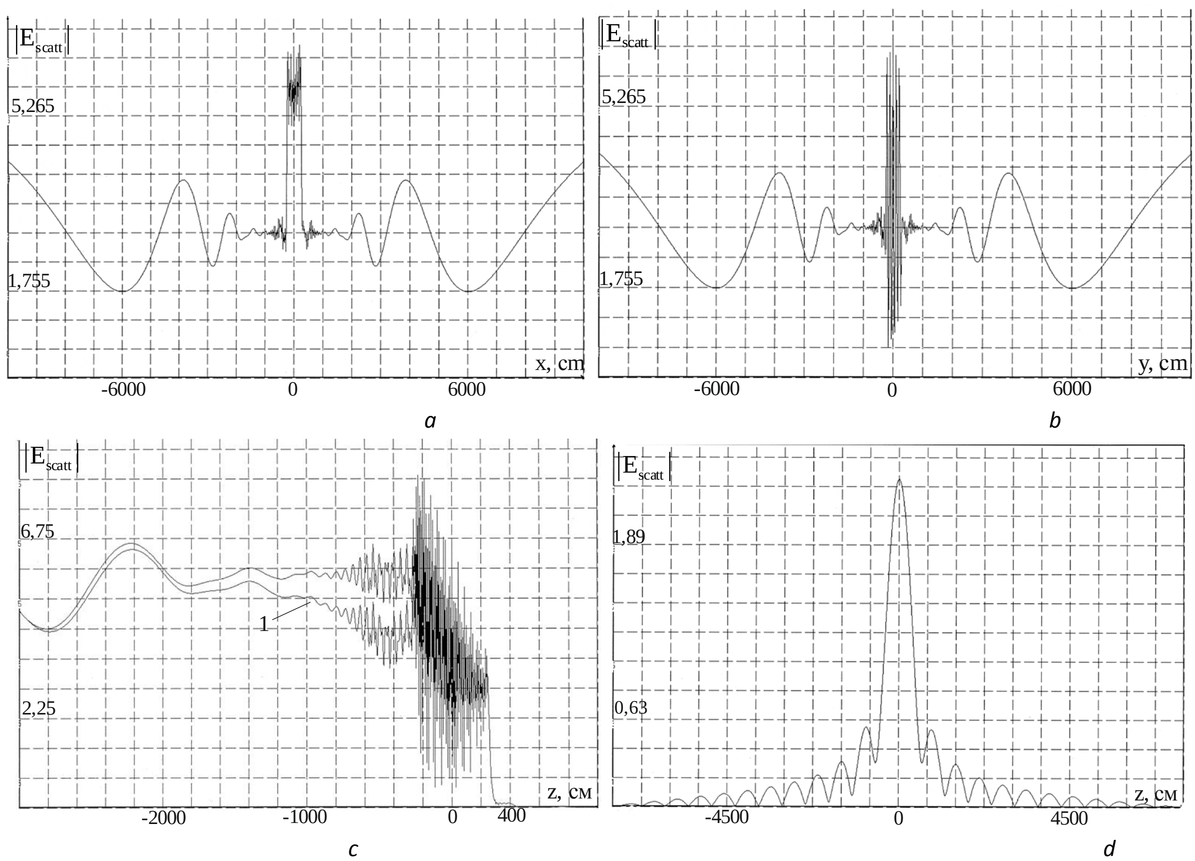

Figure 3. The field scattered in a medium with $\varepsilon_0 = \mu_0 = -1$.

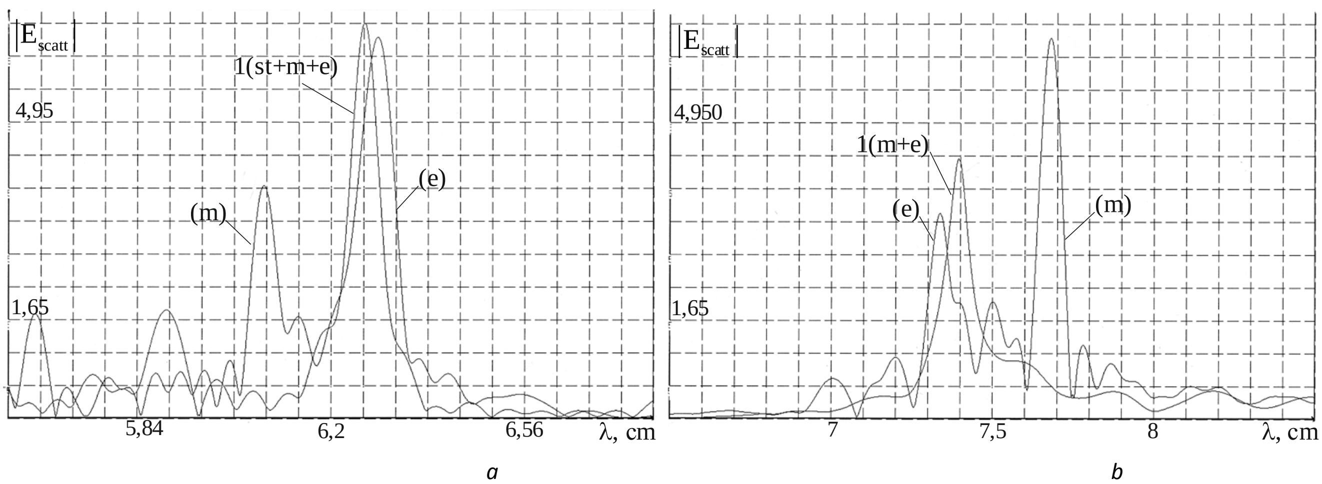

Figure 4. The influence of environment on cancellation of the resonant cubic crystal degeneracy

3. Conclusions

The resonant degenerate crystal located in a magnetodielectric medium with varying parameters ε_0 and μ_0 can be used for creation of resonant metastructures with abnormal electromagnetic properties.

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