



Секция 5. ИНФОРМАЦИОННЫЕ СИСТЕМЫ И ТЕХНОЛОГИИ В
ЭКОНОМИКЕ И ОБРАЗОВАНИИ

MATHEMATICAL MODELING AND NUMERICAL ANALYSIS OF
SUSTAINABLE DEVELOPMENT OF NONLINEAR DYNAMICAL SYSTEMS
WITH COMPETITIVE INTERACTIONS

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The Problem to analyze a sustainable development and competitive interaction of dynamic systems in technology, economy and society, has appeared recently in the beginning of this century. Model concept of "actor" can be either "technical subsystem" or "species" and "enterprise". Some characteristic classes of competition systems, such as pipe stations, the Chamber of Commerce and Economic Union of countries and Species competition may become object of such simulation.

For the systems with competitive interaction it was necessary to define conditions of stabilisation for their development and growth. There is the task in view for models, irrespective of the physical nature of objects: it is necessary to reveal parameters of stable growth and also conditions of random. Adequacy of such models to objects either is known earlier, or is shown by the system analysis of the general properties of objects.

Objects of research are the processes of a sustainable development of multi-connected dynamical competitive and solidary systems.

Subject of research is multy-dimensional nonlinear dynamic models of competitive interaction.

The general model of competition is represented in three more special cases:

1st – $2n$ -dimensional linear model;

2nd – nonlinear 2 and 3 dimensional models of interaction;

and 3d – models with exterior periodic effect.

Research methods: analytical and numerical methods of solution of linear and nonlinear systems of differential equations. Regardless of the physical nature of objects, the method was solving differential systems to find fixed structures in phase-space, such as fixed points of repellery and attractive type, limiting cycles, torus and "strange attractors".

The general problem put into research for sustainable development of dynamic systems is actual and is reduced to the following:

– within the limits of the general model investigate dynamics of mathematical models for competitive interactions in engineering systems to detect the conditions of steady growth and elimination the conditions of chaos.

– the purpose of numerically-analytical research for all the models is to determine the areas in multy-dimensional space of parameters which values ensure either:



a) steady increment for all or the majority of system's variables on a finite interval;

b) on the contrary, the conditions of quasi-random dynamics that give the chance to exclude it.

For the given Caushee problem Runge-Kutta method was used. Fuser, the results of computing the model of competitive interaction in the model of 2 actors are presented in different cases of relations. The conclusions about stability and development is made using Liapunov's exponents and projections of phase-portraits of the considered systems.

The application of this method to the systems, either Economic Union of countries or pipe-stations, are shown. The quasi-chaotic dynamics of the model of 3 actors, including the linear model of power plant was investigated.

For the considered pattern "producer" and "user" the general conclusion in this case is the statement about parametrical stability and structural instability of the base model.

The rigid model should be substituted by a soft model, for example, the logistic one, so that general conclusions remain fair at any values of factors and even for a wide class of models with various unimodal functions in the right part of equation.

Therefore, there is suggested the modified and expanded mathematical model describing competitive processes.

This modification of the Volterra model was called the model of relations between the producer and user (or moderator). It has been modified and refined by adding of one or more producers.

Then the base autonomous set of Volterra equations is perturbed by a small periodic oscillation of the rate in one or both variables of this system.

Exterior factors can cause a periodic modification of the rate for each of the actors. We notice that the system becomes nonautonomous and looks. The frequency of perturbation is taken close to the frequency of the cycle without perturbations. Bifurcation value of the parameter n obviously is zero.

The general conclusions and outcomes of the work.

On the basis of analysis of references and characteristic indications of the objects' behavior, three classes of objects and their dynamic models are chosen for the research.

For the first time stationary sets of torus type in linear multicomponent competitive models are revealed. Their research allows to avoid the occurrence of conditions for quasi-random dynamics which lead to catastrophes.

For the first time mathematical models for exposition of competitive processes in engineering systems and systems of service of municipal affaires are offered.

The further development is made for mathematical models of competing systems of several ($n > 2$) actors that include two sets of actors of different types at various parameters of competition.

The method of modelling of random dynamics is improved for close to resonance periodic exterior action. It allows to receive the obvious image of a random attractor, or its projections in the expanded phase space.