RESONANT PHENOMENA IN A PERIODIC ARRAY OF SLITS FILLED WITH A METAMATERIAL

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Abstract — The TM-wave diffraction problem by a periodic perfectly conducting grating is solved for the single-mode approximation of the slit field. The physical reason of the observed effects is discussed for the case when slits are filled with a conventional positive index magnetodielectric. The obtained solution allows analyzing the periodic arrays of slits filled with an arbitrary metamaterial.

РЕЗОНАНСНЫЕ ЯВЛЕНИЯ В ПЕРИОДИЧЕСКОЙ СИСТЕМЕ ЩЕЛЕЙ, ЗАПОЛНЕННЫХ МЕТАМАТЕРИАЛОМ

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Аннотация — Решена задача дифракции ТМ-волны на периодической проводящей решетке в одномодовом приближении поля в щелях. Рассмотрены причины наблюдаемых эффектов в случае заполнения щелей магнитодиэлектриком с положительным коэффициентом преломления. Полученное решение позволяет проводить анализ решеток, щели которых заполнены метаматериалом с произвольными материальными параметрами.

I. Introduction

Since 1998, when optical extraordinary transmission (ET) through subwavelength hole arrays was reported, this physical phenomenon has attracted a great amount of scientists' interest due to its promising applications. Initially, terms of Plasmonics [1] was employed for the explanation of the effect. According to this theory, the main physical reason of ET is that a real metal acting as a lossy dielectric supports field modes localized in the vicinity of the air-metal interfaces. These confined modes were called surface-plasmon-polaritons (SPPs).

However, later it was discovered that metallic gratings exhibit the similar effect of enhanced transmission at microwaves and terahertz waves [2]. Since at these spectrum regions real metals are successfully treated as perfect conductors, there is only negligible field penetration into a metal volume. Thus, dielectric properties of real metals do not reveal the nature of the effect and the conception of SPPs makes no physical sense at low frequencies. Therefore, recent papers are mainly focused on the rigorous analytical and numerical electrodynamic methods. Firstly, because of its simplicity, theoretical analyses considered the two-dimensional (2D) geometries: individual slits and slit arrays perforated in a metal screens [3]. Subsequently, researchers embarked on the study of the hole gratings [4]. Discovery of the good transmittance of quasiperiodic and random hole arrays as well as the significant field enhancement at the exit side of the individual holes have led to the discussion on the relative roles of the geometric shape of the individual opening and the grating periodicity toward the extraordinary transmission [5]. This fact reinvigorates interest in diffraction problems by simple 2D slit geometries.

So far it is known that a slit perforated in a metal screen acts as the Fabry-Perot cavity. Its fundamental mode, TEM-mode, experiences a periodic sequence of resonances with increasing of a slit depth. They emerge due to constructive interference between the forward and backward waves propagated in a slit. It should be noted that the slit fundamental mode has not a cut-off frequency; therefore it is propagating at all frequency ranges. It is also discovered that Fabry-Perot resonances redshifts from their expected positions [3, 5, 6].

The redshift of resonances leads to the fact that the resonant slit depth is slightly reduced. The aim of the presented paper is to show the frequency independence of the shortening of the resonant depth over a wide frequency range below the first Rayleigh minimum defined by the grating period. Moreover, the obtained single-mode solution subsequently allows analyzing reflection and transmission coefficients for the case when slits are filled with a metamaterial of dielectric and magnetic constants which can be both positive and negative values.

II. Main Part

Fig. 1 shows schematically the system under study. Slits of width $2d$ are perforated in a perfectly conducting screen of thickness h . A grating period is $2b$.

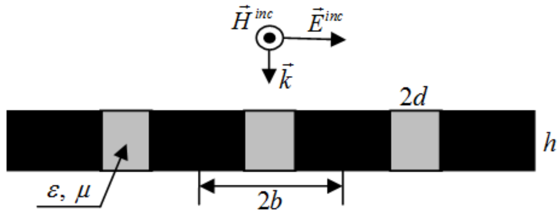

Fig. 1. A periodic grating of slits filled with a magneto-dielectric.

Рис. 1. Периодическая решетка, щели которой заполнены магнитодиэлектриком

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The system is illuminated by a normally incident TM-polarized plane wave with wavelength λ. The slits are filled with a medium with parameters ε and μ. Scattered fields inside and outside the slits are expanded in Fourier series. By matching the appropriate boundary conditions, the reflection and transmission coefficients for the zeroth diffraction order can be expressed as

$$R = -1 + \frac{2i\xi Z \sin\varphi(\cos\varphi - igZ\sin\varphi)}{1-(\cos\varphi - igZ\sin\varphi)^2}, \quad (1)$$

$$T = \frac{2i\xi Z \sin\varphi}{1-(\cos\varphi - igZ\sin\varphi)^2}, \quad (2)$$

where $\xi = d/b$ is the grating fill factor, $Z = \sqrt{\mu}/\sqrt{\varepsilon}$ is the wave resistance of the material which fills the slits, $\varphi = kh\sqrt{\varepsilon\mu}$ is the phase shift along the slit depth, $g = \xi(1-i\sigma)$ is the reactivity of entrance and exit sides of the slits, $\sigma = 2\sum_{m=1}^{\infty} \mathrm{sinc}^2 m\pi\xi \Big/ \sqrt{m^2(\lambda/2b)^2 - 1}$.

As an example, Fig. 2 shows the energy coefficients versus the slit depth h normalized by the wavelength λ_0. As can be seen, $|R|^2$ and $|T|^2$ satisfy the energy conservation law $|R|^2 + |T|^2 = 1$ with good accuracy. As expected, the dependence exhibits a periodic sequence of the Fabry-Perot resonances with increasing of the slit depth. It should be pointed out that the full transparency mode is realized at these resonances which are slightly redshifted due to the capacitive reactivity of slits.

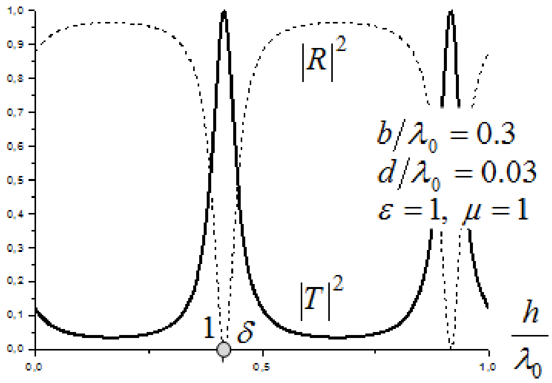

Fig. 2. Energy coefficients versus a slit depth.

Рис. 2. Зависимость энергетических коэффициентов от глубины щели

By using the resonance condition $\mathrm{Im}\,T = 0$, one can derive the analytical expression for the resonant depth:

$$h_n = \frac{n\lambda}{2\sqrt{\varepsilon\mu}} - \Delta = \frac{n\lambda}{2\sqrt{\varepsilon\mu}} - \frac{\lambda}{2\pi\sqrt{\varepsilon\mu}} \arctan\frac{2\alpha\sigma}{1+\alpha^2(1-\sigma^2)},$$

where n is the order of Fabry-Perot resonance, $\alpha = \xi Z$.

The dependence of the resonant slit depth h_1 upon the wavelength λ is depicted in Fig. 3. Point 1 in Fig. 3 corresponds to one in Fig. 2. Note that the absolute value of the resonant depth shortening $\Delta = \delta\lambda_0$ is practically independent of frequency at the wide range below the first Rayleigh minimum ($\lambda_1 = 2b$). We confirmed this fact by analyzing $\Delta(\lambda)$ at the low-frequency mode:

$$\Delta\big|_{\lambda\to\infty} = 2b\frac{1}{\pi\sqrt{\varepsilon\mu}}\frac{2\alpha}{1+\alpha^2}\sum_{m=1}^{\infty}\frac{1}{m}\mathrm{sinc}^2 m\pi\xi \neq f(\lambda).$$

This effect can be physically explained by taking into account the quasistatic nature of the grating near field.

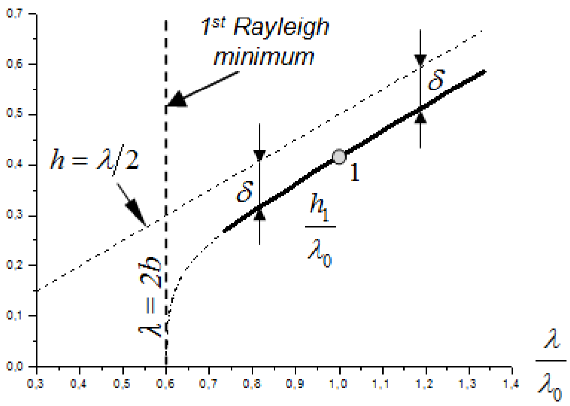

Fig. 3. The resonant slit depth versus wavelength.

Рис. 3. Зависимость резонансной глубины щели от длины волны

Analysis of the array of slits filled with a negative index metamaterial also shows the existence of the Fabry-Perot resonances. It is interesting that in this case the opposite resonances shift is observed.

III. Conclusion

The TM-wave diffraction problem by a periodic grating of slits filled with a magnetodielectric was solved analytically in the case of single-mode propagation in slits. The well-known features of resonant transmission through such structures (Fabry-Perot resonances and their redshift) are confirmed. This fact indicates good accuracy of the presented expressions for scattered field coefficients. The frequency independence of the shortening of the resonant slit depth was discovered at the wide frequency range below the first Rayleigh minimum. The obtained solution allows analyzing not only the systems of periodic slits filled with conventional magnetodielectrics but also gratings which contain a metamaterial with negative material parameters.

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